Forecasting Livestock Prices with an Artificial Neural Network versus Linear Time Series Models

by

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Introduction

Price forecasting is an integral part of commodity trading and price analysis. Quantitative accuracy with small errors along with turning point forecasting power are both important in evaluating forecasting models. Numerous studies have found that univariate time series, such as Box-Jenkins ARIMA models, are as accurate as larger multivariate econometric or vector autoregressive models [Bessler and Brandt; Dorfman and McIntosh; Gill and Albisu; and Harris and Leuthold]. Recent developments in the study of neural networks show that feedforward neural networks are nonlinear universal mapping structures that can approximate any arbitrary function (Cybenko; Hecht-Nielsen; Hornik et al.). Therefore, such a flexible model may be superior to ARIMA models especially for nonlinear time series data. But even if data is linear the ARIMA models may still be at a disadvantage to neural network models since they require restrictive regression assumptions regarding the error term, including the zero mean and constant variance and the assumption of being independent of explanatory variables, in order to be valid for modelling price behaviour. Neural network models are less sensitive to violations of these assumptions.

The objective of this study is to determine whether neural network models can outperform traditional ARIMA models in forecasting commodity prices. Specifically, a neural network is used to forecast monthly US cattle prices. We then compare the results with the ARIMA model as a benchmark. The remainder of the study is organized as follows. The traditional univariate time series approach to forecasting is described followed by the neural network architecture and analysis. Evaluation methods for comparing the two forecasting approaches and data and forecast procedures are then discussed, followed by the results from both the ARIMA and the neural network forecasts. Finally, a brief conclusion is presented.

ARIMA Time Series Model

Dorfman and McIntosh suggest that "structural econometrics may not be superior to time series techniques even when the structural modellers are given the elusive true model." Therefore, a common approach to forecasting is the Box-Jenkins time series approach which is used here for comparison with neural networks. It has attracted researchers because it is a parsimonious approach which can represent both stationary and non-stationary stochastic processes (Harvey). It is used here is to build an Autoregressive Integrated Moving Average model (ARIMA) which adequately represents the data generating process. The basic Box-Jenkins model has the following form:

\[ y_t = \alpha_0 + \sum_{i=1}^{p} \gamma_0 y_{t-i} + \sum_{j=1}^{q} \beta_j e_{t-j} \]  

\[ i=1,...,p \text{ and } j=0,...,q \]

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where $y_t$ is a stationary stochastic process with non-zero mean, $\alpha_0$ is a constant term, and $e_t$ is a white noise disturbance term. The second and third terms in the right hand side of equation 1 are autoregressive and moving average parts of the model. Equation 1 is denoted by ARMA($p,d,q$) in which "d" stands for the number of differencing performed on $y_t$ before estimating the above model. Box-Jenkins methods involves the following four-step iterative cycle:

(i) model identification,
(ii) model estimation,
(iii) diagnostic checking, and
(iv) forecasting with the final model.

Forecasting with the estimated model is based on the assumption that the estimated model will hold in the time horizon for which the forecasts are made. The AR part of the model indicates that the future values of $y_t$ are weighted averages of the current and past realizations. Similarly, the MA part of the model shows how current and past random shocks will affect the future values of $y_t$.

The Neural Network Approach

Neural networks, or parallel distributed processing, are computational structures modeled on the gross structure of the brain (Hecht-Nielsen 1988). Neural networks are powerful methods for pattern recognition, classification, and prediction. As far as its applications in economics are concerned, they have been primarily used to address financial economic problems. Typical applications in finance have included mortgage risk assessment, economic prediction, risk rating of exchange-traded fixed-income investments, portfolio selection/diversification, simulation of market behaviour, index construction, and identification of explanatory economic factors (Trippi and DeSieno). For example, the US government in 1989 "embarked on a five-year, multi-million dollar program for neural network research, but financial services organizations have been the principal sponsors of research in neural network applications." (Trippi and DeSieno).

There are a number of studies in which, along with conventional methods, neural networks are used to address financial economic problems (c.f., Schoneburg; Kamijo and Tanigawa; Kimoto and Asakawa; Stephens et al; Odom and Sharda; Surkan and Singleton; Tam and Kiang; Trippi and DeSieno). For instance, Surkan and Singleton found that the neural network model outperforms "multivariate discriminate analysis" (MDA) for bond rating. In their study, the neural network model provided 88% correct classification compared with, at most, 56.6% by the MDA method. Odom and Sharda set up both MDA and neural network models for predicting bankruptcy for various companies listed in the Wall Street Journal. They also found that neural networks were over 20% more accurate than MDA. Trippi and DeSieno compared a neural network based trading strategy in S&P 500 Index futures with a passive buy and hold strategy. They found that the neural network model strongly outperformed the buy and hold strategy by as much as 228%, even after the inclusion of brokerage charges.

A neural network consists of a collection of input units and processing units, neurons, that are arranged in several layers (Figure 1). Each neuron at the input layer receives the data, multiplies them by a connection strength called a "weight" and sends them to the neurons in the next layer. The processing units which are immediately after the input layer are self-sufficient when processing the information they receive. Processing in each individual neuron takes place in parallel with other units according to a transfer function. If there is more than one layer, then the results from each neuron are sent to all the neurons in the next layer. The relationship
between each neuron and other neurons in the next layer is generally "feed-forward".

Neural networks are different from computers in the sense that they learn to solve problems. The learning takes place either by supervised training or unsupervised training. A supervised network is allowed to compare the results of its analysis with the desired output. The discrepancies between the desired output and that of the network error are then minimized up to a tolerance level which is determined by the researchers. In unsupervised training there are no actual outputs. The network learns to classify patterns by its own clustering scheme.

**Error Back Propagation Neural Networks (BP)**

Artificial neural networks differ according to their "learning laws incorporated in their transfer functions, the topology of their connections, and the weights assigned to their connections" (Hecht-Nielsen 1988). One of the most widely used networks is the "back-propagation" (BP) network. This network has been popular due to its success in solving practical problems. Hornik et al rigorously showed that the standard BP network using an arbitrary transfer function is capable of approximating any measurable function in a very precise and satisfactory manner, provided that a sufficient number of hidden neurons are used. Hecht-Nielsen (1989) has also shown that a three layer BP is capable of approximating any arbitrary continuous mapping.

BP is a supervised learning network with a feed forward topology (Figure 1). In BP, each neuron is activated according to the weights and the transfer functions in the preceding layers. Transfer functions include, for example, step functions, Gaussian functions, and sigmoid functions. Since sigmoid transfer functions are both nonlinear and continuously differentiable, they exhibit many desirable properties when they are used in neural networks. In setting up the BP, if the sigmoid transfer function is selected, inputs are weighted and sent to the next layer in which they are processed according to the following function:

\[ f(x) = \frac{1}{1 + \exp(-x/T)} \]

where \( f(x) \) is the output of a neuron to the neurons in the next layer, \( x \) is the weighted input and \( T \) is a constant which determines the steepness of the function.

**Training in Back Propagation Neural Networks**

During the training phase, BP calculates the difference between its results and the target output and tries to minimize this difference up to the predetermined level. This process takes place in the following steps (Zurada):

1. **Step 1.** Weights are initialized at small random values.
2. **Step 2.** Training step starts by computing the output, \( f(x) \).
3. **Step 3.** The difference between \( f(x) \) and the desired output is calculated:

\[ E = \frac{1}{2}(d_k - o_k)^2 \]

where \( E \) is the error, \( d_k \) is the desired output, and \( o_k \) is the network's output.
Step 4. Error signal vectors $\delta_o$ and $\delta_h$ of the hidden layer and output layer are calculated:

\begin{align}
\delta_{ok} &= 1/2(d_k - o_k)(1 - o_k^2) \\
\delta_{hi} &= 1/2(1 - h_i^2)\sum\delta_{ok}w_{ki}
\end{align}

Step 5. Output layer weights are adjusted.
Step 6. Hidden layer weights are adjusted.
Step 7. If $E$ is less than or equal to the tolerance level, the training cycle is completed. If $E$ is greater than the specified level then a new training cycle is initiated by going to step 2.

The speed of learning in a BP network can be increased without leading to oscillation. This is achieved by modifying the BP learning rule to include a momentum term (McClelland and Rumelhart). The rule is

\begin{align}
\Delta W_{ij}(n+1) &= \varepsilon(\delta_i a_{pj}) + \alpha W_{ij}(n)
\end{align}

where the subscript $n$ denotes the presentation number, $\varepsilon$ is the learning rate (the speed by which weights are changed), $\delta_i$ is the error, $a_{pj}$ is the output of the transfer function, and $\alpha$ is the momentum term, where $\alpha$ determines how past weight changes should affect the current weight changes.

**Evaluation Methods**

Three criteria will be used to make a comparison between the prediction power of the time series model and the neural network model. The first is the mean squared error, MSE, which measures the overall performance of a model. The formula for the MSE is

\begin{align}
MSE = \frac{1}{T}\sum(P_t - A_t)^2
\end{align}

Where $P_t$ is the predicted value at time $t$, $A_t$ is the actual value at time $t$, and $T$ is the number of predictions.

The second criterion is the absolute mean error, AME. It is a measure of the average error for each point forecast made by the two methods. AME is given by

\begin{align}
AME = \frac{1}{T}\sum|P_t - A_t|
\end{align}

While MSE and AME are good measures of the deviations of the predicted values from the actual values, they do not say much about the power of the models in predicting turning points. For many traders and analysts the market direction and turning points are as important as the value forecast itself. "In these markets, money can be made simply by knowing the direction in which the series will move" (McIntosh and Dorfman). A correct turning point forecast requires:

\begin{align}
sign(P_t - A_{t-1}) = sign(A_t - A_{t+1})
\end{align}
The ability of a model to forecast turning points can be measured by a third method developed by Cumby and Modest which is a version of Merton’s test. Merton’s test is as follows: define a forecast variable \( F_t \) and an actual direction variable \( A_t \) such that

\[
A_t = 1 \text{ if } \Delta A_t > 0 \quad \text{and} \quad A_t = 0 \text{ if } \Delta A_t \leq 0
\]

\[
F_t = 1 \text{ if } \Delta P_t > 0 \quad \text{and} \quad F_t = 0 \text{ if } \Delta P_t \leq 0
\]

where \( \Delta A_t \) is the amount of change in the actual variable between time \( t-1 \) and \( t \) and \( \Delta P_t \) is the amount of change in the forecasted variable for the same period.

The probability matrix for the forecasted direction of changes in the actual value conditional upon the direction of changes in the forecasting variable \( F_t \) is

\[
P_1 = \text{Prob}[F_t = 0 \mid A_t = 0]
\]

\[
1 - P_1 = \text{Prob}[F_t = 1 \mid A_t = 0]
\]

\[
P_2 = \text{Prob}[F_t = 1 \mid A_t = 1]
\]

\[
1 - P_2 = \text{Prob}[F_t = 0 \mid A_t = 1]
\]

In other words, (12) and (14) is the probability that the forecasted direction has actually occurred and (13) and (15) is the probability of a wrong forecast.

By assuming that the magnitude of changes in \( F_t \) and \( A_t \) are independent, Merton (1981) showed that a necessary and sufficient condition of market timing ability is that

\[
P_1(t) + P_2(t) > 1
\]

i.e. the forecaster on average has to be right in more than half of the time that forecasts are made. So the null hypothesis to be tested is

\[
H_0 : \quad P_1 + P_2 - 1 \leq 0 \quad \text{vs} \quad H_1 : \quad P_1 + P_2 - 1 > 0
\]

Cumby and Modest showed that the above hypothesis can be tested through the regression equation:

\[
X_t = \alpha_0 + \alpha_1 A_t + \epsilon_t
\]

where:

- \( X_t \) is the change in the actual price from previous period at time \( t \)
- \( A_t \) is the realized price direction variable defined in (10)
- \( \epsilon_t \) is the error term,
- \( \alpha_t = P_1 + P_2 - 1 \),

and an \( \alpha_t \) significantly different from zero is needed to prove forecasting ability.
**Data and Procedure**

Monthly cash prices ($/100 lb.) of US cattle (900-1100 lb) traded in Omaha are used to test the prediction power of the two approaches. Data are obtained from the CRB Commodity Year Book, various issues, and cover the period 1974-1990. Monthly data are used to estimate the three time series models from 1974 through 1987, 1988, and 1989 respectively. The estimated coefficients from each of the above three models are then used to forecast cattle prices out of sample and twelve steps ahead without updating. The forecasted values from the models fitted over 1974-1987, 1974-1988, and 1974-1989 are then compared with the out of sample actual prices for 1988, 1989, and 1990 respectively.

A multi-layer backpropagation neural network with one hidden layer was set up which uses both actual and differenced data as inputs. To make the comparison with the time series models, twelve lags of both actual and differenced prices were assumed to be sufficient as inputs to the network. The hidden layer was set up with 18 neurons, 9 for levels and 9 for differenced prices. This is 75% of the number of neurons in the input layer following Baily and Thompson's suggestion. The output layer consisted of one neuron representing next month's cattle price. The same neural network specification were used for each model over the three years.

For forecasting, a moving window of twelve months lagged prices was created. For the first step the weight matrix was applied to the twelve monthly cattle prices in 1987 to obtain a price forecast for the first month of 1988. Without the network weights being changed, the forecasted prices then became part of the inputs to the network and the last prices in the previous input series were dropped. The above process of forecasting and back substitution continued until all twelve forecasts were made.

**ARIMA Time Series Results**

**Identification and Estimation Results**

Results of the identification step suggest that the ARIMA models below can best represent the price behaviour over the various periods. The maximum likelihood estimate of the three models are:

**1974-1987 model**

\[
y_t = 0.12646 + 0.25788y_{t-1} - 0.26293y_{t-3} - 0.17033y_{t-7}
\]

(18) \hspace{2cm} (0.79) \hspace{2cm} (3.69) \hspace{2cm} (-3.63) \hspace{2cm} (-2.34)

**1974-1988 model**

\[
y_t = 0.14611 + 0.21051y_{t-1} - 0.21857y_{t-3} - 0.21488y_{t-4} - 0.21100y_{t-7}
\]

(19) \hspace{2cm} (1.18) \hspace{2cm} (3.09) \hspace{2cm} (-3.10) \hspace{2cm} (-3.01) \hspace{2cm} (-2.97)

**1974-1989 model**

\[
y_t = 0.15720 + 0.26187y_{t-1} - 0.26205y_{t-3} - 0.16774y_{t-7}
\]

(20) \hspace{2cm} (1.06) \hspace{2cm} (3.99) \hspace{2cm} (-3.84) \hspace{2cm} (-2.44)
T statistics in parentheses show that all coefficients are significant other than the constant terms. However, since the means are not subtracted from differenced data the constant terms are kept in the models for the forecasting steps.

**Diagnostic Checking**

Plots of autocorrelation of the estimated residuals were inside two standard error bands. This indicates a white noise error term in the estimated model and proper modelling procedure in that, by ARIMA’s standards, all information has been extracted from the error terms. Ljung-Box test statistics reported in Table 1 show that all estimated probabilities are greater than 1%. Therefore, equations 18 to 20 can be considered an acceptable representation of the data generating processes for the ARIMA models.

**Forecasting Results**

Results of Box-Jenkins forecasts using equations 18 to 20 are shown in Table 2. Results show root mean squared errors of 4.39 for 1988, 2.10 for 1989, and 2.30 for 1990 forecasts for the ARIMA model. Absolute mean errors indicate that forecasted prices by ARIMA were on average, $3.56, $1.65, and $1.77 for 1988, 1989, and 1990.

**Neural Network Results**

Results of the neural network are shown in Table 2. Results show root mean squared errors of 1.99 for 1988, 2.98 for 1989, and 1.17 for 1990. Absolute mean errors indicate that forecasted prices by the neural network were on average, $1.53, $2.17, and $0.98 for 1988, 1989 and 1990, respectively.

**Evaluation and Comparison**

**Quantitative Evaluation**

In terms of quantitative forecasts, Table 2 results show that in 1988 and 1990 the neural network outperformed the ARIMA model. In terms of forecast error, in the above period, the neural network outperformed the ARIMA model by 97% to 233%. In terms of error variance, the neural network forecasts’ variances were 4 to 5 times lower than the ARIMA’s forecasts. In 1989, however, ARIMA forecasts errors were on average $0.52 lower than that of the neural network and also had lower forecast error variance. In order to rigorously compare the neural network and the ARIMA forecasts, t tests for mean and F tests for variance were used. For 1988, the t and F values in Table 2 indicate that the neural network had a lower mean and variance of forecast errors than ARIMA. For 1989, results indicate that the forecasts by the ARIMA and neural network models were not statistically different. For 1990, the above statistics show lower variance of errors for the neural network and this was significant at the 1% level.
Turning Point Evaluations

The formal statistical test of turning points for both models is performed by estimating equation 17 and the results are shown in Table 3. The t ratios of slope coefficients, \( \alpha_i \), show that the ARIMA model did not have significant turning point forecasting power. The negative signs in 1988 and 1989 also imply that the ARIMA models were even giving wrong signals about the turning points in the above periods. In contrast, for the neural network predictions the \( \alpha_i \) was 0.80 in both 1988 and 1989 and significantly different from zero. The value of slope parameter shows that the neural network was correct 80\% of the time. In 1990, forecasts of neither the ARIMA nor the neural network had significant turning point forecasting power. This may be expected if prices were truly stochastic in 1990.

Conclusion

The purpose of this study was to determine the performance of the neural network when both actual and differenced prices are fed into the model and compare it to traditional Box-Jenkins ARIMA methods as a benchmark. The advantage of a neural network may be that the price levels pick up the trend in the data while the price differences capture the prices changes around the trend. In contrast, ARIMA models may lose valuable price information when data must be differenced and trends removed. Results of this study show that price information may not be fully captured by the traditional time series models. Results also indicate that feedforward neural network models have the ability to both identify and forecast time series examined here with considerable quantitative and qualitative accuracy. The neural network generally outperformed the ARIMA model in both forecast error and turning point prediction. This supports the theoretical proofs by Hornik and also Hecht-Nielsen that a feedforward neural network with only one hidden layer can approximate any continuous function.
Figure 1. Topology of a Neural Network

Table 1. Ljung-Box Test Results for Autocorrelation of Residuals From The Estimated ARIMA Models.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td>6.08</td>
<td>0.11</td>
<td>0.35</td>
<td>0.84</td>
<td>7.73</td>
<td>0.05</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>14.21</td>
<td>0.12</td>
<td>9.25</td>
<td>0.32</td>
<td>18.73</td>
<td>0.03</td>
</tr>
<tr>
<td>18</td>
<td>15</td>
<td>18.24</td>
<td>0.25</td>
<td>13.86</td>
<td>0.46</td>
<td>22.57</td>
<td>0.09</td>
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<tr>
<td>24</td>
<td>21</td>
<td>21.44</td>
<td>0.43</td>
<td>16.27</td>
<td>0.70</td>
<td>24.99</td>
<td>0.25</td>
</tr>
<tr>
<td>30</td>
<td>24</td>
<td>24.14</td>
<td>0.62</td>
<td>19.91</td>
<td>0.80</td>
<td>28.79</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Note: A probability value greater than 0.05 indicates that the estimated model is a reasonable representation of data generating process.
Table 2. Results of US Monthly Cattle Price Forecasts by ARIMA and Neural Network Models, 1988-1990.

<table>
<thead>
<tr>
<th>Month</th>
<th>1988</th>
<th></th>
<th>1989</th>
<th></th>
<th>1990</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecast error ( ^a )</td>
<td>ARIMA</td>
<td>NN</td>
<td>Forecast error</td>
<td>ARIMA</td>
<td>NN</td>
</tr>
<tr>
<td>1</td>
<td>0.68</td>
<td>-0.16</td>
<td>0.17</td>
<td>-0.75</td>
<td>0.48</td>
<td>-0.61</td>
</tr>
<tr>
<td>2</td>
<td>3.06</td>
<td>1.15</td>
<td>-0.14</td>
<td>-1.7</td>
<td>0.47</td>
<td>-1.54</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>2.27</td>
<td>3.05</td>
<td>0.53</td>
<td>2.63</td>
<td>0.38</td>
</tr>
<tr>
<td>4</td>
<td>6.62</td>
<td>2.22</td>
<td>2.99</td>
<td>0.4</td>
<td>3.65</td>
<td>2.39</td>
</tr>
<tr>
<td>5</td>
<td>9.11</td>
<td>4.92</td>
<td>2.78</td>
<td>1.11</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>6</td>
<td>4.48</td>
<td>1.28</td>
<td>0.21</td>
<td>-0.04</td>
<td>-0.03</td>
<td>0.81</td>
</tr>
<tr>
<td>7</td>
<td>-0.31</td>
<td>-2.61</td>
<td>-0.84</td>
<td>-0.69</td>
<td>-0.85</td>
<td>-0.18</td>
</tr>
<tr>
<td>8</td>
<td>0.67</td>
<td>-1.29</td>
<td>-0.73</td>
<td>-1.33</td>
<td>1.01</td>
<td>1.23</td>
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<tr>
<td>9</td>
<td>1.29</td>
<td>-0.82</td>
<td>-3.64</td>
<td>-5.6</td>
<td>0.34</td>
<td>-0.56</td>
</tr>
<tr>
<td>10</td>
<td>2.72</td>
<td>0.28</td>
<td>-2.75</td>
<td>-6.03</td>
<td>1.65</td>
<td>-0.82</td>
</tr>
<tr>
<td>11</td>
<td>3.58</td>
<td>0.63</td>
<td>-0.26</td>
<td>-4.69</td>
<td>3.79</td>
<td>-0.43</td>
</tr>
<tr>
<td>12</td>
<td>4.54</td>
<td>0.7</td>
<td>2.18</td>
<td>-3.12</td>
<td>4.55</td>
<td>-1.04</td>
</tr>
</tbody>
</table>

RMSE\(^b\)  
MAE\(^c\)
var. of errors

| t value\(^d\) | 2.34* | -0.71 | 1.64 |
| F value\(^e\) | 4.10* | 2.47 | 5.51** |

\(^a\)Mean prices for 1988, 1989, and 1990 were $69.54, $72.52, and $77.40 (per 100lb) respectively.

\(^b\)Root Mean Squared Errors.

\(^c\)Mean of Absolute Errors.

\(^d\)Significant values reject the null hypothesis that mean of forecast errors are the same.

\(^e\)Significant values reject the null hypothesis that variances of errors in the two forecasts are the same.
<table>
<thead>
<tr>
<th>Year</th>
<th>Model</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988</td>
<td>ARIMA</td>
<td>1.00**</td>
<td>-0.20</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.54)</td>
<td>(-0.07)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neural Network</td>
<td>0.0</td>
<td>0.80*</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.58)</td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>ARIMA</td>
<td>0.40</td>
<td>-0.46</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.97)</td>
<td>(-1.72)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Neural Network</td>
<td>0.20</td>
<td>0.80**</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.58)</td>
<td>(4.83)</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>ARIMA</td>
<td>0.40</td>
<td>0.31</td>
<td>0.10</td>
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<tr>
<td></td>
<td></td>
<td>(1.74)</td>
<td>(1.05)</td>
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<tr>
<td></td>
<td>Neural Network</td>
<td>0.40</td>
<td>0.31</td>
<td>0.10</td>
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<td></td>
<td></td>
<td>(1.74)</td>
<td>(1.05)</td>
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</tbody>
</table>

* = significant at 5% level  ** = significant at 1% level  t values in parentheses

$X_t = \alpha_0 + \alpha_1 A_t + \epsilon_t$ is the turning point test equation.
References


