A Portfolio-Based Discrete Optimal Hedging Technique with an Application to Cattle Feeding

by

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A general definition of hedging is the simultaneous holding of positions in both cash and futures markets for the purpose of reducing risk due to price fluctuations. Since livestock futures were introduced in the mid-1960s, many cattle feeders have become interested in hedging as a way of reducing income volatility.

A number of strategies for timing the placement of hedges on live cattle [Farris (1972); Purcell, Hague and Holland (1972); Menzie and Archer (1972); Leuthold (1975); McCoy and Price (1975); Erickson (1978)] and on combinations of feeder cattle, feed grain, and live cattle [Shafer, Griffin and Johnston (1978); Leuthold and Mokler (1979); Caldwell, Copeland and Hawkins (1982)] have been published. Hedging strategies are decision tools for determining the proper times to hold fully hedged and unhedged positions, and preclude the use of partial hedges. Determining how much of the cash position should be hedged, and how many futures contracts are required to achieve some desired level of risk reduction, is a separate decision. This latter decision is referred to as the optimal hedging problem, and is the subject of this paper.

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The purpose of this paper will be to develop a portfolio-based optimal hedging technique in which the hedger holds investments in various cash positions and searches for suitable futures positions which reduce the total risk of the portfolio to some desired level. Then, the risk-return performances of several optimal hedging formulations will be compared to those of traditional hedging techniques for a hypothetical commercial cattle feedlot.

Principles of Portfolio Theory

Portfolio theory was developed by Markowitz (1952, 1959) and is concerned with the selection and combination of individual investments so that, for some level of risk aversion, when taken as a group they maximize total return subject to some level of total risk. Conversely, that same group will also minimize total risk subject to some level of total return. Traditionally, portfolio return and portfolio risk are represented by the mean and variance, respectively, of historical rates of return on the group of investments.

In portfolio analysis the traditional method is to optimize a quadratic utility function representing an investor's risk-return preferences. A general form such as:

$$\min \lambda \sum_{ij} X_i X_j \sigma_{ij} - \sum_i X_i \bar{R}_i \qquad (1)$$

where: λ = risk aversion parameter,

X_i = percentage of the total market value of the portfolio invested in i,

σ_{ij} = variance of returns on the ith investment, i=j, or covariance of returns on the ith and jth investments, i≠j,

$\bar{R}_i$ = mean of returns on the ith investment,

illustrates the tradeoff between risk and return, with λ indicating the investor's degree of relative risk aversion. If $\lambda = 0$ the investor is risk neutral, and if $\lambda = \infty$ the investor is perfectly risk averse.

Given some set of means, variances and covariances, a variety of portfolios may be specified simply by varying the value of λ. Portfolio construction methods automatically determine the optimal combination of investments since some type of optimization technique is used to obtain a solution. If the investments are cash commodities and futures contracts, then the portfolio approach can be used to determine the optimal futures position for a hedger, given his cash market position and his value of λ.

The principle of diversification, upon which portfolio theory is based, states that the total risk on any pair of investments will be less than the weighted average of the risks on the individual investments, provided that the returns on the two investments are less than perfectly positively correlated. Minimum, or zero, risk reduction occurs when the correlation of returns is +1, and maximum risk reduction occurs when the correlation is -1, with intermediate levels of correlation leading to corresponding reductions in risk.

Returns on long (short) positions in both cash and futures for any commodity have a nearly perfect positive correlation. Hedging, which typically involves a long (short) position in the cash commodity and a short (long) position in the corresponding futures contract, will then lead to a nearly perfect negative correlation of returns, as well as a substantial reduction in risk.

With long (short) positions in multiple cash commodities, some degree of risk reduction occurs automatically from the less than perfect correlation of returns on those cash positions. When long positions are held in some cash commodities and short positions are held in others, further reductions in total risk may result. If these cash positions are then hedged with opposite positions in the corresponding futures contracts, the diversification within the cash commodities may reduce the need for hedging, and therefore reduce the number of futures contracts required to achieve some desired level of risk reduction, compared to that required to hedge each cash commodity individually.

Feedlot Specifications

A typical commercial feedlot buys feeder cattle weighing approximately 600 pounds and feeds them a ration consisting primarily of corn or other cereal grains until the cattle reach a slaughter weight of around 1050 pounds. Using feeding recommendations for beef cattle published by the National Research Council (1976, p. 23), a feeder steer fed a ration consisting of 75-80% grain will consume approximately 3400 pounds (61 bushels) of corn during the entire feeding period. This ration will produce an average daily gain of 2.5 pounds, so a 600 pound steer requires 180 days to gain the necessary 450 pounds.

The major cash commodities in cattle feeding - feeder cattle, corn, and live cattle - have corresponding futures contracts. If we assume that the factors and products of production are fixed in a ratio of one 600 pound feeder steer and 61 bushels of corn to one 1050 pound fed steer, then the smallest multiple of cash commodities that permits each to be fully hedged is 490 head of feeder cattle, 30,000 bushels of corn, and 490 head of live cattle. This corresponds to seven 42,000 pound contracts of feeder cattle, six 5000 bushel contracts of corn, and thirteen 40,000 pound contracts of live cattle.

Stages in the Feeding Process

For the purposes of this project the entire feeding process will be divided into three consecutive three-month periods, or stages. This permits liquidation of futures positions upon purchase or sale of the respective cash commodities and allows calculation of new optimal hedges for the remaining cash positions.

The first, or planning, stage covers the period from three months prior to the start of feeding up to the point at which feeding actually begins. During this time anticipatory hedges could be placed simultaneously on all four commodities - feeder cattle, the first lot of corn (corn A), the second lot of corn (corn B), and live cattle. When feeding begins, all 490 head of cash feeder cattle and half, or 15,000 bushels, of the necessary cash corn are purchased, and any feeder cattle and corn A hedges are lifted. Hedges on the remaining 15,000 bushels of corn and on the 490 head of live cattle are replaced by revised optimal hedges for the start of the second, or initial feeding, stage.

After three months of feeding, the remaining 15,000 bushels of corn are purchased and any corn B hedges are lifted. The hedge on the 490 head of live cattle is replaced by a revised optimal hedge for the start of the third, or final feeding, stage. Three months later, at the end of stage three, the cash live cattle are marketed and any futures position is liquidated.

This procedure is repeated for a total of 76 pens of cattle, with a new pen entering stage 1 each month. To illustrate, pen 1 enters stage 1 in March 1975 and leaves it in June 1975, when it begins stage 2. It completes stage 2 and moves into stage 3 in September 1975, finishing it in December 1975. Pen 2 enters stage 1 in April 1975 and leaves it in July 1975, when it begins stage 2. It completes stage 2 and moves into stage 3 in October 1975, finishing it in January 1976. All of the other pens follow this pattern, with each pen lagging by one month the pen before it. This sequence ends with pen 76, which enters stage 1 in June 1981 and leaves it in September 1981 when it begins stage 2. It completes stage 2 and moves into stage 3 in December 1981, finishing it in March 1982.

Collection of Price Data

Representative prices were collected for each cash commodity and futures contract from the first full business week (Monday-Friday) of each month, from January 1954 to March 1982. The weekly average of daily price quotations for USDA choice (USDA medium frame #1 beginning December 1979), 600-700 pound feeder steers at Sioux City, and for USDA choice, yield grade 2-4, 900-1100 pound slaughter steers at Omaha, were gathered from Livestock, Meat, Wool Market News. The midpoint of Thursday bids for US #2 yellow corn at Chicago were collected from Grain Market News. The three markets - Sioux City, Omaha, and Chicago - were the most consistently active markets for their respective commodities for the period covered by this study, and serve as proxies for local cash prices. They also are par delivery points for the corresponding futures contracts. In addition, the specifications for each cash commodity match those for the corresponding futures contract, so no spatial or quality differentials between cash and futures prices should exist.

Futures prices were recorded for corresponding dates for the contract expiring in the month in which a purchase or sale of the corresponding cash commodity would occur. If no contract was deliverable in that month, the next available (nearby) contract was used. Weekly average settlement prices were calculated for the appropriate dates for feeder cattle and live cattle futures contracts, using daily settlement prices for the first full business week of the month published in the Chicago Mercantile Exchange Yearbook. Thursday settlement prices for corn in the first full business week of the month for the appropriate dates and contracts were gathered from the Chicago Board of Trade Statistical Annual.

Since corn is a storable commodity, any two cash prices within a given crop year are linked by the cost of storage. This means that when cash corn prices for two different points in time are compared, the difference must be reduced by the total storage cost to find the net gain or loss over that period.

Holding a position in cash corn spanning two crop years also complicates matters because cash prices are highest just prior to harvest and lowest just after harvest. To overcome both of these problems, a futures price adjusted for storage charges was used to represent the expected net cash price three months in the future. This expected price can be compared with the actual cash price three months later to determine the net holding period gain or loss.

Calculation of Returns and Input Statistics

After all prices were collected, and adjusted in the case of cash corn, three-month holding period rates of return to a long position were calculated with the following formula:

$$R_i = \frac{P_{i,t+3}}{P_{i,t}} - 1 \tag{2}$$

where: R_i = holding period rate of return on the ith investment,

$P_{i,t+3}$ = price of the ith investment at the end of the three-month holding period, and

$P_{i,t}$ = price of the ith investment at the beginning of the three-month holding period.

A three-month holding period is used because it corresponds to the three-month duration of each stage in the feeding process.

The expected cash corn prices discussed above would be represented by $P_{i,t}$, while the actual cash corn prices would be represented by $P_{i,t+3}$.

Means, variances, and covariances of holding period rates of return are the traditional input statistics for portfolio analysis under the Markowitz model (expression 1) used in this study. For this feedlot the relevant historical period for planning purposes is probably no greater than twelve months, so means, variances, and covariances were calculated using the holding period rates of return for the 12 most recent overlapping three-month holding periods. This corrects for any seasonal effects so that any temporal price differences have been eliminated, in addition to the spatial and quality differences discussed above. These statistics serve as forecasts of the expected levels and relationships of returns during the subsequent three-month hedging period and are the only elements of forecasting used, since inclusion of formal forecasts exceeds the scope of this study.

Nonlinear Optimization

The objective function used in this study (expression 1) can best be solved via quadratic programming. One problem with quadratic programming is that the solutions are continuous, while futures contracts are indivisible, discrete investments.

The use of rounded-off continuous solutions as approximations of discrete solutions may be suboptimal, and the implications of this problem have been noted by Heifner (1966); Baum, Carlson and Jucker (1978); Robison and Barry (1980); Rausser (1980); and Anderson and Danthine (1981). To overcome these problems, this study used the discrete modified complex (DMC) algorithm developed by Fox and Liebman (1981).

To determine whether the use of rounded-off continuous solutions to approximate discrete solutions results in suboptimal solutions, a second version of the optimal hedge uses small increments, or pseudointervals, of .01 contract with the DMC routine to approximate a continuous solution for the optimal hedge. Full-covariance continuous and full-covariance discrete optimal hedges will be compared to determine the extent of the indivisibility problem. Since the set of subprograms used for the continuous optimal hedge is identical to the one used for the full-covariance discrete optimal hedge, any differences in the solutions are due solely to the use of the pseudointerval and therefore a direct comparison is possible.

Calculation of Percentages

Positions in futures feeder cattle, futures corn A, futures corn B, and cash live cattle are long, while positions in cash feeder cattle, cash corn A, cash corn B, and futures live cattle are short. Using the convention of positive signs for long positions and negative signs for short positions means that the quantities of futures feeder cattle, futures corn A, futures corn B, and cash live cattle all are nonnegative, while the quantities of cash feeder cattle, cash corn A, cash corn B, and futures live cattle are nonpositive.

Also by convention, market prices for all investments at the time the hedges were placed were used in the calculation of the total portfolio value as well as in the calculation of the percentage of the portfolio value invested in each commodity. A convention suggested by Lintner (1970) was used, so that X_i was defined as the percentage of the total portfolio exposure invested in i without regard to market position. As a result, a long position worth \$1000 and a short position worth \$1000 would have X_i values equal in magnitude but with opposite signs. Then:

$$X_i = \frac{P_i Q_i}{\sum_i \left| P_i Q_i \right|} \tag{3}$$

where: X_i = percentage of total portfolio exposure invested in i,

P_i = market value of investment i, per unit,

Q_i = number of units of investment i,

so that:

$$\sum_i \left| X_i \right| = 1 \,. \tag{4}$$

Upper and Lower Bounds

Upper and lower bounds on the Q_i were set to limit the range of hedging levels between 0% and 100%. This range was selected since few lenders would permit a borrower to hold speculative positions for greater than 100% or for less than 0% of the cash position. Furthermore, relaxing these bounds in several test runs resulted in impractical and/or unbounded solutions.

For feeder cattle, corn A, corn B, and live cattle, upper bounds were set at 7, 3, 3, and 0 contracts, respectively, corresponding to hedging levels of 100%, 100%, 100%, and 0%, respectively. Lower bounds for the same investments were set at 0, 0, 0, and -13 contracts, respectively, corresponding to hedging levels of 0%, 0%, 0%, and 100%, respectively.

Risk Aversion Parameter Values

Two values were selected for λ, the risk aversion parameter. One of these, designated the high level of risk aversion, was set at 1×10^{5} and represents the situation where portfolio risk dominates portfolio return to such a degree that portfolio return plays an insignificant role in determining the optimal hedge. The other value of λ, set at 1×10^{-5} and designated the low level of risk aversion, represents the opposite situation where portfolio return dominates portfolio risk to such a degree that portfolio risk plays an insignificant role in determining the optimal hedge. These two λ values represent the extreme cases of pure risk minimization and pure return maximization, respectively.

Hedging Models

The features discussed above were incorporated into two versions of the optimal hedging problem - discrete and continuous - each at two levels of risk aversion - high and low - for a total of four optimal hedging models that were solved by the DMC algorithm. All of these used a complete variance-covariance matrix with elements to represent all possible interactions between pairs of investment returns. Variance (diagonal) terms represent price risk, and covariance (off-diagonal) terms represent basis risk. Basis risk here refers not only to the variation in returns on a cash commodity with respect to the returns on its corresponding futures contract and vice versa, but also to variation in the return on any investment i with respect to the returns on any other investment j, where $i \neq j$. These hedges also consider the mean returns on each investment in the portfolio, and solve for the number of futures contracts, treating the quantities of the cash commodities as fixed and given.

Another type of hedge was examined which did not use the DMC algorithm, and therefore by definition is not a true optimal hedge. The perfect forecast hedge combines features of the naive hedge and the zero hedge with a perfect forecast of commodity prices. The naive or full hedge is based on the traditional definition of hedging where futures positions are equal in size but opposite to the cash positions [Hieronymus (1977, p. 328)]. Both cash and futures positions are predetermined, with futures always covering 100% of the cash positions. The zero hedge is one in which no futures position is ever held and deals only with cash market positions. Cash and futures positions are predetermined, with futures covering 0% of the cash position. Using predetermined cash positions, the perfect forecast hedge establishes futures positions <u>ex ante</u> based upon <u>ex post</u> information about cash price movements over the three-month duration of the hedge. If the forthcoming price change for a cash commodity is favorable no hedge is placed, and if the forthcoming price change for a cash commodity is unfavorable a naive hedge is placed.

The optimal hedges use improvements in determining the futures positions (diversification) to provide better hedging performance, and employ naive forecasts. In contrast, the perfect forecast hedges use improved forecasts to provide better hedging performance, and employ naive and zero hedges. Comparing the results of the perfect forecast hedges to any of the optimal hedges provides an indication of the relative gains in hedging performance from improvements in forecasting and futures position determination, respectively.

Results

Since each of the five hedging models was used in each of the three stages on each of the 76 feeding periods, or pens, for a total of 1140 hedges, it is not possible to discuss the hedges themselves in detail, although the information is available from the authors. Instead, the optimal and perfect forecast hedges have been classified as naive, partial, or zero hedges as shown in Table 1. Overall, there were only three cases out of 228 (1%) in which the discrete and rounded-off continuous solutions differed, each time by only a single contract. These differences occurred in partial hedges, so the classifications in Table 1 are identical for both discrete and continuous optimal hedges and the results are listed only once for each level of risk aversion in each stage.

Since this paper treats hedging as a type of risk response, a high level of risk aversion would be associated with a high level of hedging, such as the naive hedge. However, for both the discrete and continuous optimal hedges in stage 1 at the high level of risk aversion, only 18 (24%) of the pens had naive hedges on all four commodities. Instead, some combination of full and partial (or no) hedging of the individual commodities was optimum. In any one of the four commodities a majority of the pens had naive hedges. This occurred for feeder cattle in 41 pens (54%), for corn A in 61 pens (80%), for corn B in 67 pens (88%), and for live cattle in 54 pens (71%).

From the line of reasoning used above, a low level of risk aversion would be associated with a low level of hedging, such as the zero hedge. Any use of futures contracts at the low level of risk aversion occurs because of their ability to increase returns rather than their ability to reduce risk. Only three (4%) of the pens had zero hedges on all four commodities. Instead, some combination of full and zero hedging of the individual commodities was generally optimum, and no partial hedging was observed. For any one of the four commodities there were many pens with zero hedges. An unhedged position was optimum for feeder cattle in 37 pens (49%), for corn A in 51 pens (67%), for corn B in 48 pens (63%), and for live cattle in 51 pens (67%). There was also one pen in which all four commodities were fully hedged.

For the stage 1 perfect forecast hedges there were 5 (7%) of the pens with full hedges and 10 (13%) with zero hedges on all four commodities; the remainder of the pens had some combination of full and zero hedges. For individual commodities, full hedges were placed on feeder cattle in 42 pens (55%), on corn A in 28 pens (37%), on corn B in 28 pens (37%), and on live cattle in 38 pens (50%).

In stage 2 at the high level of risk aversion, of the 76 pens, 49 (64%) had naive hedges on both commodities, and all of the pens had naive hedges on at least one of the commodities. Corn B was fully hedged in 68 pens (89%), and live cattle was fully hedged in 57 pens (75%); there were only 4 cases of individual commodities with zero hedges, with the remainder being partial hedges.

At the low level of risk aversion, unhedged positions on both commodities were optimum for 42 (55%) of the pens, and fully hedged positions on both commodities were optimum for 10 pens (13%). The remaining 24 pens (32%) had full hedges on one commodity and zero hedges on the other; no partial hedging occurred. An unhedged position was optimum for corn B in 60 pens (79%) and for live cattle in 48 pens (63%).

Among the perfect forecast hedges, 10 pens (13%) had naive hedges and 21 (28%) had zero hedges on the two commodities, with the remainder having some combination of fully hedged and unhedged positions. For the individual commodities, naive hedges were placed on corn B in 25 pens (33%) and on live cattle in 40 pens (53%).

In stage 3 at the high level of risk aversion, 50 (66%) of the 76 pens had naive hedges, and partial hedging occurred in the remaining 26 pens (34%). At the low level of risk aversion there were 44 pens (58%) which had optimal hedges at the zero level, and the remainder were fully hedged; no partial hedges were placed. Among the perfect forecast hedges, 38 pens (50%) were fully hedged and an equal number of pens were unhedged.

Summary of Optimal Hedges

Using the portfolio theory approach to hedging results in optimal hedges at the high level of risk aversion that often differed from the traditional naive hedge. This is especially true when multiple commodities are involved and diversification is achieved in the cash positions. Likewise, optimal hedges at the low level of risk aversion were generally different from the zero hedge. These differences were more numerous as the number of commodities being hedged increased. Numerous partial hedges occurred at the high level of risk aversion, but none was observed at the low level of risk aversion.

There were only a few minor differences between full-covariance discrete and continuous optimal hedges at the same level of risk aversion, suggesting that the indivisibility problem may not be as severe as one might expect. From the results presented in this paper, it appears that continuous nonlinear programming solutions may be acceptable in similar applications.

Finally, perfect forecast hedges were considerably different from the pure naive and pure zero hedges, suggesting that the use of accurate forecasts can have a substantial effect on the selective use of fully hedged and unhedged positions.

Feeding Margins

To evaluate the risk-return performance of the hedges just presented, actual dollar returns for each cash and futures position for each pen for all three stages were calculated to determine what would have been the outcome had each hedging method been used. The values of all inputs - cash feeder cattle, cash corn A, and cash corn B - were treated as expenses, and were adjusted by gains or losses on the corresponding futures market positions, if any. In a similar manner, the value of the output - cash live cattle - was treated as revenue, and was adjusted by gains or losses on any futures market positions. Combining total net (cash + futures) revenues and total net (cash + futures) expenses for each pen produced the feeding margin, treated here as a gross profit margin. Then, feeding margin means and variances were calculated across all 76 pens for each hedging method.

Means and variances of the feeding margins for all seven hedging methods are listed in Table 2 and are plotted in Figure 1. Since the discrete and continuous optimal hedges at the high level of risk aversion were different in only 3 out of 288 cases, each by only a single contract, then the feeding margin results should also be similar. The continuous version produced slightly lower mean and variance values than the discrete version.

At the low level of risk aversion the discrete and continuous models had identical optimal hedges, and therefore had identical means and variances. These two formulations, which completely ignored risk and sought to maximize returns, succeeded in both objectives. Their means were higher than any other optimal hedging method, and the variances were highest of the seven methods. The zero hedge, which involved no futures positions, lies between the optimal hedges at both levels of risk aversion. As expected, the zero hedging model had both a higher mean and a higher variance than the naive method. Also, optimal hedges at the high level of risk aversion had higher mean feeding margins than did naive hedges, but at a somewhat higher variance level.

The perfect forecast hedge, which placed naive or zero hedges *ex ante* based upon *ex post* information of cash market returns, had the highest returns of any hedging method, but at a lower level of risk than the optimal hedges at the low level of risk aversion. The improvement in hedging performance from forecasting indicates that it can be a valuable tool for the hedger. By combining optimal hedging with accurate forecasting, it may be possible to produce hedges with lower risk and higher returns, and with overall performance superior to that of any hedging method examined in this study.

Conclusions

Treating the hedging process as a special case of the general portfolio problem has been shown to be a valid approach, both as a theoretical device for understanding hedging from the decisionmaker's standpoint and as a practical tool for improving hedging performance. This study combined portfolio theory with a discrete nonlinear optimization algorithm to provide a general framework for solving the optimal hedging problem. Of particular interest is its capacity to simultaneously handle multiple short and long positions in both cash and futures markets with solutions expressed in multiples of full contracts. With these multiple positions, the traditional method of fully hedging all positions is very unlikely to occur. Combinations of partial, full or no hedging are most likely to be optimal.

Since discrete solutions were nearly identical to the corresponding rounded-off continuous solutions, for problems of this type it appears that the need to use a discrete optimization technique is not as critical as one might expect.

Using the feeding margin as a proxy for cattle feeding income, optimal hedging based on naive forecasts provided risk and return levels comparable to those of traditional hedging methods. The use of accurate forecasts with the optimal hedging techniques developed in this paper may provide hedging performance superior to that of hedging methods currently in use.

Table 1. Classification of Optimal Hedges

Stage and Hedging Model	Feeder Cattle	Corn A	Corn B	Live Cattle	All Commodities
STAGE 1, HIGH LEVEL					
Naive hedges	41 (54)*	61 (80)	67 (88)	53 (70)	18 (24)
Partial hedges	18 (24)	3 (4)	1 (1)	23 (30)	0 (0)
Zero hedges	17 (22)	12 (16)	8 (11)	0 (0)	0 (0)
STAGE 1, LOW LEVEL					
Naive hedges	39 (51)	25 (33)	28 (37)	25 (33)	1 (1)
Partial hedges	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Zero hedges	37 (49)	51 (67)	48 (63)	51 (67)	3 (4)
STAGE 1, PERFECT FORECAST					
Naive hedges	42 (55)	28 (37)	28 (37)	38 (50)	6 (8)
Partial hedges	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
Zero hedges	34 (45)	48 (63)	48 (63)	38 (50)	10 (13)
STAGE 2, HIGH LEVEL					
Naive hedges	--	--	68 (89)	57 (75)	49 (64)
Partial hedges	--	--	4 (5)	19 (25)	0 (0)
Zero hedges	--	--	4 (5)	0 (0)	0 (0)
STAGE 2, LOW LEVEL					
Naive hedges	--	--	16 (21)	28 (37)	10 (13)
Partial hedges	--	--	0 (0)	0 (0)	0 (0)
Zero hedges	--	--	60 (79)	48 (63)	42 (55)
STAGE 2, PERFECT FORECAST					
Naive hedges	--	--	25 (33)	40 (53)	10 (13)
Partial hedges	--	--	0 (0)	0 (0)	0 (0)
Zero hedges	--	--	51 (67)	36 (47)	21 (28)
STAGE 3, HIGH LEVEL					
Naive hedges	--	--	--	50 (66)	50 (66)
Partial hedges	--	--	--	26 (34)	26 (34)
Zero hedges	--	--	--	0 (0)	0 (0)
STAGE 3, LOW LEVEL					
Naive hedges	--	--	--	32 (42)	32 (42)
Partial hedges	--	--	--	0 (0)	0 (0)
Zero hedges	--	--	--	44 (58)	44 (58)
STAGE 3, PERFECT FORECAST					
Naive hedges	--	--	--	38 (50)	38 (50)
Partial hedges	--	--	--	0 (0)	0 (0)
Zero hedges	--	--	--	38 (50)	38 (50)

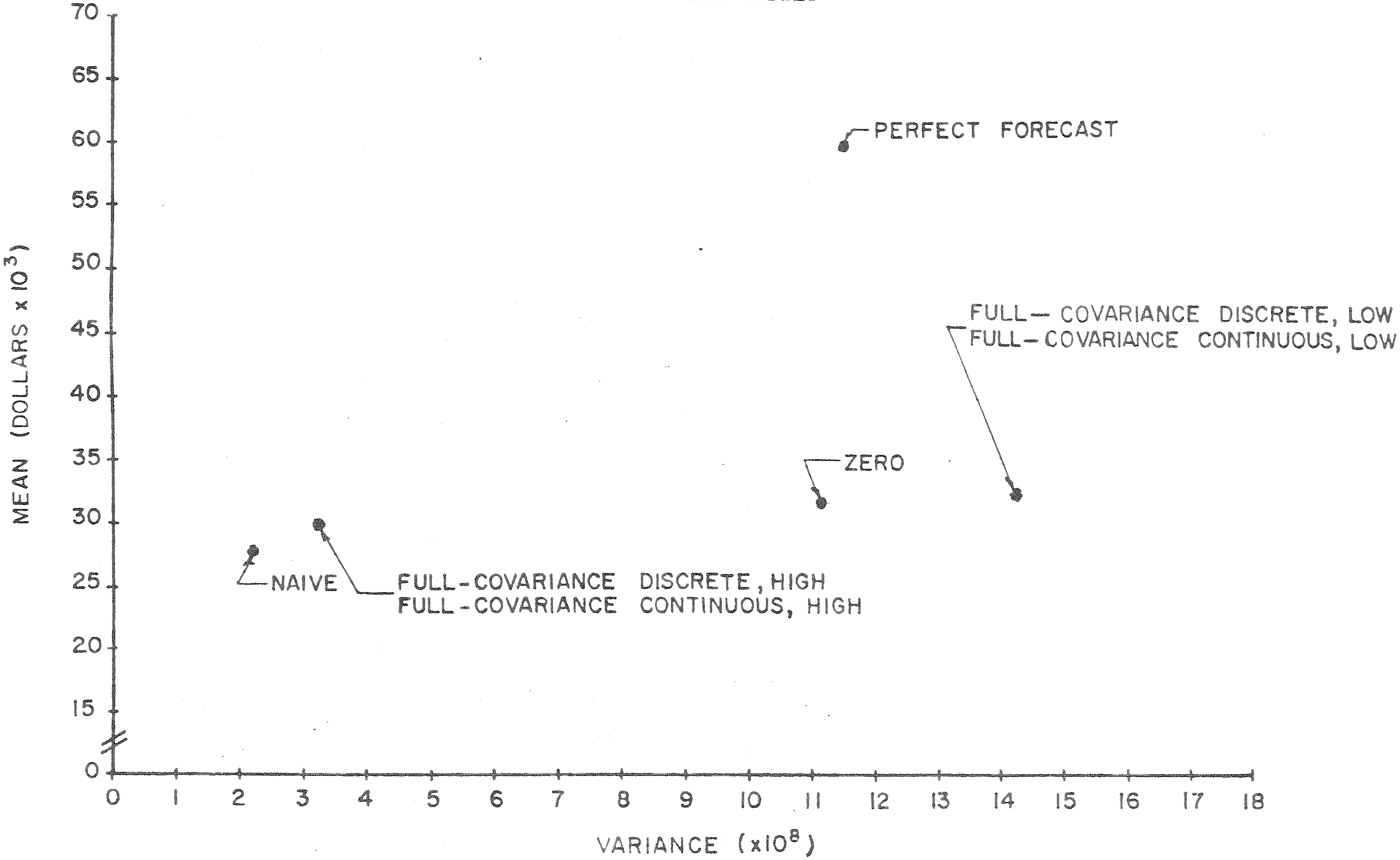
*Number of hedges, with percent of total in parentheses.

Table 2. Means and Variances of Feeding Margins

HEDGING MODEL	MEAN OF FEEDING MARGIN	VARIANCE OF FEEDING MARGIN
Full-covariance discrete High level of risk aversion	$30,333	334,049,626
Full-covariance discrete Low level of risk aversion	33,322	1,423,216,625
Full-covariance continuous High level of risk aversion	30,321	333,533,522
Full-covariance continuous Low level of risk aversion	33,322	1,423,216,625
Perfect forecast	59,537	1,148,428,283
Naive	27,612	222,805,524
Zero	32,140	1,116,173,710

FIGURE 1

MEANS AND VARIANCES OF FEEDING MARGINS
FOR ALL HEDGING MODELS



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