Parametric Disaggregation of a Beef Cattle Model and Application to Regional Supply Response

by

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PARAMETRIC DISAGGREGATION OF A BEEF CATTLE MODEL AND APPLICATION TO REGIONAL SUPPLY RESPONSE

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Disaggregation of national-level econometric commodity models to a regional basis can add interest and immediacy to predictions and analyses based on such models. Since many agricultural commodities are produced, processed, and consumed in a spatial context, and none more so than beef cattle, the ability to focus on regional implications of commodity market predictions and analyses should be of real interest to most clientele groups. This capability is especially valuable for those in research and extension roles in state experiment stations.

Regional disaggregation can also be useful in the validation of national-level models. A tacit assumption in such models is that their structural specification applies to the whole market. However, it is easy to envision a situation in which supply, for example, responds to different sets of variables in different regions, and that the national-level function is specified in terms of national average relationships which have no real applicability to its constituent regions. Disaggregation based on the parameters of the model rather than just proportional allocation of national-level quantities should be able to detect this kind of specification error. If a parametrically disaggregated national model does not seem to conform to its regions, then the assumption of uniformity of structure is suspect.

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Multi-regional analysis is usually done by means of mathematical programming or, when econometric methods are used, by proportional allocation of national-level predicted values among regions (Milne, Adams, and Glickman). The approach suggested here is to disaggregate to regions on a parametric basis so as to develop sets of regional functions which have parameters that are consistent with their national-level counterparts. So far as is known, this procedure has not been used before. Regional functions can be used to disaggregate national model predictions, possibly with more accuracy than proportional methods, and regional issues can be addressed in the context of a model where linkages to the national market are established. How this is done is discussed below and is then applied to the disaggregation of a beef cattle industry model. The paper concludes with a further application of its results to an analysis of the effects of acreage set-asides on regional beef cow inventories.

Disaggregation Procedure

Parametrically disaggregated regional functions are estimated by means of joint least-squares regression with linear, cross-equation constraints patterned on the method suggested by Theil (pp. 42-45). Consider the national-level structural equation

(1) $Y_t = Z_t B + E_t$

and its counterpart in the ith region,

(2) $y_{it} = z_{it} b_i + e_{it}$,

where Y_t, y_{it} = national and regional-level values for an endogenous variable,

Z_t, z_{it} = vectors of national and regional-level explanatory variable consisting of included endogenous, predetermined, and exogenous variables,

B, b_i = vectors of national and regional-level parameters, and

E_t, e_{it} = national and regional-level disturbance terms.

Explanatory variables are the same at both levels, although observed values can be different.

Estimation for N regions requires joint estimation of all regional parameters, subject to the constraint that they aggregrate to their national-level counterparts. The N-equation estimation model is expressed as

(3) $R = Xb + U$,

subject to the constraints

(4) $Wb = B$,

where R = $NT \cdot 1$ observation matrix of y in N regions for T periods

X = $NT \cdot KN$ observation matrix containing the $T \cdot K$ matrices z_i as block diagonals with all other elements being 0,

b = $KN \cdot 1$ regional parameter matrix,

U = $NT \cdot 1$ disturbance matrix,

W = $K \cdot NK$ matrix of parameter weights, and

B = $K \cdot 1$ matrix of national-level parameter estimates for Y.

The constraints in equation (4) are imposed as Lagrangean functions on equation (3) so that the least-squares estimator for b is:

(5) $\hat{b} = (X'X)^{-1} X'R + [(X'X)^{-1}W')] [W(X'X)^{-1}W']^{-1} [B-W(X'X)^{-1}X'R]$

Obviously, prior estimation of B at the national level is required to do this. Consistency of estimation at the national level will be transmitted to the regional parameters by means of the aggregation constraints. Thus, if B_k is a consistent estimate of a simultaneous relationship between two endogenous variables, the parameters b_{ik} will also be consistent by virtue of the requirement that they aggregate to B_k. Consistent estimates can be obtained from disaggregation of one structural equation at a time, even though systems of equations methods may have to be employed at the national level.

The weighting process designated by matrix W is a "black box" which deserves further attention. Types of weights used depend on the nature of the explanatory variables. For variables such as price, which are not themselves disaggregated when measured at regional levels, the aggregation constraint requires that

(6) $\sum b_{ik} = B_k$

so that the weights are simply 1's. For a two region model with a price parameter $B_k = 400$, $b_{1k} = 150$ and $b_{2k} = 250$ would be a feasible solution. Spatial price differentials are permissable in the regional data.

Variables which are disaggregated at the regional level require the following constraints:

(7) $\sum W_{ik}b_{ik} = B_k$, $\sum W_{ik} = 1$

In this case, the national level parameter is treated as a weighted average of the regional parameters, with the weights being regional mean proportions or simply 1/N. If the explanatory variable is, say, cropland acreage, and the national-level parameter is 20, then the solution $b_{1k} = 15$ ($W_1 = 2/3$), $b_{2k} = 30$ ($W_2 = 1/3$) would be feasible. Of course, there is an unbounded number of feasible solutions, but the additional criterion of least-squares yields a unique set of regional parameters that meet the aggregation constraints.

Limits on Applicability

Application of parameteric disaggregation requires observations of requisite data by regions (or by states, which can then be aggregated to desired regions). Unfortunately, such data are not available in many instances. For example, the beef demand function in this study can not be disaggregated for lack of state or regional consumption data. In some cases, regional data exist but can not be linked to the model for lack of transshipment data. Cattle and calf slaughter data are available by states, and so can be aggregated to regional configurations, but supporting data on interregional shipments of feeder cattle to feeding areas would be required to tie regional slaughter functions into the model.

These data limitations restrict application of parametric disaggregation to those functions where data are available and where interregional movements are minimal. Fortunately, in the case of the beef cattle industry, some very important functional relationships qualify, namely those for beef cow inventory investments, for dairy cow inventories (omitted here), for the annual calf crop, and, with somewhat less assurance, for beef heifer inventories.

Choice of Regions

Choice of regions is based primarily on beef cattle production characteristics and secondarily on the desire to limit the number of regions because of lack of experience with parametric disaggregation. Six regions have been chosen. These are shown in Figure 1. Except for the Great Plains region, these regions are the same as, or aggregates of, farming regions designated by the USDA (1981, p. 474).

Regional Function Estimates

Table 1 presents the parametric regional disaggregation of a national-level beef cow inventory function, the parameters for which are also shown in the table. Variable identifications are given in the Appendix. National-level parameter estimates have been estimated by a systems of equations method developed by Dhrymes and Taylor, and independently by Hatanaka, to obtain consistent estimates for systems of dynamic, simultaneous equations with autoregressive disturbances. Data for the regional explanatory variables were aggregated from state and sub-regional data except for the feed price index (Z1), which was approximated from regional corn and protein supplement prices. Autoregressive parameters are fixed at the national level and used in conjunction with proportional allocations of disturbances among regions. This procedure is followed for the other functions reported here. Standard errors for regional parameter estimates are computable, but since aggregation constraints emanating from a non-least-squares procedure are imposed, it is not clear what the sampling distributions for these parameters are. Standard errors are not shown for this

Figure 1. Beef Cattle Industry Model Regions

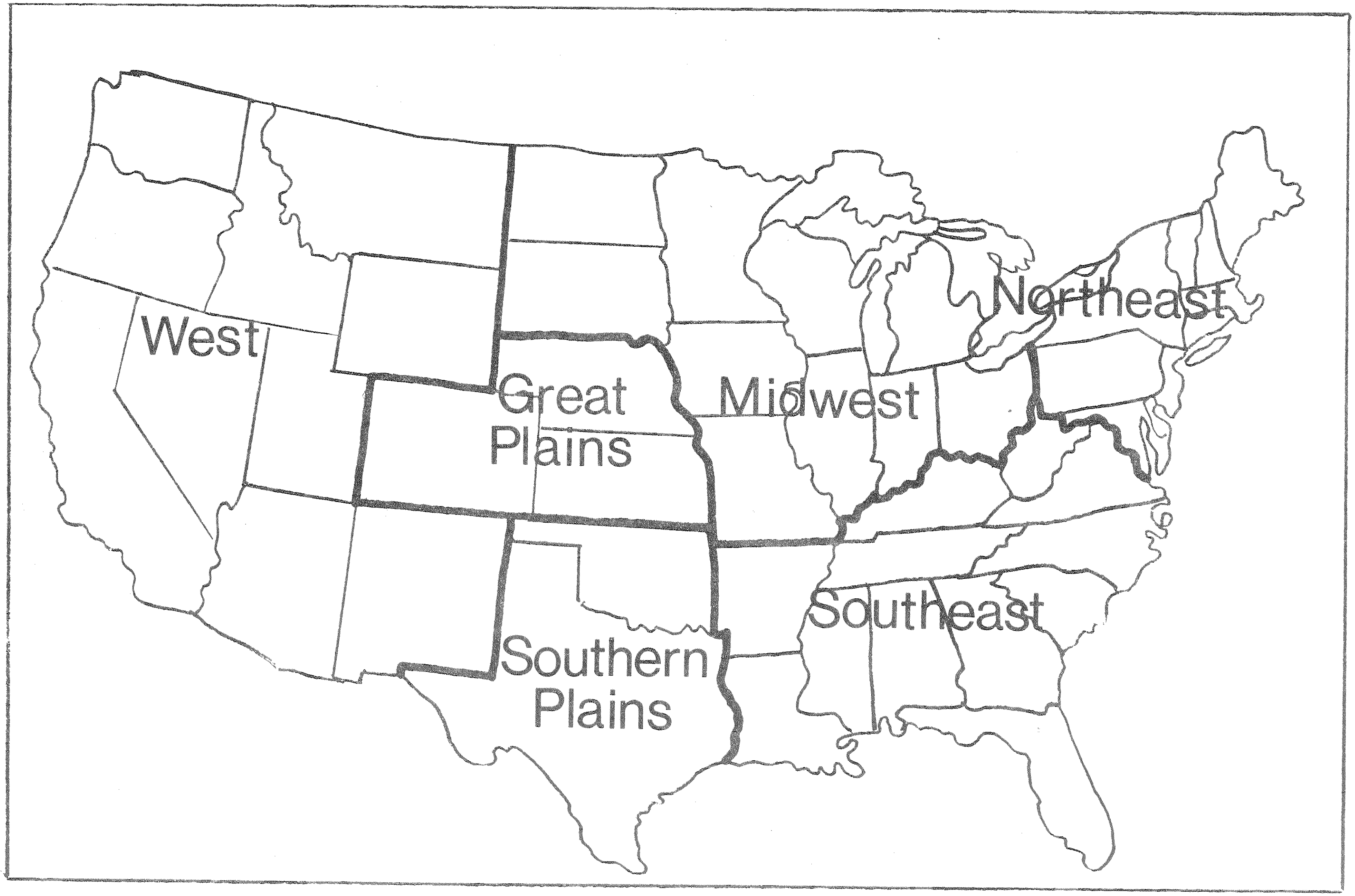

Table 1. U.S. and Regional Beef Cow Inventory Investment Function Estimates, 1954-80

Parameter[a]	U.S.	Region					
		Southeast	Southern Plains	Great Plains	Northeast	Midwest	West
Constant	1782.56	-181.725	1630.62	1388.19	-717.907	1016.06	-1260.36
$Y4_t$	235.952	24.8806	21.7574	41.9518	28.1109	43.9288	75.3225
$Y1_{t-1}+Y2_{t-1}$	0.804075	0.809865	0.735441	0.746848	0.433924	0.827468	0.904651
$Y4_{t-1}$	199.524	70.9732	48.4681	21.2158	-1.60179	53.2846	7.25443
$Z1_t$	-16.5593	0.807586	-4.00191	-7.35477	1.77560	-2.81137	-4.97444
$Z2_t$	1561.39	-36.1719	-60.1434	-44.5006	9.4348	-26.0388	-24.1431
$Z3_t$	-0.34521	27.9659	408.736	329.824	24.7261	255.710	514.428
ρ[b]	-0.34521	-0.34521	-0.34521	-0.34521	-0.34521	-0.34521	-0.34521
RMSPE[c]	2.322	4.694	4.169	3.600	24.050	3.667	3.372
RMSPE, Proportional Allocation	-----	6.015	3.851	6.534	19.302	5.518	9.665

a/ See the Appendix for variable identification.

b/ Parameter held uniform among regions.

c/ RMSPE = root mean square percentage error for parameter-based predictions.

reason. Rather than testing individual parameters, evaluations of the goodness of fit of the entire functions are made using the root mean square percentage error (RMSPE) statistics in Table 1.

These tests are made to indicate how well the structure fits individual regions. If a region shares the national model's structure, but its parameters differ in value, then the RMSPE from the parameterized function should be less than the corresponding RMSPE of a proportional allocation from the national model. This is the case for beef cow inventories in the Southeast, the Great Plains, the Midwest, and the West. In the Southern Plains, the RMSPE's are virtually equal, indicating no essential difference (except for scale) between the regional and the national functions. Thus, the national model structure appears to be compatible in these five regions.

In the Northeast, however, structural incompatibility is indicated by the RMSPE comparison. The region's beef cow inventory investment function evidently responds to variables other than those specified in the national model, so that parametric disaggregation provides a poorer fit than proportional allocation. Neither fit well by comparison to other regions' results. Since the Northeast has so few beef cattle, its structural incompatibility is of little practical concern so far as prediction and analysis for the industry is concerned. For this reason, no action is taken to determine what variables should be added to the national model to make it compatible in the Northeast. In other circumstances, however, such mismatches could compel respecification. In this sense, parametric disaggregation provides a stern test of the validity of a national model's specification and can contribute to its improvement.

One sign reversal occurs among the other five regions' parameters. The Southeast's feed price parameter is a small, positive, number, indicating an apparent lack of importance of feed prices on beef cow inventories in that region. All other parameters for these regions conform to the signs of their national-level counterparts.

Parametrized beef heifer inventory functions for the six regions are presented in Table 2. Comparison of RMSPE's reach much the same conclusions about structural coverage as do the beef cow functions. They confirm the structure for five of the six regions, but the Northeast's structure evidently is different from the national model. Of course, it follows that if the beef cow inventory structure is different, the heifer inventory structure will be too.

Parameters for current and lagged beef cow inventories and beef cattle prices in the other five regions reflect problems caused by the composite nature of the dependent variable and interregional shipments. Both replacement and feeder heifers are included in the variable, the latter being widely transshipped among regions. Shipments are reflected in these parameters, which are likely to be stable only so long as shipment patterns are.

Despite these problems, regional beef heifer functions seem to provide sufficient additional information to warrant their use. The pattern of signs of the feed price (Z1) parameters is evidence of this. The positive national-level feed price coefficient is interpreted as reflecting the stretching-out of total feeding periods (between weaning

Table 2. U.S. and Regional Beef Heifer Inventory Investment Function Estimates, 1954-80

Parameter[a]	U.S.	Region					
		Southeast Plains	Southern Plains	Great	Northeast	Midwest	West
Constant	267.156	-419.94	-1691.25	1294.78	309.159	1818.86	-1040.21
$Y1_t$	0.183834	0.359588	0.071617	-0.033491	-0.216789	0.352151	0.074522
$Y4_t$	49.6259	-1.35607	19.1072	9.62190	-8.78727	12.1779	18.8621
$Y1_{t-1}$	0.125536	-0.120289	0.237897	0.397158	0.06064	-0.015183	0.2443
$Y4_{t-1}$	-49.5513	-9.19242	16.9927	-7.30614	-1.50657	-44.9920	-3.54678
$Z1_t$	11.3509	5.86270	6.53777	-5.30366	1.72078	-2.28087	4.81421
$Z3_t$	-2931.97	-184.053	-518.502	-854.911	-218.469	-872.724	-283.312
ρ[b]	0.194126	0.194126	0.194126	0.194126	0.194126	0.194126	0.194126
RMSPE[c]	5.013	5.173	7.477	9.853	39.986	5.852	5.037
RMSPE, Proportional Allocation	-----	9.515	17.605	11.824	31.129	6.915	14.774

a/ See the Appendix for variable identifications.

b/ Parameter held uniform among regions.

c/ RMSPE = root mean square percentage error for parameter-based predictions.

and slaughter) due to heifers being kept on pasture and other lower cost feedstuffs when feed prices increase. Total feeding periods stretch out as rates of gain decrease. The regional parameters support this interpretation. FIS slaughter data by region and class (USDA, 1982, p. 75) indicate that 70% of heifer slaughter takes place in FIS regions corresponding to the Great Plains and Midwest regions of this study. Presuming that heifers are slaughtered where they are fed, the negative signs on the feed price variable in these regions reflect demand for placements on feed, and the positive signs in the other regions reflect corresponding rates of change in retention on pasture.

Regional calf crop function estimates are presented in Table 3. Additional restrictions are applied in these functions for lack of data to disaggregate the national calving rate parameter and the national parameter on forage availability. The restrictions take the form of fixing these parameters at their national-level values in all regions. The calving rate is fixed for lack of regional data on biological sources for variation in calving rates. Climate and breed differences are known to affect cow fertility and reproductive efficiency (Minish and Fox, pp. 51-52), and both vary among regions. However, there are no independent data on climate and breed distributions. Using lagged cow numbers as instruments for themselves and for the unobserved region-specific variable leads to an unacceptably large dispersion of calving rates among regions.

The forage parameter is also fixed for lack of data. The USDA forage production index is used at the national level, but it is not available at state or regional levels.

Table 3. U.S. and Regional Calf Crop Function Estimates, 1954-80

Parameter[a/]	U.S.	Region					
		Southeast	Southern Plains	Great Plains	Northeast	Midwest	West
Constant	-4570.55	-972.529	-1260.45	-389.086	-364.507	-1305.50	-277.334
$Y1_{t-1}+Y3_{t-1}$[b/]	0.882018	0.882018	0.882018	0.882018	0.882018	0.882018	0.882018
$Y4_{t-1}$	360.516	80.7079	51.2336	48.4783	35.2943	97.5947	47.2067
$Z1_{t-1}$	-46.3902	-13.9777	-2.84137	-8.18060	-3.69916	-8.77725	-8.91412
$Z4_{t-1}$[b/]	15.6029	15.6029	15.6029	15.6029	15.6029	15.6029	15.6029
ρ[b/]	0.346925	0.346925	0.346925	0.346925	0.346925	0.346925	0.346925
RMSPE[c/]	2.111	4.440	4.516	4.130	4.163	2.659	3.154
RMSPE, Proportional Allocation	-----	5.882	9.353	5.215	19.615	6.114	3.963

a/ See the Appendix for variable identification.

b/ Parameter held uniform among regions.

c/ RMSPE = root mean square percentage error from parameter-based predictions.

No structural mismatches are detected by comparison of the RMSPE's. Even though they are truncated, the parameterized regional functions seem to provide more information than proportional allocations. No sign reversals from the national model parameters occur.

Beef Cow Response to Crop Set-Asides

Evaluation of the impact of crop set-asides on beef cow inventories is a useful and timely application of parametrized regional functions. The competitive relationship between crop acreage is interpreted to be causal, so that price support and acreage control policies which changed crop acreage in the past also affected beef cows. inventories to change. Now the U.S. is embarking on another episode of crop acreage control, and already there is scattered concern about effects on beef cattle production (Farm Journal, 1983 a,b).

Regional functions are particularly useful in addressing this issue, because programs tend to be crop-specific and crops have uneven regional distributions. To handle this, crop-specific acreage changes can be converted to region-specific data which are then used to predict beef cow inventory changes. These can be aggregated to national totals. This approach provides valuable quantitative information, and it should help to promote economic understanding of the ramifications of acreage contral programs.

Effects of long-run acreage cuts in three major crops, cotton, corn, and wheat, are analyzed in this way. Long-run effects are emphasized, because resources obviously can not be immediately redeployed from crops to beef cattle. Regional distributions of crops based on 1978 Census of Agriculture data. Each crop's acreage is ımed to be cut by one million acres. Cuts are distributed portionally according to each crop's distribution and then multiplied regional cow/crop acreage parameters to estimate inventory changes. Table 4 presents these changes for five of the six regions ;ignated. No estimates are made for the Northeast. These estimates ə the direct, but partial, effects of crop acreage cuts. They do not clude secondary effects from feed and beef cattle price changes that ght be induced.

Beef cow inventories are found to be most responsive to cuts in otton acreage, and inventories expand most in the Southern Plains, due ɔ the region's comparatively large substitution coefficient and its nportance in cotton. Corn has the least effect on a per-million cre basis. The West would experience the least change because of its omparatively small substitution coefficient and low crop acreage. This nalysis can, of course, be extended to different sizes of acreage cuts nd to different crops.

Aggregate beef cow inventory is essentially what would have been redicted from the national-level model. The aggregate change is 13,000 head/-3,000,000 acres, or -37.7 thousand head per million acres, hich is very close to the national-level parameter of -37.3 thousand ead per million acres. This consistency is reassuring so far as ccuracy of the regional coefficients is concerned and really should be emanded of the model, given the national importance of these three rops.

Table 4. Changes in Beef Cow Inventories by Region and Crop from One Million Acre Cuts in Cotton, Corn and Wheat.

Region	Cotton	Corn	Wheat	Total Three Crops
	....................1,000 head.................			
Southeast	10	4	1	15
Southern Plains	34	1	11	46
Great Plains	0	5	12	17
Northeast	0	--	--	--
Midwest	0	18	8	26
West	4	0	5	9
Totals, Five Regions[a/]	48	28	37	113

a/ Six regions for cotton.

Regional changes in beef cow numbers imply changes in the regional distribution of feeder cattle and other facets of the industry. Thus, capability to generate region-specific information is valuable for purposes of market prediction, and the commodity-specific information can be useful in policy analysis. In addition, the causal relationships between commodities within regions that it is able to elicit should be of great interest and of real educational value to farmer clientele groups.

Concluding Remarks

Parametrically disaggregated regional functions have several advantages over alternative means of getting regional estimates from a commodity market model. Compared to proportional allocation of national level predictions, they can, as shown here, reduce prediction error by taking regional differences in parameters into account. It is also possible to provide some assessment as to whether the structural relationships embodied in the national model even apply to a region. There is no reason why every region must be uniform in structure, but an assumption of uniformity is implicit in the proportional allocation approach.

Finally, as demonstrated with the analysis of beef cow inventory response to crop acreage changes, it may be possible to take advantage of the consistent aggregation properties of parameterized functions to work directly with region-specific changes and then aggregate back to the national level. This can not be done in any meaningful way with proportional allocations. With them, no additional information is gained beyond what is known from the regional distribution of the change in an exogenous variable. Of course, both approaches are superior to using independently estimated regional functions because of their inherent specification error problems.

Data availability seems to be the biggest obstacle to the application of parameterized disaggregation. Even for an important commodity like beef cattle, the data base only allows the disaggregation of a few functions. Fortunately, these are important ones, but there are other important relationships which can not be touched at present.

REFERENCES

Dhrymes, P.J. and J.B. Taylor. "On an Efficient Two-Step Estimator for Dynamic Simultaneous Equations with Autoregressive Disturbances." International Economic Review. 17(1976): 362-376.

Hatanaka, M. "Several Efficient Two-Step Estimators for the Dynamic Simultaneous Equations Model with Autoregressive Disturbances." Journal of Econometrics. 4(1976): 189-204.

Milne, W.J., F.G. Adams and N.J. Glickman. "A Top-Down Multiregional Model of the U.S. Economy." Modeling the Multiregional Economic System. F.G. Adams and J.J. Glickman, Editors. Lexington, Mass.: Lexington Books, 1980, pp. 133-145.

Minish, G.L. and D.G. Fox. Beef Productional Management. Reston, Va.: Reston Publishing Co., 1982.

"Outlook South." Farm Journal. March, 1983, p. 10.

Theil, H. Principles of Econometrics. New York: John Wiley, 1971.

U.S. Department of Agriculture. Agricultural Statistics 1981. Washington: U.S. Government Printing Office, 1981.

U.S. Department of Agriculture. Livestock and Meat Statistics; Supplement for 1981. Washington: U.S. Government Printing Office, Statistical Bulletin 522, October 1982.

U.S. Department of Commerce, Bureau of the Census. 1978 Census of Agriculture. Volume 1, Part 51. Washington: U.S. Government Printing Office, 1981.

"Will More Farmers Decide to Diversify?" Farm Journal. February, 1983, p. 3 (livestock).

APPENDIX

Variable Definitions.

$Y1_t$: Beef cows and heifers having calved (beef cows 2 years and older before 1970) on December 31, year t, 1,000 head.

$Y2_t$: Beef replacement and "other" (non-dairy) heifers (no replacement category before 1970) on December 31, year t, 1,000 head.

$Y3_{t-1}$: Dairy cows having calved (2 years and older before 1970) on December 31, year t-1, 1,000 head.

$Y4_t$: Price received by farmers for beef cattle, year t, $ per 100 pounds, deflated by the CPI (1967 = 100).

$Y5_t$: Number of calves born in year t, 1,000 head.

$Z1_t$: Index of feed prices paid (1967 = 100), year t, deflated.

$Z2_t$: U.S. crop acreage harvested, year t, adjusted by the USDA index of agricultural productivity, (1967 = 100), 1 million acres.

$Z3_t$: Dummy variable to account for 1970 change in inventory definition (1 prior to 1970, 0 since).

$Z4_t$: Index of forage output, adjusted by the index of agricultural productivity (1967 = 100 for both indexes).