Vector Autoregressions on U.S. Hog Prices

by

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Most model builders and perhaps many model users recognize the difference between quantitative models constructed for prediction versus those constructed for hypothesis testing. The consequences of this recognition are real and go to the heart of what one can and cannot do in a model-building exercise. Indeed, textbooks on the former stress simple or parsimonious models; while tests on the latter stress full specification, in agreement with prior theory. A model constructed for hypothesis testing may accordingly not perform well (or as well as some "simpler" alternative) when evaluated in a particular forecasting setting.

As one illustration of our point, consider recent work on modeling vector autoregressions. Two distinct approaches have recently received considerable attention. First are models similar in structure to those considered by Tiao and Box. Here, theory is used to suggest series to be modeled -- the data and test statistics of a first (or second, or third, ...) fit are used for explicit model specification. Insignificant coefficients on an earlier fit are dropped in secondary or tertiary model fitting. Since one is not explicitly interested in hypothesis testing, the failure to control significance levels on the resultant model is not of prime importance. In these models the test of the pudding is actual out-of-sample prediction. T-tests or F-tests are not convincing to viewers of these models, as generous portions of pre-testing are used.

As an alternative to the Tiao and Box procedures for model building, one might consider the vector autoregressions (VARs) of Sims and his colleagues (Litterman, Litterman and Weiss). In these models, hypothesis testing is of prime importance -- no real consideration is given to prediction. Consequently,

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large models (Sims calls them profligatly-fit models) are fit whose purposes are to capture the regularities in the data. Here, once a set of data is selected to be studied in a vector autoregression, one does no (or very little) pretesting. All coefficients, whether significant from zero or not, are kept in the final model. This allows users of the "Sims-type" econometrics to make hypothesis tests with approximately valid significance levels. However, they oftentimes do not predict very well, as compared with alternative model forecasts (see Fair).

This distinction between prediction and hypothesis testing is certainly not new. It is a general distinction which has been with us since early writings on science. Yet builders and evaluators of econometric models often forget this rather basic principle.

In this paper, we illustrate the use of vector autoregressions with an agricultural example from the U.S. hog market. The "simplicity" postulate which we described above, will perhaps be illustrated with our example. That is, we find that a "seemingly" properly specified vector autoregression does not perform as well as a simple univariate autoregression in out-of-sample tests. Here our results agree with those found by earlier writers. Nerlove et al., find, after an elaborate study of multiple relationships in the cattle market, that "by every measure ... the multiple time-series models perform marginally worse than the ordinary single time-series ARIMA model" (p. 260).

Our paper is organized into four additional sections. First we present results on modeling VARs. Next we present results or estimation of a VAR using the procedures suggested by Tiao and Box. Results from out-of-sample forecasts are presented -- along with forecasts of a univariate ARIMA model and expert opinion. In our final section we discuss the implication these results have for forecasting.

Autoregressive Modeling of a Vector Stochastic Process

From the theory of stationary stochastic processes (see Anderson) it is well known that an m-component, zero mean, covariance stationary stochastic process $X = \{X_t\}$ has the moving average representation of[1/]

$$(1) \quad X_t = \zeta_t + \Theta_1 \zeta_{t-1} + \Theta_2 \zeta_{t-2} + \cdots$$

where ζ_t is a white noise innovation vector such that

$$E\{\zeta_t\} = \underset{(m \times 1)}{0}$$

$$E\{\zeta_t \zeta_s'\} = \begin{cases} \underset{(m \times m)}{0}, & t \neq s \\ \underset{(m \times m)}{\sigma_j^2 I}, & t = s,\ j = 1, \ldots, m. \end{cases}$$

and Θ_k $(k = 1, 2, \ldots)$ are m x m matrices of moving average parameters.

The moving average representation is of both theoretical and practical importance (see Feige and McGee); however, it is often useful to approximate X by either its vector autoregressive (Parzen) or vector autoregressive-moving average form (Tiao and Box). In particular, under fairly general conditions, the model given in (1) can be demonstrated to be equivalent to:

$$(2) \quad \zeta_t = X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} - \cdots$$

where ϕ_i $(i = 1, 2, \ldots)$ are m x m matrices of autoregressive parameters and X are defined as in (1).

The model given in (2) is an infinite series. In applications it must be approximated; to do so is not a trivial task. Numerous procedures have been suggested in the literature. In a univariate contest (m = 1), Anderson sets

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1/ Here we argue that any linear deterministic component (which is present in Wold's decomposition) say η_t can be subtracted from the original process to give us X_t which is nondeterministic.

up the selection problem as a multiple testing problem. More specifically, he shows that, for a suitable transformed model, a Neyman-Pearson type constrained optimum problem can be specified, which results in a series of tests that the process is not of order i, starting at some predetermined upper possibility and continuing until either a particular hypothesis is rejected or until we fail to reject the final hypothesis -- usually that the series of observations are independent through time.

Anderson's procedure is not often used, even for univariate time series. More importantly, for multiple time series we are not aware of any generalizable test which proports to be optimal (in a classical sense). Tiao and Box do, however, suggest a likelihood ratio statistic based on Bartlett's early work on multiple regression. They suggest testing the null hypothesis $\phi_k = 0$ against the alternative $\phi_k \neq 0$ when an autoregressive model of order k is fit. Writing the sum of squares and cross products error matrix of order k as

$$S(k) = \sum_{t=k+1}^{T} (X_t - \hat{\phi}_1 X_{t-1} - \cdot\cdot\cdot - \hat{\phi}_k X_{t-k}) \times (X_t - \hat{\phi}_1 X_{t-1} - \cdot\cdot\cdot - \hat{\phi}_k X_{t-k})',$$

the likelihood ratio will be given by the ratio of successive determinants

$$U = \{|S(k)|\} \Big/ \{|S(k-1)|\}.$$

The statistic

$$M(k) = -(T - 1/2 - k \cdot m) \ln U$$

is asymptotically distributed chi-squared with m^2 degrees of freedom. Here T is the number of data points over which we fit the parameter $\hat{\phi}$.[2/]

After a tentative order AR process is selected (using the likelihood ratio test), Tiao and Box suggest one re-estimate the vector AR process deleting statistically insignificant matrix elements. This will reduce the parameter numbers in the final model. To circumvent the seemingly ad hoc nature of hypothesis tests in selection of autoregressive orders, numerous authors have suggested using explicit loss functions where the loss is defined directly as a function of the autoregressive order fit. Probably the best known work of this type is that of Akaike (1969), (1974), and (1977). He defines the final prediction error (FPE) as the expected variance of the prediction error when an autoregressive model of order k is fit to another independent realization of the process. An estimate of FPE for each autoregressive model within a sufficiently wide range of models can be made with the following for univariate autoregressive processes:

$$FPE(k) = \frac{T+k+1}{T-k-1} \left[\sum_{t=k+1}^{T} (X_t - \hat{X}_t(k))^2 / T \right]$$

where T, k, X_t are defined as above and $\hat{X}_t(k)$ is the t^{th} predicted observation of the process X_t which has been formed from an autoregression of order k. A multivariate extension of FPE is given in Caines, et al. (1981).

Our work (Bessler and Binkley) and that of others (Hannan and Quinn) suggests that FPE tends to overfit autoregressions -- for both real (empirical) data and Monte Carlo generated data. Alternative selection criteria, suggested for univariate order selection are given by Hannan and Quinn, Geweke and Meese, and

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2/ We are unaware of whether this likelihood ratio test possesses optimal properties in Anderson's sense. In particular we have not yet studied the sequential nature of its application and the selection of the candidates for maximum lag length.

Schwarz. Our initial Monte Carlo results suggest that all of these alternatives do a better job in selecting the proper order of autoregression. We have not investigated the use of these loss functions in the context of multiple time series.

A final method for fitting vector autoregressive models can be found in recent work of Sims. He chooses an order based on sample size and fits an unconstrained vector autoregression to his data points. Sims' method results in a model with $m^2 \times k$ parameters, which in general will be more than the number found with the procedures of either Tiao and Box or Caines, et al.

Estimation of the Forecasting Models

The hog price forecasting models were estimated over the sixty-four quarter period 1960 through 1975. Over most of this period, prices tended to cycle in reasonably regular pattern with an upward trend seemingly due largely to inflation. Over the sixteen year fit period hog prices (all barrows and gilts at seven terminal markets) ranged from a low of $13.92 per hundredweight in the first quarter of 1960 (60-1) to a high of $58.83 in the third quarter of 1975 (75-3). However, prior to 1973, the highest price observed was $28.89 (72-4). Since 1973, hog prices (as well as those of many other agricultural commodities) have begun to reflect greater instability.

Vector Autoregressive Model

With attention focused on forecasting hog prices (PHOG), four other time series thought to affect or at least be related to hog prices were identified for investigation. These series were chosen either because of their presumed biological or economic relationship with hog prices. The number of sows farrowing (SF) is considered to be biologically related to hog price movements.

The market hog feeding period from birth to slaughter requires six to eight months; thus it was hypothesized that changes in the sows farrowing series would cause a reaction in hog prices two to three quarters later. The price of corn (PCRN), price of slaughter cattle (PCAT), and disposable income (INC) are all considered to be economic variables related through supply costs or demand shifters. In addition to the estimated relationships generated between these variables and the price of hogs, the effects of interactions between all combinations of variables are obtained during the estimation process.

The vector AR model was fit over the 1960-1975 period using the stepwise AR program described in Tiao, et al. Following the discussion in the previous section, likelihood ratio statistics (M(k)) were calculated and are provided in Table 1 for nine lags. The statistics suggest that AR matrices through lag 6 are different from zero at the 5 percent level of significance. These results support, in general, earlier findings by Bessler and Brandt (1982b) and Bessler and Binkley (1980) who found causal relationships between hog prices and related time series at six lags. The indicator matrices on the estimated coefficients associated with the model at six lags suggested thirty-two coefficients were significantly different from zero. (The test was $\pm 2/\sqrt{N}$ where N was the number of observations.) This suggests that approximately 21 percent of the potential coefficients (32/150) were important in explaining movements in the five time series. Of the thirty-two coefficients, twenty-four or three-fourths, were off-diagonal elements, suggesting evidence of dependence among the time series and some degree of feedback. The fitted vector AR model is shown in Table 2.

Those factors found to affect hog prices include a three-quarter lag of its own series, three and four quarter lags in the sows farrowing series, and one and three quarter lags in income. Curiously, neither corn prices nor cattle prices

Table 1. Likelihood ratio statistics (M(k)) on vector autoregressions of order (k).

(k)	M(k)
1	91.24*
2	56.88*
3	145.13*
4	90.99*
5	23.49
6	40.07*
7	19.56
8	34.88
9	17.79

Critical chi-square values with 25 degrees of freedom are 37.7 (.05) and 44.3 (.01). An asterisk indicates significantly different from zero at the five percent level.

Table 2. Fitted Vector AR Model of Hog, Corn, and Cattle Prices, Sow Farrowings, and Income, 1960-1975 (quarterly)

$$
\begin{bmatrix}
(1 + .\overset{*}{4}505B^3) & \text{---} & \text{---} & (.0037B^3 + .00\overset{*}{1}2B^4) & (-.11\overset{*}{9}8B^1 + .11\overset{*}{8}3B^3) \\
(.0\overset{*}{1}7B^1 - .0\overset{*}{2}94B^4) & (1 + .3\overset{*}{8}88B^4) & (-.00048B^2) & \text{---} & \begin{array}{l}(-.0039B^1 - .00\overset{*}{5}7B^3 \\ -.00094B^5 + .00\overset{*}{7}1B^6)\end{array} \\
(.2\overset{*}{3}68B^3) & (5.\overset{*}{4}424B^1 + 4.\overset{*}{8}319B^6) & (1) & (.00057B^3 + .060\overset{*}{7}1B^4) & \text{---} \\
(-28.\overset{*}{2}39B^3) & \text{---} & (-20.\overset{*}{8}8B^4 - 14.414B^6) & \begin{array}{l}(1 - .0169B^1 + .1238B^2 \\ - .92\overset{*}{9}7B^4 - .1289B^6)\end{array} & \text{---} \\
(-.4\overset{*}{9}58B^1 - .2487B^5) & (-6.3458B^2 + 24.\overset{*}{4}46B^4) & (.3549B^3) & \text{---} & (1 - .1180B^1 - .8\overset{*}{9}821B^6)
\end{bmatrix}
\begin{bmatrix} PHOG_t \\ PCRN_t \\ PCAT_t \\ SF_t \\ INC_t \end{bmatrix}
=
\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ e_{5t} \end{bmatrix}
$$

Note: The estimated coefficients were based on indicator matrices associated with six lags. These thirty-two coefficients fell outside the range $(-2/\sqrt{N}, + 2/\sqrt{N})$ where N equals the number of observations. An asterisk (*) over the coefficient indicates it was significantly different from zero at the .05 level.

were found to be important factors explaining hog price movements. This is in contrast to most econometric models which suggest that the price of corn and the price (or quantity) of cattle affect movements in hog prices. Further investigation of these relationships is needed.

Column one of Table 2 suggests that hog prices are important in explaining movements in all other time series examined. The combining of column one and row one indicates feedback between hog prices and sows farrowing and between hog prices and disposable income. These results tend to support those found previously (Bessler and Brandt, 1982a).

Univariate ARIMA Model

As suggested above, the hog price series exhibited evidence of nonstationarity over the estimation period. First differencing the time series removed the nonstationary aspects. In addition, a fifth order moving average process was estimated which generated a residual series which cannot be distinguished from white noise (a random series). The estimated univariate model and associated performance statistics (t value in parentheses) are:

$$HP_t = HP_{t-1} - \underset{(-2.97)}{.4293}\, e_{t-5} \qquad R^2 = .87 \qquad Q_{(23)} = 21.61^{3/}$$

Hog Price Forecasts and Performance Evaluation

Quarterly cash price forecasts for hogs were generated over the seven year period, 1976-1982. Actual prices ranged from a low of $31.18 (80-2) per hundredweight to a high of $61.99 (82-3) over the 28 quarter period (Table 3). Hog production has moved from a low point in the cycle in 1976 to record levels in 1980 and back to low production in 1982. Prices, on the other hand, have moved with

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3/ The chi-squared statistic for 23 degrees of freedom at the 95 percent confidence interval is 35.17.

Table 3. Quarterly Hog Price Forecasts and Performance Evaluation, 1976-1982.

	Actual Prices[a/]	Vector Autoregression	Univariate ARIMA	Expert Judgment[b/]
	-------dollars per hundredweight-------			
76-1	47.99	44.23	48.81	47.00
2	49.19	44.61	49.06	48.50
3	43.88	51.66	46.73	45.00
4	34.25	45.02	39.78	35.00
77-1	39.08	34.58	35.19	35.00
2	40.87	41.00	39.12	32.50
3	43.85	46.86	40.09	44.00
4	41.38	43.64	45.11	37.50
78-1	47.44	40.66	43.84	36.00
2	47.84	44.26	45.58	42.00
3	48.52	50.21	47.01	51.00
4	50.05	47.32	47.12	45.00
79-1	51.98	50.79	51.83	51.00
2	43.04	52.05	50.26	48.00
3	38.52	40.67	41.96	43.00
4	36.39	37.84	37.80	32.50
80-1	36.74	41.96	34.99	32.50
2	31.18	38.46	36.67	37.00
3	46.23	29.39	34.64	42.00
4	46.44	46.60	47.88	45.00
81-1	41.13	53.57	47.11	50.50
2	43.63	33.20	40.03	44.00
3	50.42	43.29	46.26	52.50
4	42.63	54.49	44.87	48.00
82-1	48.17	41.06	43.32	48.50
2	56.46	38.84	51.04	51.00
3	61.99	59.62	54.86	60.00
4	55.31	63.47	60.00	61.00
Mean	45.16	44.98	44.68	44.45
Mean Squared Error	---	59.52	19.53	21.88
Mean Absolute Error	---	6.14	3.69	3.66

a/ Price for all barrows and gilts at seven terminal markets.

b/ Expert forecasts are based on monthly predictions by the Department of Agricultural Economics Marketing Guides Committee, Purdue University.

less regularity over this seven year period. This reflects a continuation of the observed relative instability of prices which began in the early 1970's.

Prices forecasts from the vector autoregression and univariate models are provided in Table 3.[4/] For comparison, forecasts based on a less-mechanical, more-qualitative approach, which we call expert judgment, are also given.[5/] The means of the four price series in Table 3 are similar in magnitude. In fact, the mean of the vector AR forecasts is closest to the mean of the actual price series, only 18 cents below, or less than one-half percent. However, in terms of a mean squared error (MSE) criterion, the vector AR forecasts are unquestionably the least desirable. This criterion penalizes large errors more severely than small ones, a rule we believe most decision makers would likely follow. The MSEs from the forecasts of the univariate ARIMA process and the expert judgment approach are similar in magnitude and approximately two-thirds lower than the MSE of the vector AR forecasts.

Two other performance criteria are also provided. The Mean Absolute Error (MAE) measure is based on a linear (rather than quadratic as the MSE) loss approach. The results are comparable to those of the MSE evaluation. The expert judgment and univariate ARIMA forecasting approaches generate similar MAEs (with the expert showing a very slight advantage). The MAE of either of these methods is about sixty percent of the MAE of the vector AR.

The third criterion evaluates performance in terms of the ability of the forecasting approach to anticipate turns in the price series. For the decision maker interested in longer term price moves, this evaluation approach may be more important than relative closeness of the price forecast to the price observed.

4/ These are one-quarter ahead forecasts. That is, the models are updated with new data as it becomes available each quarter.

5/ Obviously, a substantial degree of judgment is used in the arbitrary selection of variables to be included in the vector autoregression and in the final choice of model specification. However, once estimated, both the vector AR and univariate approaches become mechanical in their forecast generation.

Table 4 illustrates the outcomes of the three forecasting techniques. High numbers in the diagonal (1, 1 and 2, 2) elements are desirable. Clearly, the univariate ARIMA process and the expert outperform the vector AR model, with 58 percent correct "direction" predictions (versus 34 percent correct for the vector AR).

Thus, it appears that based on three alternative performance evaluations, the vector autoregressive forecasting approach does not measure up to the univariate model or the expert. These results were not surprising in light of the warning provided by Sims regarding the profligate use of vector autoregressions for hypothesis testing analysis of industry structural characteristics rather than for economic forecasting. In addition, the results reported here for hogs agree with those offered by Nerlove, Grether, and Carvalho for the cattle industry. Those authors (p. 351) suggest, "From the overall indicators of fit, the single-time-series approach to the expectation formation models used appears to perform somewhat better (than the multiple-time-series). . . In short, one cannot regard the use of forecasts generated by the multiple-time-series models . . . as a significant improvement in this context over the simple single-time-series models."

Summary

Vector autoregressive models designed for forecasting are new relative to the various types of econometric models used by forecasters for decades. Vector autoregression offers the advantages of incorporating several time series which seem theoretically "correct" for inclusion in a forecasting model yet at the same time allowing the data themselves to determine the explicit dynamic (lead-lag) relationships between the series. Univariate models are by definition

Table 4. Turning Point Prediction Performance by Forecasting Technique.

Forecasting approach	Forecast direction of price movement	Actual direction of price movement	
		Change	No change
Vector Autoregression	Change	5	9
	No change	8	4
Univariate ARIMA	Change	7	5
	No change	6	8
Expert judgment	Change	7	5
	No change	6	8

limited to a single time series. Structural econometric models have the dynamic (lagged) relationships of the explanatory variables to the dependent variable arbitrarily assigned by the model builder during the specification phase (prior to the estimation process).

Based on the results found in this analysis, one might be tempted to dismiss the use of vector autoregression models for forecasting purposes. The measures used to evaluate forecasting performance in this study suggest that the parsimonious univariate model (and the expert judgment approach) outperformed the vector AR approach. It is gratifying to forecast model-builders to learn that occasionally (frequently ?) simple models forecast more accurately than more sophisticated approaches.

We wish to point out, however, that the vector AR does allow the researcher to study the dynamic interrelationships between time series directly. In addition, forecasts for each of the five series used in this analysis were generated directly. (Only the hog price forecasts were examined in this study.) Alternatively, if a researcher was to generate forecasts for each of these time series using a univariate approach, five separate equations (models) would need to be identified, estimated, and diagnostically checked.

The lack of significant empirical relationships between hog prices and corn prices and cattle prices was a bit disturbing. Almost without exception, structural and forecasting models of the hog sector have included one or (usually) both series as explanatory variables. The empirical regularities, however, do not suggest an important influence. This may be due to the multicollinearity between past hog prices and past corn (and cattle) prices or other variables in the system. Further investigation of these relationships is currently underway.

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