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STRUCTURAL KNOWLEDGE, INFORMATION, AND EXPECTATIONS IN A DYNAMIC COMMODITY MARKET

Greg Hertzler*

The study of market efficiency typically focuses upon two things: information and the formation of expectations. For example, rational expectations by its usual definition deals with asymmetric information held by market participants, and in its alternate definition determines consistent expectations of the future (Jordan). Economic agents are assumed to independently and simultaneously discover the "correct" structural model of the economy as specified by the researcher.

This paper introduces the concept of structural knowledge as a decision-making technology. Just as farmers don't discover new crop varieties, nor ranchers develop methods for embryo transplants, producers must use the available techniques for making decisions. Economists are not viewed as passive observers, but rather shapers of the technology which determines market behavior.

The present structural knowledge of producers is based upon static economic theory and ad hoc extensions that try to deal with time. Although markets are inherently dynamic, a true theory of allocation over time is a recent discovery. In its pure form, dynamic theory cannot predict observed behavior because it describes markets which don't exist. As new structural knowledge is adopted by producers, markets may gradually become more efficient.

To demonstrate these ideas, a capital investment model is constructed and aggregated into a market model. The model describes markets for commodities which require long-term investments such as

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beef, pork, or tree crops. Information about prices and capital stocks is often publicly available and the method of expectation formation is not crucial. Where a specific example is important, reference is made to the beef market which has always been plagued by long-term cycles and more recently by other instabilities. These problems can be eliminated if beef producers adopt a more efficient decision-making technology derived from dynamic economic theory.

A unique feature of the model to be presented treats the structure of the market as a variable. Dynamic stability properties of the market change as the proportion of producers using different decision technologies changes. For example, the beef cycle exists and will continue to exist until a new structural knowledge is adopted. More recent instabilities in the beef market are also related to the current method of decision-making by producers.

The Model

Briefly stated, the decision problem of a representative producer is:

$$(1) \quad \text{Max} \sum_{t=0}^{T-1} r^t(\pi_t - A_t) + r^T V_T$$

subject to

$$S_{t+1} - S_t = I_T - L_t$$

$$I_t \geq 0;\ I_t \leq e\alpha S_{t-k};\ L_t \geq I_{t-d};\ L_t \leq S_t .$$

where π is profits, A is adjustment costs, S is the capital stock, I is investment in capital stock, L is liquidation of capital stock, V is a known terminal condition, r is a discount factor, e is equal to 1/2 for animal and 1 for plant production, α is the reproduction rate, k is the

lag from the time of investment to the time of realized production from that investment, d is the useful life of capital stock, T is the fixed terminal time, and t is a time subscript.

Profits π are defined by

(2) $\pi_t = P_t(\alpha S_{t-k} - I_t + r^d c L_t)$

where P_t is the net current price and $\alpha S_{t-k} - I_t + r^d c L_t$ is the quantity marketed. In beef, for example, αS_{t-k} is the number of offspring produced, I_t is the number of replacement heifers diverted to investment in the breeding herd and not marketed, and $r^d c L_t$ is an allowance for beef cows liquidated. For tree crops, on the other hand, c equals zero because trees cannot be sold in the commodity market.

When producers invest or liquidate, they affect more than just the biological capital stock of breeding herds, or orchards. In beef, for example, investing requires more feedlot and packing plant capacity, and liquidation may mean abandonment of feedlots or a switch to different kinds of packing plants. Rather than specifically model the changes in facilities over time, the adjustment cost method is used as an approximation.

(3) $A_t = 1/2 a_I I_t^2 + 1/2 a_L L_t^2$

where a_I and a_L are coefficients. An assumption in 3 is the facilities necessary to accommodate investment differ from those for liquidation. The quadratic form is used for convenience.

The first constraint in 1 is the equation of motion for biological capital stock. It is the specific inclusion of resource changes over time which distinguishes dynamic theory from static. The capital stock

is called the state variable, and the present value of that capital stock for future production is a dual variable called the costate. A dynamically optimizing producer will look forward into the future using the costate to anticipate the effect past and current decisions will have on the value of future production.

The second constraint in 1' restricts investment to be nonnegative and hence irreversible. The third constraint restricts investment to be less than the biological capital stock available for investment. The fourth constraint requires capital stock at the end of its useful life to be liquidated, and the fifth constraint prevents more liquidation than the available stock. In practice, only the constraint requiring normal liquidation of aging stock is likely to be binding.

Behavorial Equations

After a Langrangian or Hamiltonian is formed, first order conditions can be derived. These plus a price equation describe the behavior of the market. A detailed derivation is found in Hertzler.

Stock equation

The first order condition with respect to the costate variable gives

$$(4) \quad S_{t+1} = S_t + I_t - L_t \ .$$

Price equation

Representative producer decisions are aggregated into a market model by including the price equation,

$$(5) \quad P_t = q(\alpha S_{t-k} - I_t + r^d c L_t) + Z_t$$

where q is the negative slope of price as a function of quantity marketed, and Z contains exogenous demand parameters such as income,

population, habits, and consumption of other foods. Endogenously including demand interactions between different foods would greatly complicate the analysis and detract from the essential points.

Investment equation

The first order condition for investment is

$$(6)\quad I_t = \frac{m}{a_I}\left(-P_t + \frac{\psi_t}{r^t} - \frac{\mu_{t+d}}{r^t}\right)$$

where m is a structural parameter to be discussed which equals one for optimal investment, ψ is the costate, and μ is the Lagrange multiplier for liquidation of older capital stock.

The interpretation of 6 is straight forward. The present value of holding stock for future production is the current value costate, ψ/r. This value of holding stock is compared to the revenue to be obtained by marketing immediately. For beef, producers can either sell heifers immediately, or invest for the future.

Liquidation equation

The first order condition for liquidation is

$$(7)\quad L_t = \frac{m}{a_L}\left(r^d{}_c P_t - \frac{\psi_t}{r^t} + \frac{\mu_t}{r^t}\right) .$$

As with investment, producers making liquidation decisions compare the value of holding stock for the future with the current market value.

Value of holding stock equation

The first order condition with respect to the state variable gives the equation for the costate

$$(8)\quad \frac{\psi_t}{r^t} = r\,\frac{\psi_{t+1}}{r^{t+1}} + r^{1+k} P_{t+1+k}{}^{\alpha} .$$

The change in the value of holding stock equals the discounted marginal value product in the future. Using the transversality condition $\psi_t = 0$, 8 can be solved backwards from the terminal time to give the value of holding stock as the sum of future marginal value products,

$$(9) \quad \frac{\psi_t}{r^t} = \sum_{\tau=t+1}^{T} r^{\tau-t+k} P_{\tau+1+k} \alpha$$

where τ is a time subscript.

Exogenous vs Endogenous Expectations

As specified, the model in equations 4-8 is a two-point boundary value problem. In other words, the optimal values of stock must be solved forward in time from the known initial stock and the costates must be solved backward in time from the known terminal value. The model can't be written in a simple matrix form as can an econometric model.

In Hertzler, the problem was solved by optimal control for the beef market, given expectations only of exogenous variable Z. While this gives the most efficient possible market outcome, it will be many years before producers also have the technology to compute optimal solutions. Instead of solving backward in time for the costates, future endogenous variables can be treated as if they were exogenous and expectations formed. Under the expectations assumption,

$$(9') \quad \frac{\psi_t}{r^t} = \sum_{\tau=t+1}^{T} r^{\tau-t+k} E[P_{\tau+k}] \alpha$$

where E is the expectations operator and ψ is now redefined to be the expected value of holding stock for the future.

If producers simply try to predict future prices by time-series analysis or by listening to market experts who have already done the

extrapolation, expectation are termed exogenous. All future endogenous variables are treated as if they were exogenous. The crucial point for achieving market efficiency is that a great deal may already be known about future prices. Specifically, the level of the current stock and perhaps current investment and liquidation may be published in U.S.D.A. reports or other sources.

Substituting the price equation at all future times into 9' gives

$$(9'') \quad \frac{\psi_t}{r^t} = \sum_{\tau=t+1}^{T} r^{\tau-t+k} E[q\alpha^2 S_\tau] + \sum_{\tau=t+1}^{T} r^{\tau-t+k} E[-qI_{\tau+k} + qr^d cL_{\tau+k} + Z_{\tau+k}]\alpha$$

Note that from the equation of motion on stock, $S_\tau = S_t + I_t - L_t + \Sigma_{s=t+1}^{\tau-1}(I_s - L_s)$. For the situation where current stock, investment, and liquidation are known, these variables can be included endogenously in the expected value of holding stock which becomes

$$(9''') \quad \frac{\psi_t}{r^t} = q\alpha^2 S_{t+1}\left(\sum_{\tau=t+1}^{T} r^{\tau-t+k}\right) + E_t$$

$$= nRq\alpha^2 S_{t+1} + E_t \,.$$

where n is a structural parameter to be discussed, R is the value of the convergent series of discount factors, and E_t is a new variable denoting the expectations at time t of all unknown future variables which are treated as exogenous.

Market Stability

Equations 4, 5, 6, 7, and 9''' can now be combined into matrix form. Assume production lag k=1, then

$$(10)\quad \begin{bmatrix} 1 & . & -1 & 1 & . & . \\ . & 1 & q & -qr^{d}c & . & . \\ . & \frac{m}{a_I} & 1 & . & \frac{-m}{a_I} & . \\ . & \frac{-mr^{d}c}{a_L} & . & 1 & \frac{m}{a_L} & . \\ -nRq\alpha^{2} & . & . & . & 1 & . \\ . & . & . & . & . & 1 \end{bmatrix} \begin{bmatrix} S_{t+1} \\ P_t \\ I_t \\ L_t \\ \psi_t/r^{t} \\ S_{Dt} \end{bmatrix} =$$

$$\begin{bmatrix} 1 & . & . & . & . & . \\ . & . & . & . & . & q\alpha \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & . & . & . & . & . \end{bmatrix} \begin{bmatrix} S_t \\ P_{t-1} \\ I_{t-1} \\ L_{t-1} \\ \psi_{t+1}/r^{t+1} \\ S_{Dt} \end{bmatrix} + \begin{bmatrix} . & . \\ 1 & . \\ . & . \\ . & . \\ . & 1 \\ . & . \end{bmatrix} \begin{bmatrix} Z_t \\ E_t \end{bmatrix}$$

where S_D is a dummy variable to allow stock at time t-1 to be included in the price equation and the Lagrange multipliers have been omitted. The reduced form matrix of coefficients on the lagged endogenous variables determines the stability properties of the model. This matrix is

$$(10')\quad \begin{bmatrix} K_1 & . & . & . & . & K_2 \\ K_3 & . & . & . & . & K_4 \\ K_5 & . & . & . & . & K_6 \\ K_7 & . & . & . & . & K_8 \\ K_9 & . & . & . & . & K_{10} \\ 1 & . & . & . & . & . \end{bmatrix}$$

where the K_1 through K_{10} are constants. The characteristic equation of 10' captures the essence of the market's dynamic stability and is

$$(10'') \qquad \lambda^2 - \lambda K_1 - K_2 = 0$$

with roots

$$(10''') \qquad \lambda_1, \lambda_2 = 1/2(K_1) \pm 1/2(K_1^2 + 4K_2)^{1/2} .$$

If all roots are real and less than unity in absolute value, the market converges toward dynamic equilibrium. Positive roots converge uniformly while negative roots cause oscillating convergence. If any root is real, but greater than unity in absolute value, the market is unstable and diverges from equilibrium. The market is neither convergent nor divergent if the absolute value of the largest real root equals unity.

A cycle is possible only if the roots are a complex conjugate pair when $K_1^2 + 4K_2 < 0$. This requires K_2 to be negative and sufficiently large in magnitude. The stability of a cyclical market depends upon whether the modulus, $(1/4(K_1^2 - 4K_2 - K_1^2))^{1/2} = (-K_2)^{1/2}$ is less than, equal to, or greater than unity.

Constants K_1 and K_2 are found to be

$$K_1 = \frac{1 - qm(\frac{1}{a_I} + \frac{(r^d c)^2}{a_L})}{1 - qm(\frac{1}{a_I} + \frac{(r^d c)^2}{a_L}) - qmnR\alpha^2(\frac{1}{a_I} + \frac{1}{a_L} - qm(1 - r^d c)^2(\frac{1}{a_I a_L}))}$$

$$K_2 = \frac{- qm\alpha(\frac{b}{a_I} + \frac{r^d c}{a_L})}{1 - qm(\frac{1}{a_I} + \frac{(r^d c)^2}{a_L}) - qmnR\alpha^2(\frac{1}{a_I} + \frac{1}{a_L} - qm(1 - r^d c)^2(\frac{1}{a_I a_L}))}$$

Structural parameters m and n can now be discussed. If all producers have investment and liquidation behavior in 6 and 7 based upon the theoretically correct signs, then parameter m = 1. Suppose, on the other hand, all producers react opposite to the correct sign and m = -1. While it may seem curious to suppose producers buy high and sell low this is precisely hypothesis of previous studies of commodity cycles in agriculture (see for example French and Matthews). Previous research has justified the hypothesis through ad hoc extensions of static theory. Reading between the lines, the real reason for postulating such backward behavior is it predicts cycles and, as will be shown, is the only possible market outcome when expectations of the future are exogenous.

Structural parameter n may or may not be related to m. Whenever expectations are exogenous, n=0, and when they are endogenous, n=1. Three types of producers are defined. Cyclical producers set m=-1 and n=0; countercyclical producers set m=1 and n=0; and efficient producers set m=1 and n=1.

Cyclical producers

Setting m=-1 and n=0 for cyclical producers gives K=1 and $K_2 = q\alpha(1/a_I + r^d c/a_L)/(1 + q(1/a_I + (r^d c)^2/a_L))$. A cycle will exist $K_2 < -1/4$. Normally the denominator of K_2 should be smaller than one because q is negative, but still be positive. Thus $K_2 \leq 0$.

Parameter estimates obtained for the beef market are $r^d c = 1.4983$, $1/a = 0.1526$, $1/a_L = 0.2190$ and $q = -0.9284$. For $m = -1$ and $n = 0$, $K_1 = 1$ and $K_2 = -0.9549$. The market is cyclical with complex roots λ_1, $\lambda_2 = 1/2 \pm 0.8396i$, and convergent because $(-K_2)^{1/2} < 1$. This is a simplified description of the beef market since other lags exist in addition to the production lag. Even so, the basic result remains true that when producers act backwards setting $m = -1$ and do not include current information about stocks, investment, and liquidation endogenously in their expectations by setting $n = 0$, a cyclical but stable market results.

Countercyclical producers

Countercyclical producers are defined as setting $m = 1$ and $n = 0$. Parameter $K_1 = 1$ and $K_2 > 0$ for the beef market because $q < 0$. There is no cycle but unfortunately the market is explosive. The dominant root is $\lambda_1 = 1/2 + 1/2(1 + 4K_2)^{1/2}$ which is greater than unity simply because K_2 is positive. The secondary root, $\lambda_2 = 1/2 - 1/2(1 + 4K_2)^{1/2}$ is smaller in absolute value but negative so the market will diverge in sawtooth fashion.

Common sense would say m should equal one meaning buy low and sell high in investment and liquidation equations 6 and 7. A closer examination using dynamic theory shows such a conclusion to be misleading. So long as expectations are exogenous, the market must be

cyclical to be stable. A market of producers trying to behave countercyclically is pathological.

Efficient producers

Efficient producers have the structural knowledge to use all currently available information. Data on current stock, investment, and liquidation may be available to cyclical and countercyclical producers as well but they haven't the necessary decision-making technology. When m = 1 and n = 1, constant K_2 must be positive since q is negative and all other parameters are positive. A cycle is impossible. Further, K_1 is now less than 1 so the dominant root in 10''' can be less than unity allowing the market to noncyclically converge toward equilibrium.

In the beef market example, $K_1 = (1.5981)/(1.5981 + 0.2604R)$ and $K_2 = (0.3838)/(1.5981 + 0.2604R)$. Parameter R, the sum of discount factors, equals $\Sigma_{\tau=t+1}^{T} r^{\tau-t+k} = (r^2 - r^{T-t+2})/(1-4)$ for $r < 1$. The magnitude of R depends upon discount rate r and the length of the planning horizon T. The discount rate found to be consistent with observations of the beef market is r = 0.9188.

When R = 1.4737, the characteristic roots are $\lambda_1 = 1$ and $\lambda_2 = -0.5968$. For this value of R, the planning horizon must be T = t + 1.8052 years. If producers look farther into the future, T increases causing R to increase and the market to become convergent. In the limit as T approaches infinity, R = 10.3912 and the roots become $\lambda_1 = 0.5372$ and $\lambda_2 = -0.1660$. Forward looking producers who treat expectations of the future as endogenous comprise an efficient market which converges quickly toward equilibrium.

Test of the Efficient Market Hypothesis

The existence of a cycle is sufficient to reject the hypothesis of an efficient market. Figure 1 shows total cattle numbers less beef cows since 1867. Between the years 1934 and 1975 a growth trend and a cycle are evident. When the data for cattle numbers are detrended by subtracting $\exp(3.6885 + 0.02580(t - 1934)$ from totals for each year, a stationary series of deviations from trend remains.

Statistical evidence of a cycle during the years 1934 and 1975 is contained in the power spectrum graphed in Figure 2. In the frequency domain the power spectrum is the total variance attributable to a regular cycle at each frequency. An F statistic is formed by summing the variances at frequencies 3/42, 4/42, and 5/42 (cycle lengths 14, 10 2/5, and 8 2/5 years), dividing by the variance at all other frequencies. The F statistic with 6 and 35 degrees of freedom is $35.8500/4.3456 = 8.2497$. The critical value at the 1% level of significance is 3.37 and the null hypothesis that no cycle existed with a length between 8 2/5 and 14 years is rejected.

Prior to 1965, data reported by the U.S.D.A. lumped beef cows and replacement heifers into the same data series. It was not possible to separate investment from stock. Accurate data on liquidation of cull cows has always been problematical. The theory for markets in which producers know only current stocks or there is significant measurement error in the data on investment and liquidation was not presented in this paper. The basic result of noncyclical convergence toward dynamic equilibrium remains, however.

More recently, the beef market has been plagued by instabilities other than the cycle. In the mid-1970's an extremely severe herd

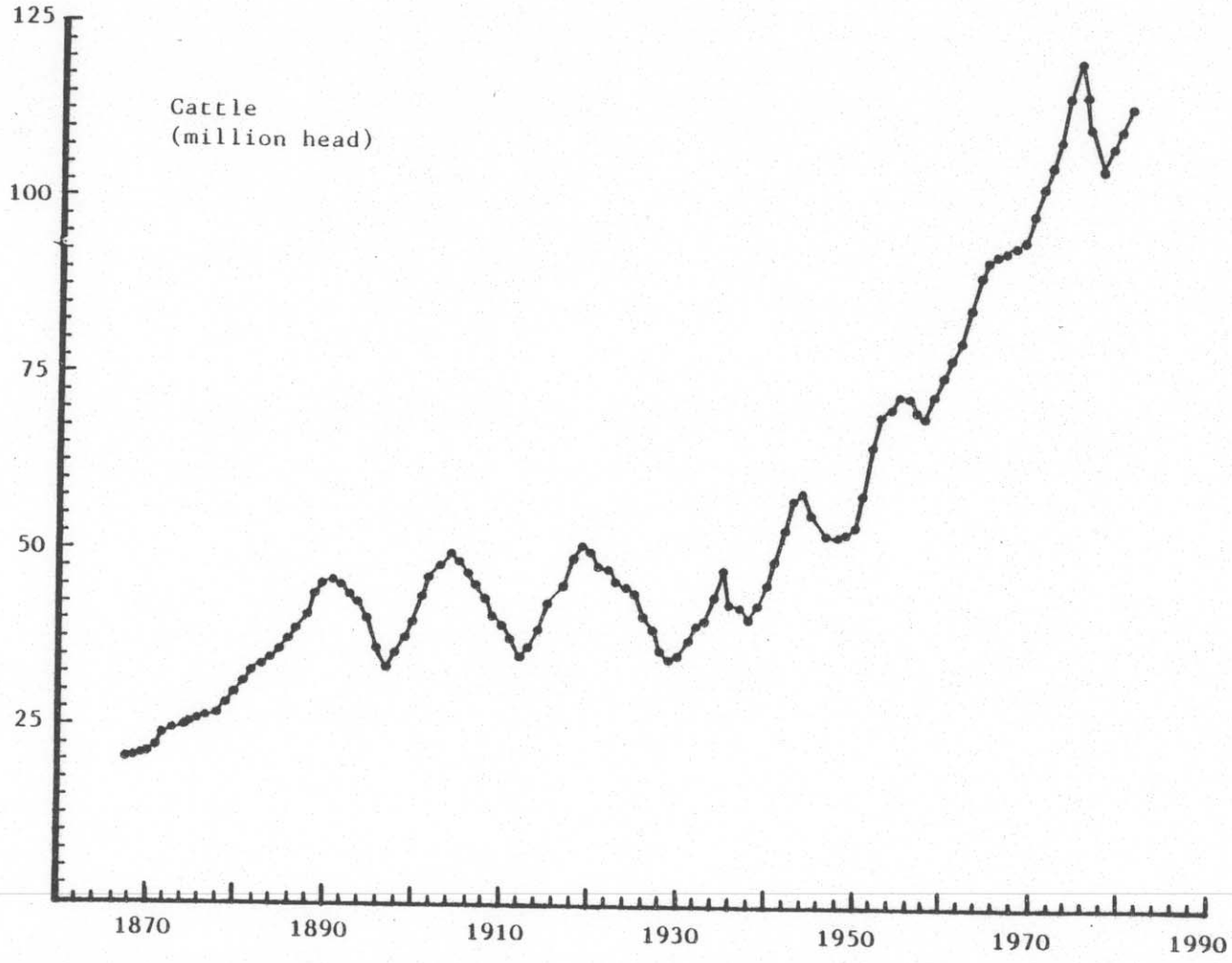

FIGURE 1 : Total Cattle Numbers Less Milk Cows on January 1, 1867 to 1981

Source: U.S.D.C., Historical Abstracts of the United States, and U.S.D.A., Agricultural Statistics

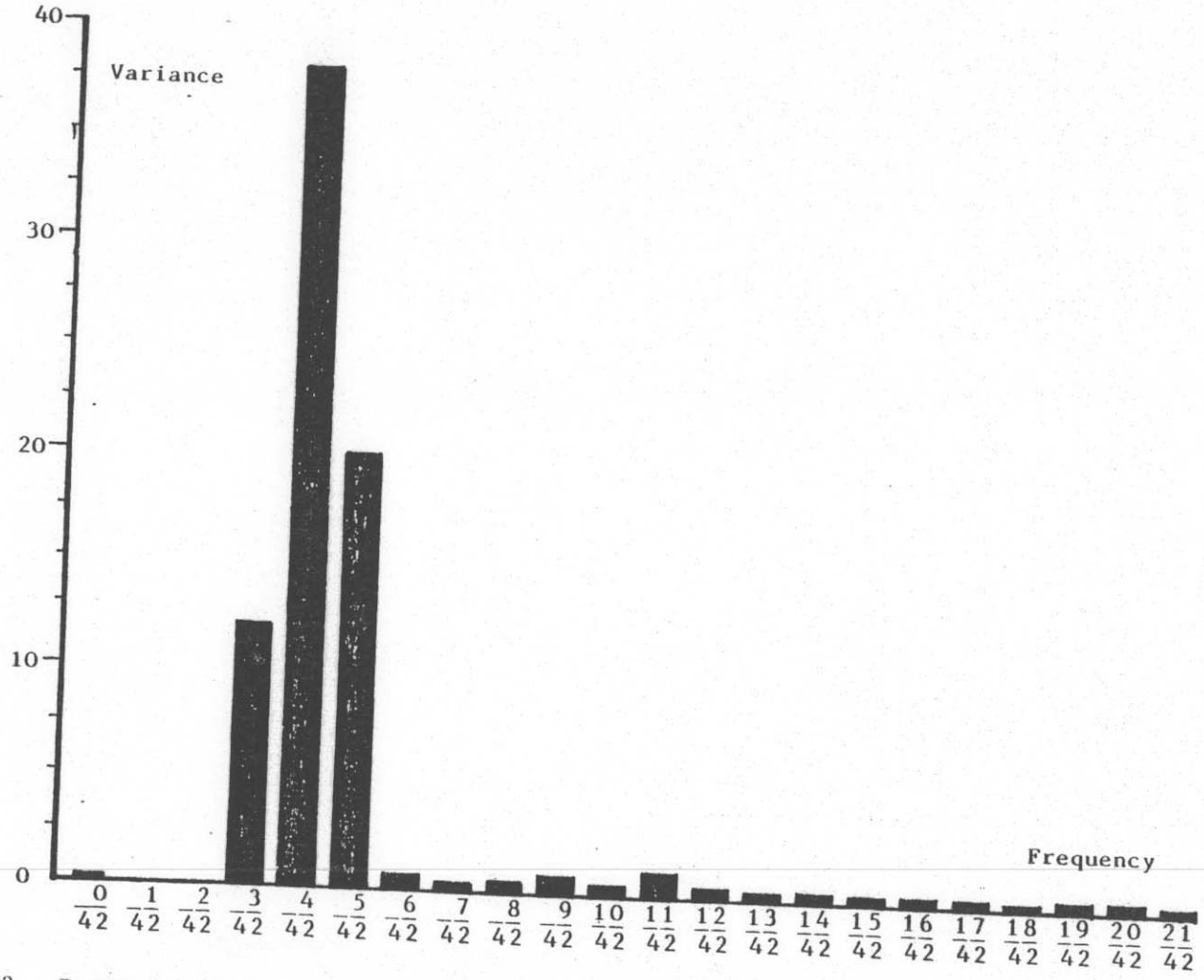

FIGURE 2: Power Spectrum for Deviations in Total Cattle Numbers Less Milk Cows
Source: Table 5-1

liquidation began. The cycle was poised for its normal downturn when consumer demand stopped growing and shocks raised production costs. In the early 1980's, prices were unexpectedly low at the time cyclical highs were predicted. These events are studied in detail in Hertzler. Briefly, the same informational inefficiency which causes the beef cycle leaves the market susceptible to unexpected changes in demand, shocks to production costs, and invasion by outside investors seeking capital gains.

Conclusions

Economists are not passive observers of the market place. Just as engineers and geneticists develop technologies for agriculture, so do economists. The technologies developed by economists become the structural knowledge about the market which producers use to make decisions. This paper presents a new method of decision making based upon dynamic economic theory, contrasts it with current structural knowledge, and analyzes the effect the new decision technology would have upon the efficiency of markets in converging toward dynamic equilibrium.

An investment model for commodities which are produced using biological capital stock is constructed and three types of production are identified: cyclical, countercyclical, and efficient. A unique aspect of the model is the treatment of market structure as a variable, depending upon which decision making technology is adopted by producers.

The key feature distinguishing cyclical and countercyclical markets from an efficient market is whether producers can effectively use non-price information. Cyclical and countercyclical producers

treat everything about the future as exogenous. They may form expectations about future prices through time-series forecasts or the extrapolation of market analysts. Efficient producers, on the other hand, endogenously incorporate data on the current stock, investment, and liquidation into their decisions. They know non-price data conveys a great deal of information about future prices and the value of holding stock for future production.

The persistence of cycles in certain commodity markets has been difficult for neo-classical economists to rationalize. This study shows cycles may exist for a very good reason. A cycle can be the only feasible market alternative when expectations are exogenous. Even though non-price information on stocks, investments, and liquidations may be available, the decision technology of producers doesn't allow such information to be used. When only price information enters into decision, producers in a dynamic market face a dilemna.

In the beef market for example, a high price could either mean a favorable demand situation requiring more investment or it could mean supply is down because producers are already investing by holding offspring off the market. Producers with exogenous expectations must assume high prices are a signal for investment. The beef market is cyclical for this reason. What is more, the market will continue to be cyclical, even after the turbulent events of the 1970's and 1980's and without significant shifts in demand. It is not possible to arbitrage the cycle through countercyclical behavior simply because no additional information is incorporated into the market place. A new structural knowledge is needed.

This raises a key question about the role of market analysts. Are they providing the best current price and non-price information, giving guidance on the demand outlook, and, most importantly, instructing producers to endogenously include non-price information in the decision process? Or are they trying to "outguess" the market and build a reputation on being accurate?

The first approach will speed the adoption of new decision making techniques and enhance market efficiency, the second will not. Even the most accurate predictions of future prices do not provide enough information for markets to be efficient. Producers must resolve the dilemna of whether a favorable price is an investment or a liquidation signal. They must begin using dynamic economic theory.

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