An Alternative Parameter Estimation Approach for Risk Management Decision Models

by

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Traditional agricultural economic forecasting models are often used to aid producers in making management decisions. A classic example is the decision of whether to store or sell wheat. An econometric model is first used to forecast expected future wheat prices. The forecasted wheat price is then used in a storage decision model. Generally the decisiom model framework consists of some form of budgeting activity where storage costs are compared to expected revenues derived from the forecasted wheat price. If an adequate positive return to storage is indicated the decision to store follows.

The procedure described in the preceding paragraph is typical of many integrated forecasting and decision making processes. It implicitly assumes that the statistical criteria used in developing the forecast model, i.e. minimizing error squared, is consistent with and optimal for the subsequent use of the forecasts in a decision making model. The parameter estimation process does not consider the impact of the forecasting errors upon the decisions made and the resulting profits. In many decision making cases the sensitivity of the decision to changes in the forecasted value varies over the range of forecasts to be made. For example, in the wheat storage decision case, the accuracy of forecasts that generate expected returns near the break even level is quite critical, while those that show large expected profits or losses

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need not be as accurate. In developing forecasting models for such cases, a methodology is needed that is capable of recognizing this and placing more emphasis/weight on accuracy within critical ranges. The objective of this paper is to develop and apply such a methodology.

Methodological Development

Econometric forecasting models can in essence be viewed as generalized forecasting models. They weight the forecasting errors according to the general criteria of error squared. What is sought here is a specialized forecasting model whose errors are weighted according to the purpose of the model, i.e. in the case of a decision model integrated with a forecast model, the weights may be based upon the profits and losses generated. The parameters sought for such an integrated forecast/decision model are those that link a set of known variables to a set of prescribed decision alternatives in an optimal manner. Optimal, in most decision making models, would be defined as maximization of the profits associated with the decisions. Estimation of parameters for models with such a structure and objective function combination can be achieved using Linear Programming or Generalized Non-linear Optimization. Linear Programming will be used here.

V. A. Sposito has demonstrated the use of Linear Programming to estimate parameters of a model (equation). The objective function he demonstrates is similar to that of Ordinary Least Squares, except that deviations from the observed value are measured in terms of absolute values instead of squared values. Letting e_{1i} denote positive deviations and e_{2i} negative deviations, X a matrix of independent variable values, Y a vector of dependent variable values, and b a vector of j parameters any fitted equation can be represented as follows for the ith observation:

$$1)\ \sum_j X_{ij} b_j + e_{1i} - e_{2i} = Y_i$$

Thus the appropriate objective function and constraint equation to estimate the parameter set b_j using Linear Programming becomes:

$$2)\ \text{Minimize} \sum_{1=i}^{k} e_{1i} + \sum_{i=1}^{k} e_{2i}$$

$$\text{Subject to } Xb + Ie_1 - Ie_2 = Y$$

$$e_1, e_2 \quad 0$$

where I denotes an identity matrix. The parameter vector b is in essence a set of activity level solutions.

Sposito's specification has been modified to allow for both negative and positive parameters and displayed in tableau form in Figure 1, Tableau #1. The tableau contains one row for each observation. The first two columns (activities) provide for estimates of either a positive or negative intercept parameter. Likewise, the third and fourth columns (activities) provide for estimates of a slope parameter for the independent variable X. The technical coefficients of columns three and four are positive and negative values of the observed independent variables. The sum of the intercept and slope parameter activities is constrained to equal the dependent variable observation, Y, for each period. To the degree this is not feasible, slack activities representing absolute errors are permitted, but with a penalty to the objective function. Thus the parameters found will minimize the absolute error of a linear equation for the data set.

The use of Linear Programming to estimate equation parameter as described above is in essence an alternative to using Ordinary Least Squares. The absolute error objective function is still a generalized objective function. However, the framework of Linear Programming

Figure 1. Parameter Estimation Tableaus

Tableau #1-Minimize Absolute Error

Objective Value	0	0	0	0	1	1	. . .	1	1	1	. . .	1	
Activity	α	$-\alpha$	B	-B	e_1	e_2	. . .	e_N	$-e_1$	$-e_2$	. . .	$-e_N$	
	(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)		(10)	
Observation #1	1	-1	X_1	$-X_1$	1				-1				$= Y_1$
Observation #2	1	-1	X_2	-X		1				-1			$= Y_2$
:	:	:	:	:			.				.		:
Observation #N	1	-1	X_N	$-X_N$				1				-1	$= Y_N$

Tableau #2-Maximize Storage Profit

Objective Value	0	0	0	0	0	0	Y_1	0	0	$-Y_1$	0	0	0	$-Y_2$	. . .	Y_N	)	0	$-Y_N$	
Activity	α	$-\alpha$	B_1	$-B_1$	B_2	$-B_2$	$-R_1$	SO_1	SU_1	W_1	$-R_2$	SO_2	SU_2	W_2	. . .	$-R_N$	SO_N	SU_N	W_N	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)		(15)	(16)	(17)	(18)	
Observation #1	1	-1	$X1_1$	$-X1_1$	$X2_1$	$-X2_1$	$-Y_1$	-1	1											= 0
Forced Solution #1							1													= 1
Wrong Decision #1									1	-100										$\leq Y_1$
Observation #2	1	-1	X1	-X1	X2	-X2					Y_2	1	-1							= 0
Forced Solution #2											1									= 1
Wrong Decision #2													1	-100						$\leq Y_2$
:	:	:	:	:	:	:									. . .					
Observation #N	1	-1	$X1_N$	$-X1_N$	$X2_N$	$-X2_N$										$-Y_N$	-1	1		= 0
Forced Solution #N																1				= 1
Wrong Decision #N																		1	-10(	$= Y_N$

provides the capability to change the objective function and error weighting scheme to any of many alternative schemes. In the models to be developed here, the Linear Programming tableau will be modified to develop error weighting schemes reflective of the impact of wheat price forecasting errors upon returns to wheat storage decisions.

Alternative Model Development

Three integrated wheat price forecasting and wheat storage decision models will be developed and presented. The first model will consist of a traditional econometric forecasting model and budgeting decision model combination. The second and third models will consist of two alternative integrated forecasting/decision models whose parameters are estimated using Linear Programming. For comparison purposes all three models will be based upon the same data and function. The data used are for the period 1960 to 1979. It consists of four series describing the rate of return to wheat storage, annual wheat supply, annual wheat disappearance, and wheat carryover stock levels. The series for rate of return to wheat storage was calculated to be the return for storing wheat from June (the harvest month) to December. Over the period 1960-1979 December, on the average, was the most profitable month to sell stored wheat. Returns to storage were calculated according to Equation 3.

3) Storage Return = December Price - June Price - Storage Cost - Interest Cost

Wheat prices used were the national average mid-month price received by farmers for all wheat. Storage costs we calculated as 1.5 cents per bushel per month. Interest cost, reflecting the opportunity cost of the stored wheat, was calculated as 3 percent of the June harvest price.

These storage and interest cost were selected to be typical of average cost incurred over the 1960-1979 period.

The Econometric Model

The econometric forecast model specified and estimated is given below. The values in parenthesis are t-values for the parameters.

4) $Y = -33.94 + 14.59X_1 - 1.07X_2$

(3.6) (4.3) (2.2)

Standard Error = 17.65 $R^2 = .66$

where

Y is the rate of return to storing wheat until mid-December, i.e. storage returns divided by harvest price times one hundred.

X_1 is [1.0/Log (Supply/Demand)] where supply is total wheat production plus carryin stocks and demand is total disappearance of wheat. Natural logarithms are used.

X_2 is the change is in wheat stocks during the year as a percent of supply, i.e. carryin stocks minus carryout stocks divided by total supply and multiplied by one hundred.

The theoretical basis for the above model will not be elaborated upon since it is not the focus of this paper. The basic theory underlying the model is that of excess demand for storage as presented by Bressler and King. This theory would indicate that as the supply/demand ratio increased the returns to storage would decline, hence the inverse of the log of this ratio would be expected to be positively correlated with the rate of return to storage. The theoretically expected sign on the change in stock level variable is less clear due to its dynamic nature. A negative sign is hypothesized as expected based upon the reasoning that if stocks are declining the demand for stored wheat and hence returns to storage must be greater than when stocks are rising.

Both estimated parameter signs are as expected and statistically significant at the .025 level of confidence.

Linear Programming Model #1

The first integrated forecasting/decision model estimated will use storage profit maximization as its parameter estimation objective function. Only two decision alternatives will be considered, i.e. to either store or not store wheat from harvest until December. More alternatives could theoretically be considered but will not be in order to keep the comparisons between alternative models simpler. The storage decision will be based upon the estimated equation's net return prediction.

To estimate the parameters of an integrated forecasting/decision model that maximizes a storage profit objective function, a Linear Programming matrix that weights all forecasting errors according to their impact upon resulting storage decision profitability must be developed. The matrix must consider all possible forecast/decision combinations. For the case at hand a matrix must be developed which: a) generates a positive contribution to the objective function equal to actual storage profits when storage profits are predicted, the decision to store is made, and profits actually occur; b) generates a negative contribution to the objective function equal to actual storage losses when storage profits are predicted, the decision to store is made, but losses actually occur; c) generates no impact upon the objective function when storage losses are predicted and the decision is made not to store. Following Sposito this error weighting matrix must then be augmented with a set of activities to estimate the parameters of the model used to make the net returns predictions. The technical coefficients of these activities are the observed values for the independent variable observations. The equation derived using the above approach is given below. The variable definitions are the same as for the econometric model.

5) $Y = -4.276 + 1.287X_1 + .0001X_2$

Tableau #2 in Figure #1 illustrates the Linear Programming matrix solved to derive Equation 5 above. The matrix contains three basic types of row operations. The first type is labeled as an Observation Row. Considering Observation Row #1, the first six columns are activities to estimate the parameters of the model and are similar to the first four columns in Sposito's general matrix reported in Tableau #1. Values to the right of Column #6 describe the error weighting structure. In the case of Observation Row #1 the error structure describes a year (case) in which returns are positive. Thus within Column #7 the amount of return, Y_1, is entered in the Objective Function Row as a positive value and in Observation Row #1 as a negative value. Since Observation Row #1 is constrained to equal zero and Forced Solution Row #1 forces Activity #7 into solution, the sum of the activities in Columns #1 through #6 (the prediction equation activities) is being forced toward a positive value to offset the negated profit value entered in Activity #7. To the extent this equality is not feasible slack activity SO_1 in Column #8 allows for over-estimation of the return level, and slack activity SU_1 in Column #9 allows for under-estimation. In this case, the Objective Function is not penalized for over or under-estimation of storage returns, except when returns are under-estimated so far that negative returns to storage are predicted. This prediction would lead to an incorrect decision. Wrong Decision Row #1 monitors the error condition to determine if this has happened. If the level of SU_1 exceeds Y_1, Activity W_1 in Column #10 is forced into solution. This activity is designated as an integer activity. It causes the Objective Function to be penalized by the storage profit amount Y_1. It also releases the constraint upon the amount of under-estimation allowed.

The rows labeled Observation #2, Forced Solution #2 and Wrong Decision #2 are the same as the first three rows, but are for a year (case) in which losses were encountered on storage. Because losses were encountered the error structure activities are specified differently. The Return Level Activity, Activity R_2 in Column #11, now has a zero value in the Objective Function and positive Y_2 in Observation Row #2. The positive value for Y_2 in Observation Row #2 forces the sum of Activities #1 through #6 (the prediction equation activities) to a negative sum reflective of the actual loss level.

In general three row operations and four unique column activities are required for each observation. Activities #1 through #6, which estimate the model parameters, apply over all observations.

Linear Programming Model #2

The second integrated forecasting/decision model to be specified is very similar to the first. The only change made is to the error weighting specification. In the previous model no consideration of the accuracy of the profit and loss level forecasted beyond proper sign was given. It would appear reasonable to assume that the producer would encounter some economic costs by improperly anticipating the magnitude of profits to be received in years he chose to store wheat. With this logic in mind, a value of -.1, reflecting a 10 percent penalty on profit, was entered in the Objective Function Row of all SO_i and SU_i Activities.

The equation derived using this approach is reported below. The variable definitions are again the same as previously given for the econometric model.

6) $Y = -10.698 + 3.224X_1 + .00009X_2$

Application and Evaluation

Table 1 presents a summary of the prediction accuracies of the three models developed. As seen from the table the prediction accuracy of the Econometric Model is far superior to that of the two Linear Programming integrated forecasting/decision models referred as L.P. #1 and L.P. #2. This is as expected given the Econometric Model was estimated with the objective of minimizing forecasting error squared. Outside of the data range used for estimating the models, i.e. years 1980, 81 and 82 the error squared values are quite comparable. Also, as might be expected, L.P. Model #2 has a lower sum of errors squared than L.P. Model #1. This would be expected since L.P. Model #2 was penalized by a -.1 for profit prediction errors, while L.P. Model #1 was not.

Table 2 summarizes the storage profits generated from using each of the models to make storage decisions. The cumulative profit columns show that the two L.P. Decision Models are superior to the Econometric Model and an arbitrary Always Store Model. Despite the fact that the two L.P. Decision Models had different objective functions and parameters, they yield the same set of decisions and profits. The L.P. Decision Models are superior both within the data range used to estimate the models and outside of it. The L.P. Decision Models generate only four improper decisions while the Econometric Model makes nine wrong decisions.

The performance results reflected in Table 2 are again as expected. Since the L.P. Models were developed using measures of storage profit as

Table 1. Actual Storage Profit, Predicted Storage Profit, and Prediction Error Squared For Econometric and Decision Model Forecasts (¢/bu.)

Year	Actual Profit[a/]	Predicted Profit by Model Type			Prediction Error Squared by Model Type		
		Econometric	L.P. #1	L.P. #2	Econometric	L.P. #1	L.P. #2
1960	- 5.33	-18.40	-2.52	-6.36	170.93	7.88	1.07
1961	1.65	- 9.23	-2.45	-6.07	118.35	16.83	59.55
1962	- 6.02	- 6.89	-2.44	-6.02	.76	12.75	0.00
1963	- 1.93	9.62	-1.76	-4.16	133.19	.03	5.01
1964	-10.14	- 3.75	-1.52	-3.80	40.86	74.37	40.18
1965	- .66	20.25	- .73	-1.60	437.06	.01	.89
1966	- 7.40	22.36	- .10	- .10	885.84	53.33	53.33
1967	-15.75	- 1.10	- .77	-2.01	214.62	224.49	188.71
1968	- 8.65	-20.01	-1.72	-4.53	129.16	47.91	16.92
1969	- 3.82	-29.17	-3.82	-9.56	642.62	0.00	32.96
1970	- 3.82	7.00	-1.39	-3.34	117.09	5.91	.23
1971	-17.38	-12.65	-1.70	-4.38	22.37	245.71	169.08
1972	69.18	36.74	.35	1.16	1,052.35	4,738.11	4,626.85
1973	90.00	69.67	3.69	9.46	413.31	7,449.76	6,487.33
1974	23.33	24.96	1.39	3.42	2.66	481.28	396.41
1975	10.70	4.89	.10	.10	33.71	112.36	112.36
1976	-35.75	-22.05	-1.55	-4.15	187.69	1,169.98	998.69
1977	14.24	- 4.97	-1.50	-3.79	369.06	247.81	325.01
1978	.55	15.20	-1.95	-1.96	214.74	7,318.11	6.28
1979	- 1.93	8.61	- .60	-1.47	110.99	1.77	.21
1980[b/]	33.90	- 2.41	- .97	-2.53	1,318.42	1,215.92	1,327.14
1981[b/]	-10.10	- 4.91	- .98	-2.58	26.94	83.17	83.17
1982[b/]	- 8.17	1.26	- .92	-2.33	88.93	52.56	34.10
SUM					6,731.67	16,243.90	14,965.48

[a/] Profit calculated assuming storage until the month of December, a storage change of 1.5 cents per bushel per month and an interest rate of 6 percent applied to the June wheat price to determine opportunity cost.

[b/] These years are outside of the data used to estimate the models.

Table 2. Ex Post Returns to Storage Using Alternative Forecasting/Decision Models (¢/bu.)

Year	Always Store		Econometric Model		L.P. Model #1		L.P. Model #2	
	Single Year	Cummulative Total	Single Year	Cummulative Total	Single Year	Cummulative Total	Single Year	Cummulative Total
1960	- 5.33	- 5.33	0.00	0.00	0.00	0.00	0.00	0.00
1961	1.65	- 3.68	0.00[b]	0.00	0.00[b]	0.00	0.00[b]	0.00
1962	- 6.02	- 9.70	0.00	0.00	0.00	0.00	0.00	0.00
1963	- 1.93	-11.63	-1.93[b]	- 1.93	0.00	0.00	0.00	0.00
1964	-10.14	-21.77	0.00	- 1.93	0.00	0.00	0.00	0.00
1965	- .66	-22.43	- .66[b]	- 2.59	0.00	0.00	0.00	0.00
1966	- 7.40	-29.83	-7.40[b]	- 9.99	0.00	0.00	0.00	0.00
1967	-15.75	-45.58	0.00	- 9.99	0.00	0.00	0.00	0.00
1968	- 8.65	-54.23	0.00	- 9.99	0.00	0.00	0.00	0.00
1969	- 3.82	-58.05	0.00	- 9.99	0.00	0.00	0.00	0.00
1970	- 3.82	-61.87	-3.82[b]	-13.81	0.00	0.00	0.00	0.00
1971	-17.38	-79.25	0.00	-13.81	0.00	0.00	0.00	0.00
1972	69.18	-10.07	69.18	55.37	69.18	69.18	69.18	69.18
1973	90.00	79.93	90.00	145.37	90.00	159.18	90.00	159.18
1974	23.33	103.26	23.33	168.70	23.33	182.51	23.33	182.51
1975	10.70	113.96	10.70	179.40	10.70	193.21	10.70	193.21
1976	-35.75	78.21	0.00	179.40	0.00	193.21	0.00	193.21
1977	14.24	92.45	0.00[b]	179.40	0.00[b]	193.21	0.00[b]	193.21
1978	.55	93.00	.55	179.95	0.00[b]	193.21	0.00[b]	193.21
1979	- 1.93	91.07	-1.93[b]	178.02	0.00	193.21	0.00	193.21
1980[a]	33.90	124.97	0.00[b]	178.02	0.00[b]	193.21	0.00[b]	193.21
1981[a]	-10.10	114.87	0.00	178.02	0.00	193.21	0.00	193.21
1982[a]	- 8.17	106.70	-8.17[b]	169.85	0.00	193.21	0.00	193.21

[a] These years are outside of the data used to estimate the models.

[b] Incorrect decision.

their objective function, they would be expected to outperform an Econometric Model in this respect. The question to be posed at this point is what objective function should the model being estimated have? The Linear Programming approach to parameter estimation permits a variety of choices to be made that are not possible with traditional Econometric Models. A model which has been specified and estimated to maximize (or minimize) a certain objective function should always do so with greater ability than one specified for another purpose.

Summary and Conclusions

An alternative method of estimating model parameters has been presented. The method makes use of Linear Programming as the estimation algorithm. This allows the objective function for the estimation process to be flexible. It is contended and demonstrated that this capability can be used to improve the profits derived from wheat storage decision models.

A very simple wheat storage decision model was presented here. The capacities of the Linear Programming algorithm allow much more complexity to be developed in the model structure and objective function. The strength of its approach is in the capability to consider unique objective function for special purpose models. The disadvantages of the approach may be that the models are more difficult to specify and estimate and are dependent upon the selection of an objective function. Also no statistical properties are immediately available for the parameter estimates. It is believed that these disadvantages are outweighed by the methodologies potential to provide specialized models with more efficient performance in terms of the objective sought.

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