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Pricing Options on Hog and Soybean Futures

R. J. Hauser, D. K. Andersen, and S. E. Offutt*

The foundation of contemporary option pricing theory was developed in 1973 by Black and Scholes and by Merton. The well known Black-Scholes option pricing formula, per se, did not represent a "new" solution (Boness, for example, offered the same formula in 1964); however, the arguments used by Black and Scholes to derive the formula provided a theoretical construct which implied that the formula yielded an equilibrium premium under any risk preference structure. In 1976, Black derived an "equilibrium" pricing formula for options on futures.¹ Although this futures options formula was developed in the context of the capital asset pricing model, it is actually just a special case of Merton's continuous proportional dividend model for options on physicals.

In this paper, we will focus on Black's basic option formula but not on the arguments leading to the solution. The following section reviews the formula in an expected value context and identifies assumptions underlying the formula which may not hold for agricultural futures prices. In the next section, the validity of these assumptions with respect to hog and soybean futures is investigated. Based on this investigation, Black's pricing formula is modified and premium estimates from the modified model are compared to the premia generated under traditional assumptions. Concluding remarks are then offered.

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Pricing Options on Futures

Insight into Black's pricing formula can be gained by expressing it in expected value terms rather than in its computational form. (Appendix A contains the Black-Scholes and Black models in computational form.) In an expected value context, Black's formula for call and put premiums can be expressed, respectively, as:

$$(1) \quad C_t = e^{-r(T-t)} \int_X^{\infty} (F_T - X) L'(F_T) dF_T, \text{ and}$$

$$(2) \quad P_t = e^{-r(T-t)} \int_0^X (X - F_T) L'(F_T) dF_T,$$

where C_t is the call premium at time t ; P_t is the put premium; F_T is the futures price at expiration time T ($T > t$); X is the exercise price; r is the rate of return on a risk-free investment under continuous compounding; and L' is a log-normal probability density function. Estimates C_t and P_t are expected option values at expiration discounted at rate r to time t . Under the assumption that F_T is distributed log-normally at expiration, the integral in (1) is an expected value because C_T is $F_T - X$ if $F_T > X$, and C_T is zero if $F_T \leq X$; likewise, P_T is $X - F_T$ if $X > F_T$, and zero if $X \leq F_T$. For more detail on option formulae in this context, see Hauser and Smith.²

The characteristics of the log-normal distribution are clearly of central importance to option pricing. The expected value of F_T is F_t , regardless of whether this actually represents the market's expectation. The dispersion measurement needed for the option formula is an estimate of the variance of $\ln(\frac{F_T}{F_t})$. The estimate of this variance is based on underlying price dynamics which imply that the variance of $\ln(\frac{F_{t+\Delta t}}{F_t})$ is $\sigma^2 \Delta t$, where Δt is change in time and σ^2 (in concept) is the variance when Δt is an instant. Virtually all option pricing models assume that σ^2 is known and

constant over the life of the option contract. Given that σ^2 is constant, then the variance of $\ln(\frac{F_T}{F_t})$ is equal to the variance of $\ln(\frac{F_{t+\Delta t}}{F_t})$, $V(\Delta t)$, multiplied by $(T-t)/\Delta t$. If, for instance, Δt is one day, then $V(1)$ could be estimated by computing the variance of the log-price first differences of closing futures prices. Assuming that $V(1)$ is constant throughout the year then $V(250)$, the annualized variance given 250 trading days per year, is $V(1)$ times 250.³

Of central interest in this paper is whether $V(1)$ for hog and soybean futures is constant throughout the year. Samuelson's time-to-maturity hypothesis (for futures price variance) suggests that price volatility may increase as maturity approaches. Anderson's state-variable hypothesis suggests that volatility may change in response to seasonal factors. If a systematic change in $V(1)$ can be found, then Black's formula should be modified.

The second empirical question (which is certainly related to the first question of constant volatility) concerns the assumption of log-normality. Previous studies (e.g., Houthakker, Mandelbrot, Mann and Heifner, and Stevenson and Bear) have found leptokurtotic distributions for agricultural prices and a common reason offered for leptokurtocity is that a non-constant variance might exist over time.

The empirical analysis below investigates the distribution and variance characteristics of the log-price returns of hog and soybean futures prices. Hogs and soybean futures were chosen for analysis because of the high likelihood that options on these futures will be traded and because we wanted to examine the distributional characteristics of a storable versus a non-storable commodity.

Options on Hog and Soybean Futures

To investigate evidence of log-normality, normality tests were conducted on series of log-price first differences of daily closing prices under the assumption that the differences are independent. The March, July, and November soybean contracts traded at the Chicago Board of Trade during 1960-1983, and the February, June, and October live hog contracts traded at the Chicago Mercantile Exchange during 1970-1984 were examined.⁴ Normality tests were conducted on each contract's yearly distribution of "raw" log-price first differences. In addition, tests were conducted on "standardized" series in which each log-price difference is divided by the estimated standard deviation of the differences for the respective month. This transformation is intended to account for the effects on normality test results of possible nonconstant variance within a year. Three normality tests are used: (a) a kurtosis test, (b) a skewness test, and (c) a test using the Kolmogorov D statistic.

The test results for the 1976-1983 June hog contracts and the 1976-1982 November soybean contracts are summarized in Table 1. The results for these most recent years provide the most relevant perspective on today's price structures. The results shown essentially mirror the general results of the other contracts for the respective commodity. (For further detail on these tests and on models discussed later for soybean futures, see Andersen; see Hahn for more detail on the hog futures tests and models).

Evidence of a log-normal distribution for the hog price series is rather strong. For the raw series, the skewness statistics do not indicate non-normality for any year, and the kurtosis and Kolmogorov D statistics lead to rejection of the normality hypothesis in only two years and one year, respectively. The results for the standardized series are basically the

same as those for the raw series. For soybeans, however, there is strong evidence of non-normal yearly distributions for the raw series but, when standardized, the series usually appear to be distributed normally. This suggests that the soybean series may be distributed normally, but that the distributions change within the year as variance changes.

If variance does change, an important issue in option pricing is whether this change is systematic. We examined this issue by developing explanatory least-squares regression models of $V(1)$, using monthly binary variables, annual binary variables, and time-to-maturity variables as independent variables. For soybeans, the explanatory model specified is similar to Anderson's (p. 14) and can be expressed as:

$$(3) \quad V_{it} = a + b_1 M + b_i S_i + b_t Y_t + e_{it},$$

where V_{it} is the variance of log-price first differences of observations in month i and year t ; M is the number of months until maturity; S_i is a binary variable for month i (seasonality effect, $i=2,3,\dots,12$); Y_t is a binary variable for year t (year effect, $t=61,62,\dots,83$); and e_{it} is the error term under classical assumptions. A major difference between this model and Anderson's is that Anderson used observations across all contracts within one model whereas we run separate regressions for individual contracts because of the intertemporal price dependence of a storable commodity. Another difference is that Anderson's dependent variable is the natural logarithm of the variance of prices.

Regression results are presented in Table 2. Three general conclusions are drawn. First, the time to maturity effect does not seem to be strong; although its coefficient is of expected sign for all three contracts, it is significant at the .05 level for only the November contract.

Table 2. Regression Results for Explanatory Models^a

Variable ^b	March		July		November	
	Coefficient	t-value	Coefficient	t-value	Coefficient	t-value
Intercept	2.634	0.40	3.230	0.27	4.369	0.59
TTM	-0.785	-1.87	-0.429	-0.57	-0.866	-2.04*
S2	-4.452	-1.05	-3.278	-0.37	-3.411	-0.68
S3	-0.814	-0.13	0.454	0.05	-1.690	-0.33
S4	2.242	0.40	0.608	0.07	-2.051	-0.40
S5	1.494	0.28	-2.196	-0.26	-3.629	-0.69
S6	12.347	2.43* ^c	15.962	1.88	7.089	1.31
S7	19.506	3.98**	43.216	3.56**	14.453	2.58*
S8	11.393	2.39*	11.482	1.18	6.366	1.10
S9	4.321	0.93	3.326	0.36	-1.114	-0.21
S10	2.759	0.60	1.627	0.18	-0.800	-0.16
S11	2.244	0.49	1.092	0.12	0.433	0.07
S12	-0.040	-0.23	-1.217	0.14	-0.212	-0.04
Y61	3.398	0.43	4.555	1.09	4.784	0.65
Y62	0.174	0.02	-2.334	-0.18	1.265	0.17
Y63	0.575	0.08	0.596	0.05	7.405	1.00
Y64	8.297	1.08	5.351	0.41	5.821	0.79
Y65	5.725	0.74	8.284	0.64	3.130	0.43
Y66	1.857	0.24	2.149	0.17	8.260	1.12
Y67	5.415	0.70	0.822	0.06	0.848	0.12
Y68	-1.284	-0.17	-2.290	-0.18	0.195	0.03
Y69	-1.698	-0.22	-2.457	-0.19	0.559	0.08
Y70	-0.457	-0.06	-1.080	-0.01	6.147	0.85
Y71	6.382	0.84	0.933	0.07	6.217	0.87
Y72	9.349	0.69	1.269	0.10	5.624	0.78
Y73	9.349	1.23	54.906	4.22**	67.331	9.55**
Y74	78.718	10.52**	60.334	4.72**	41.415	5.88**
Y75	39.225	5.09**	34.497	2.65**	36.382	5.16**
Y76	29.311	3.92**	21.897	1.78	28.544	4.00**
Y77	24.659	3.30**	31.408	2.50*	35.089	3.92**
Y78	29.571	4.03**	30.381	2.47*	17.824	2.50*
Y79	15.604	2.06*	12.048	0.97	15.517	2.14*
Y80	15.729	2.13*	8.505	0.69	20.386	2.89**
Y81	23.268	3.14*	18.804	1.46	17.982	2.55*
Y82	13.047	1.76	3.488	0.28	9.926	1.41
Y83	9.958	1.36	---	---	---	---

^a For the March, July, and November models, R^2 = .59, .34, and .56; Durbin-Watson = 1.00, 1.15, and 1.24; and number of observations = 287, 275, and 281, respectively.

^b TTM = time to maturity in months; S2-S12 are binary variables for February-December, respectively; and Y61-Y83 are binary variables for 1961-1983, respectively.

^c Significance levels for hypothesis of zero parameter are: ** = 1%; * = 5%.

Second, the seasonal effect is largest during June, July, and August, corresponding with the time of year in which supply and demand uncertainty is relatively high. Third, the annual level or year effect can be divided into three general periods: 1960-1972, 1973-1978, and 1979-1983. These period classifications are most appropriate for the July contracts and perhaps reflect uncertainty emanating from the export market.

In contrast, preliminary results show little evidence of systematic variance change for hog futures. Using 1970-1983 prices, the equations using seasonal and/or time-to-maturity variables were characterized by insignificant coefficients and adjusted R^2 's always less than .05. With annual dummy variables, the adjusted R^2 's were always greater than .4 but improved little, (except for the October contract) when seasonal and time-to-maturity variables were added. These results suggest that the level of variance may change markedly from year to year but, within the year, there is little systematic change in the variance.⁵

Thus, when determining volatility estimates for options on hog and soybean futures, the explanatory models indicate that one should focus on the annual level for hogs and both the annual level and within year systematic effects for soybeans. The systematic change in variance can be incorporated into Black's model by replacing the overall variance estimate ($V(1)$ times the number of trading days until expiration based on the assumption that the instantaneous variance is constant) with an overall variance estimate found by summing different $V(1)$ estimates distinguished by time period within the year (see Ingersoll, p. 112). Regardless of whether the variance is constant, it is obvious that the variance is not known for the option contract life. Thus, the option pricing assumption of known variance is always broken in practice. Identifying systematic changes in the variance

is done to improve the overall variance forecast.

To demonstrate the differences in option premium estimates caused by incorporating systematic variance changes, we consider a put option on November soybean futures being priced at the end of April; thus $V(1)$ must be forecasted for the months May-October, inclusively. Three alternative prediction models were examined. The first model (MOD I) is an adaptation of (3) in which individual year-effect dummy variables are dropped so that a year-effect coefficient does not have to be estimated for the forecasting period; one binary variable for the period 1973-1975 is added on the basis of model performance; and the sample variance is lagged one month to represent current year effects. The second model (MOD II) is an autoregressive integrated moving average (ARIMA) model based on past variances specified as $(0,1,3) \times (0,0,1)_7$. The third forecasting model (MOD III) is a naive model which assumes that the current sample variance (April's variance) is the best variance forecast for any future month between April and November.

Variance forecasts for each month within the May-October period, inclusively, are made for 1976-1982, inclusively. The coefficients for MOD I and MOD II are estimated from observations during the period of 1960 to April of the year being considered. Thus, since seven years are considered, seven sets of coefficients for both MOD I and MOD II are estimated. For the naive model, a variance estimate from April's observations is calculated for each year. When using MOD I for forecasting, April's variance is used as the lagged variable to forecast May's variance; the forecasted May variance is then used as the lagged variable to forecast June's variance; and so on.

For the non-constant variance scenarios, it is assumed that variance

changes from month to month are continuous and linear and that a forecast is for mid-month. Linear functions connecting the mid-month forecasts are developed. The overall variance is calculated by integrating each of these functions over the mid-month to mid-month period and then summing the five (May-June, June-July, . . . , September-October) integrals.⁶

Premiums for at-the-money put options (which are the same as at-the-money call options since the options are on futures) are computed using closing futures prices on May 20, or the first subsequent trading day. The premium estimates using the three forecast variances are compared to "actual"⁷ premia generated by using the variances observed during the forecast period. The results are shown in Table 3.

In general, the ARIMA model (MOD II) outperforms the other two forecasting models when comparing the resulting premia to the "actual" premium. In 1976, 1979, and 1980, estimates using ARIMA forecasted variances are much better than estimates from MOD I and MOD III. For 1977, the ARIMA model performs about as well as the naive model. In 1981 and 1982, the MOD I and MOD III models are more accurate than the ARIMA model, but the differences in premiums are not nearly as large as those found in many of the other years. The only year in which the ARIMA model performed poorly was in 1978, when volatility decreased during the summer months relative to critical prior periods.

When the performance of the models as a group is considered, it is seen that they all consistently over or underestimate the premium when compared to that computed using the observed variances. This similarity in forecast behavior is attributable to the importance of April's price variance in each forecasting model. It is the only variable in the naive model and figures

Table 3. Calculated Option Premia Using Actual and Forecasted Variances, in Cents Per Bushel.

Year	Futures Price (\$)	Exercise Price (\$)	Annualized Interest Rate ^a (%)	Method of Variance Estimation			
				Actual	MOD I	MOD II	MOD III
1976	5.53	5.53	6.40	44.0	22.0	43.7	16.9
1977	7.22	7.22	6.53	62.7	41.6	55.4	56.2
1978	6.30	6.30	8.15	31.3	32.4	46.6	37.6
1979	7.28	7.28	10.37	44.4	33.2	43.8	18.6
1980	6.55	6.55	11.10	41.3	30.9	35.3	25.3
1981	7.85	7.85	14.08	34.4	40.0	43.4	40.0
1982	6.81	6.81	10.69	27.1	32.5	34.1	25.5

^a See footnote 8.

prominently in both the structural and the ARIMA models.

Concluding Remarks

The evidence presented above suggests that (a) log-normally distributed price returns assumed in most option pricing formulae is a good approximation for hog futures, (b) soybean futures price returns may also be distributed log-normally, but the distribution may change within the year as variance changes, (c) there is some systematic change in the soybean variance; thus, more "accurate" premiums can usually be found when accounting for this systematic change.

The incorporation of systematic variance changes into option pricing formulae may be very important for options on storable commodities. However, in any year, there are variance changes related to non-systematic events and so not foreseeable from the types of prediction models developed

above. The next step in empirical investigation might be to try to identify those factors within years and look for some connection among them which may be incorporated into forecast models. In this respect, composite models may hold promise and, in practice, traders' familiarity with past supply, demand, and technical conditions as related to price-return volatility may prove useful in modifying variance forecasts.

The importance of the market's anticipation of volatility on option premia is highlighted by the large differences among premium estimates shown in Table 3. From the perspective of agricultural producers and merchandisers using options strictly as a marketing tool, the market's implied volatility forecast will be an important factor in evaluating the cost of reducing price uncertainty. For instance, a producer who is considering the purchase of a put option on soybean futures may expect a higher volatility than that implied by the option premium. If the producer is correct, then the premium value will decrease at a slower rate as time to maturity approaches, *ceteris paribus*, than the rate implied by the market's volatility expectation. This rate of change in premium is an important factor in determining the effective price received for the soybeans since this effective price is determined by subtracting the loss from offsetting put transactions from the largest of either the exercise price or futures price (ignoring basis). Thus, the actual cost of reducing price uncertainty could depend a lot on the ability of the producer or merchandiser to forecast volatility more accurately than the market.

The evaluation of "correct" option prices is important in determining how and why producers and merchandisers should incorporate options into marketing strategies. However, this is only one of many aspects involved in the evaluation of options as a means to price agricultural commodities. To date, the financial economists have provided a wealth of option

pricing research (particularly in theory), but little has been done in the area of using options to price and market goods or services. Development of this theory and empirical research will undoubtedly soon be a product of agricultural economists.

Footnotes

- 1 Black and Scholes' and Black's models provide equilibrium solutions for European options (options which can be exercised only at expiration) but, with the exception of particular types of calls, the formulae do not yield equilibrium solutions for American options (options which can be exercised at any time before or at expiration).
- 2 Two caveats concerning expressions (1) and (2) should be noted. First, the expressions represent Black's futures option model and not Black and Scholes' model for options on physicals. Second, these types of expressions are valid for the most basic option pricing solution; however, there have been numerous extensions of these basic models developed during the last decade which cannot be represented in this manner.
- 3 In practice, $T-t$ is usually based on the number of trading days from time t to time T when calculating the volatility measurement needed in Black's formula; whereas $T-t$ for discounting purposes is based on the number of calendar days.
- 4 Price data were obtained from tapes provided by the Chicago Board of Trade Foundation, the Commodity Futures Trading Commission, MJK Associates, Inc., and Iowa State University. Observations during a contract's expiration month were not used because options on futures will expire during the preceding month.

- 5 There is some evidence that volatility increases as maturity approaches, particularly for the October contract. The time-to-maturity variable is significant for the October contract in models which include the year effect variables and in the univariate model with just time to maturity considered. Additional work is currently being done to identify systematic change for hog futures.
- 6 The use of these functions illustrates the point that, theoretically, the variance changes must be continuous for the model to hold. However, since a linear relationship is assumed, the overall variance found when calculated by using the simple average of the five variance estimates as a constant instantaneous variance is only slightly different than the overall variance found when summing the integrals.
- 7 "Actual" in the sense that each month's variance during the forecasting period is computed and, under the same assumptions of continuity and linearity as stated above, the overall variance is computed.
- 8 For each year during 1976-1980, intermediate credit bank loan rates are used (U.S. Department of Agriculture); rates for U.S. Governmental three month bills (Bureau of Economic Analysis) are used for 1981 and 1982. These rates were chosen because they were on low-risk instruments and readily available. It can be shown that relatively large changes in interest rates, given the range of other parameters used in this example, cause very small changes in option premia.

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Appendix A. Computational Formulae for Option Premia.

A general relationship described by Smith (p. 16) can be easily adapted to fit many option pricing scenarios. Assuming $L'(S^*)$ is a log-normal density function with

$$Q = \begin{cases} \lambda S^* - \gamma X & \text{if } S^* - \psi X > 0 \\ 0 & \text{if } S^* - \psi X < 0 \end{cases}$$

then

$$\begin{aligned} E(Q) &= \int_{\psi X}^{\infty} (\lambda S^* - \gamma X) L'(S^*) dS^* \\ &= e^{\rho T} \lambda S \cdot N \left\{ \frac{\ln(S/X) - \ln \psi + (\rho + V^2/2)T}{V T^{.5}} \right\} \\ &\quad - \gamma X \cdot N \left\{ \frac{\ln(S/X) - \ln \psi + (\rho - V^2/2)T}{V T^{.5}} \right\} \end{aligned}$$

where λ , γ , and ψ are arbitrary parameters; for option pricing, S is the current underlying commodity price, T is time to maturity in years, X is the exercise price, and V^2 is the annualized variance of the log-price return; $S^*/S = e^{\rho T}$; and N is the cumulative standard normal distribution function.

Defining C and P as the call and put premia, respectively, found with Black-Scholes model, C_f and P_f as the call and put premia found with the Black's model, and r as the risk-free interest rate, then:

$$C = E(Q) \text{ with } \lambda = e^{-rT}, \gamma = e^{-rT}, \rho = r, \text{ and } \psi = 1.$$

$$C_f = E(Q) \text{ with } \lambda = e^{-rT}, \gamma = e^{-rT}, \rho = 0, \text{ and } \psi = 1.$$

$$\begin{aligned} P &= \int_0^X (X e^{-rT} - S) L'(S e^{rT}) dS e^{rT}, \\ &= (X e^{-rT} - S) - \int_X^{\infty} (X e^{-rT} - S) L'(S e^{rT}) dS e^{rT}, \end{aligned}$$

$$\text{therefore, } P = (X e^{-rT} - S) - E(Q) \text{ with } \lambda = -e^{-rT}, \gamma = -e^{-rT}, \rho = r, \text{ and } \psi = 1.$$

$$P_f = (X e^{-rT} - S e^{-rT}) - E(Q) \text{ with } \lambda = -e^{-rT}, \gamma = -e^{-rT}, \rho = 0, \text{ and } \psi = 1.$$