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Variance of a Forecast Made with Predicted Values of Exogenous Variables

by

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The textbook formulas for variance of a forecast are inappropriate for many of our forecasts of future events. These formulas assume that there are two sources of error in the forecasts: (a) deviation of the realized value of the error in the equation from its assumed value, which is typically zero, and (b) differences between estimated values of coefficients and their true values. Many of our forecasts of future events contain a third source of error: (c) errors in the predicted values of exogenous variables. This paper applies two formulas for variance of a forecast that account for all three of these sources of error: one formula for an equation estimated by ordinary least squares, and one for an equation estimated by a simultaneous equation method. It also presents formulas for covariances of forecasts. The formulas presented in this paper are derived from Bohrnstedt and Goldberger's formulas for variance and covariance of products.

Variance and Covariance of Products

Bohrnstedt and Goldberger presented general formulas for the variance and covariance of products, and also formulas appropriate for specific joint distributions. Let x and y be two random variables with means E(x) and E(y) and variances V(x) and V(y) and covariance C(x,y). If x and y are stochastically independent, whether normal, or nonnormal,

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their variance is

$$(1)\quad V(xy) = E^2(x)V(y) + E^2(y)V(x) + V(x)V(y)$$

where $E^2(x) = [E(x)]^2$.

If the four variables x, y, u, and v follow a multivariate normal distribution, the covariance between product xy and product uv is

$$(2)\quad C(xy, uv) = E(x)E(u)C(y,v) + E(x)E(v)C(y,u) + E(y)E(u)C(x,v) + E(y)E(v)C(x,u) + C(x,u)C(y,v) + C(x,v)C(y,u)$$

Consistent estimates of V(xy) and C(xy, uv) are obtained by replacing expectations by their consistent estimates. For example

$$\hat{V}(xy) = \bar{x}^2\hat{V}(y) + \bar{y}^2\hat{V}(x) + \hat{V}(x)\hat{V}(y)$$

Textbook Formulas: Known Predictors

Ordinary Least Squares

Let the single-equation model be

$$(3)\quad Y = X\beta + u$$

where u is $N(0, I\sigma^2)$, X is an n by k matrix, and rank of X equals k. Denote the ordinary least squares (OLS) estimator as b: $b = (X'X)^{-1} X'Y$. And let its dispersion matrix (matrix of variances and covariances) be $D(b) = E[(b-\beta)(b-\beta)'] = (X'X)^{-1}\sigma^2$. Let the row vector X_f be a vector of known values of the regressors. Then $Y_f = X_f'\beta + u_f$, and $\hat{Y}_f = X_f'b$ is an unbiased prediction, and the variance of $\hat{Y}_f$ is

(4) $V(\hat{Y}_f) = X_f' D(b) X_f + \sigma^2 = [X_f' (X'X)^{-1} X_f + 1] \sigma^2$

Because u is normal and X_f is fixed, $\hat{Y}_f$ is normally distributed about Y_f. An (1-α) tolerance interval for Y_f is, therefore,

$$X_f'b \pm t_{\alpha/2} S(\hat{Y}_f)$$

where $S(\hat{Y}_f)$ is the square root of $V(\hat{Y}_f)$.

Suppose β_1 and β_2 in $Y_1 = X_1\beta_1 + u_1$ and $Y_2 = X_2\beta_2 + u_2$ have been estimated by OLS and the forecasts $\hat{Y}_{1f} = X_{1f}'b_1$ and $\hat{Y}_{2f} = X_{2f}'b_2$ have been computed. Use k_1 and k_2 to denote the number of elements in β_1 and β_2. If elements of u_1 and u_2 are temporarily independent, but their contemporaneous covariance is $C(u_1, u_2)$, then the covariance between b_1 and b_2 is

$$D(b_1, b_2) = (X_1'X_1)^{-1} X_1'X_2(X_2'X_2)^{-1} C(u_1, u_2)$$

and

(5) $C(\hat{Y}_{1f}, \hat{Y}_{2f}) = X_{1f}' D(b_1, b_2) X_{2f} + C(u_1, u_2)$

If u_1 and u_2 are independent, then $C(u_1, u_2) = 0$, $D(b_1, b_2)$ is a null matrix, and the covariance between the forecasts is zero.

Simultaneous Equations

Use Y(t) and X(t) to represent a G-element column vector of values of G endogenous variables at time t and a K-element column vector of exogenous variables at time t. And let β and α be G by G and G by K matrices of coefficients. Write a set of structural equations as

$$\beta Y(t) = \alpha X(t) + u(t).$$

Write the estimated equations as $BY(t) = AX(t) + \hat{u}(t)$. Then the derived reduced form equations are $Y(t) = B^{-1}AX(t) + B^{-1}\hat{u}(t) = PX(t) + v(t)$. Write the reduced form equation for the r-th endogenous variable as $Y_{rt} = \Pi_r X_r(t) + v_{rt}$ and its derived estimate as $\hat{Y}_{rt} = P_r X_r(t)$. The variance of $\hat{Y}_{rf}$ is

$$(6) \quad V(\hat{Y}_{rf}) = X'(f)D(P_r)X(f) + V(v_r)$$

where $D(P_r)$ is the dispersion matrix of P_r. If $\hat{Y}_{sf}$ is also predicted, the covariance is

$$(7) \quad C(\hat{Y}_{rf}, \hat{Y}_{sf}) = X'(f)D(P_r,P_s)X(f) + C(v_r, v_s)$$

$D(P_r,P_s)$ is the matrix of covariances between the coefficients in the two reduced form equations.

Stochastic Predictors

Ordinary Least Squares

Now assume that some or all elements of X_f must be predicted. Let $\hat{X}_f$ be an unbiased and normal (or at least consistent and asymptotically normal) estimator of X_f with k by k dispersion matrix $D(\hat{X}_f) = E(\hat{X}_f - X_f)(\hat{X}_f - X_f)'$. And assume that the estimates of β and X_f are independently distributed.[1/] Now $\hat{Y}_f = \hat{X}_f'b$ and by using equations (1) and (2), we can express the variance of the forecast as

$$(8) \quad V(\hat{Y}_f) = \hat{X}_f'D(b)\hat{X}_f + \sigma^2 + b'D(\hat{X}_f)b + \text{tr}\,[D(b)D(\hat{X}_f)]$$
$$= (\hat{X}_f'(X'X)^{-1}\hat{X}_f + 1)\,\sigma^2 + b'D(\hat{X}_f)b + \text{tr}\,[D(b)D(\hat{X}_f)]$$

where tr [A] is the trace of matrix [A]. The right-hand side (RHS) of equation (8) equals the RHS of (3) plus two terms, both containing $D(\hat{X}_f)$.

Suppose that a forecast of farm marketings is to be used in forecasting farm price from the equation $P_t = \sum_{i=1}^{k} \beta_i X_{it} + u_t = \sum_{i=1}^{k-1} \beta_i X_{it} + \beta_k q_t + u_t$. Assume values of X_{if} are known but q_f must be predicted. Then $\hat{P}_f = \Sigma b_i X_{if} + b_k \hat{q}_f$. $D(\hat{X}_f)$ is a k by k matrix having $V(\hat{q}_f)$ as its k-th diagonal element and having zeroes elsewhere. Then all elements of $\hat{X}_f$ except $\hat{q}_f$ are known, and

$$V(\hat{p}_f) = (\hat{X}_f'(X'X)^{-1} \hat{X}_f + 1) \sigma^2 + (b_k^2 + V(b_k)) V(\hat{q}_f).$$

If q_f is known,

$$V(\hat{p}_f) = (X_f'(X'X)^{-1} X_f + 1) \sigma^2.$$

The variance of the price forecast obtained by using the predicted value of q_f exceeds the variance of the forecast obtained by using an equal, but known, value of q_f.

If Y_{1f} and Y_{2f} are predicted using forecasted values of predictors, their covariance is

$$(9) \quad C(\hat{Y}_{1f}, \hat{Y}_{2f}) = \hat{X}_{1f}' D(b_1, b_2) \hat{X}_{2f} + b_1' D(\hat{X}_{1f}, \hat{X}_{2f}) b_2 + \text{tr}\,[D(b_1, b_2)\, D(\hat{X}_{1f}, \hat{X}_{2f})] + C(u_1, u_2)$$

Compare (9) with (5).

Simultaneous Equations

If X(f) is not known, the predicted value of the r-th endogenous variable is obtained from $\hat{Y}_r(f) = P_r \hat{X}(f)$. Its variance is

$$(10)\quad V(\hat{Y}_r(f)) = X'(f)D(P_r)X(f) + P_r D(\hat{X}(f))\, P'_r$$

$$+ \text{tr}\,[D(P_r)D(\hat{X}(f)] + V(v_r)$$

Compare (10) with (6). If $Y_s(f)$ is forecast from $\hat{Y}_{sf} = P_s\hat{X}(f)$, the covariance between the two forecasts is

$$(11)\quad C(\hat{Y}_r(f),\, \hat{Y}_s(f)) = X'(f)D(P_r,P_s)X\,(f) + P_r D(\hat{X}(f))\, P'_s$$

$$+ \text{tr}\,[D(P_r,P_s)D(\hat{X}(f)] + C(v_r,v_s)$$

Compare (11) with (7). Feldstein previously derived expressions (8) through (11). The last term in Feldstein's version of (11) is k^2 $C(v_r,v_s)$. I think the k^2 is a mistake.

When predictors are known, the t-distribution can be used to compute confidence intervals for a forecast. When predictors are stochastic, this is not possible because the predicted value of Y_f, being a sum of products of random variables, is not normally distributed. Feldstein suggested using Tchebychev's inequality to determine a tolerance interval.

Implications

Before looking at some empirical applications of (8) through (11), let us look at some of their implications.

(A) When we use predicted values of exogenous variables to predict endogenous variables and use any of equations (4) through (7) to measure reliability, we tend to overstate the reliability of the forecast.

One striking consequence of (8) is that a forecast of Y_f can have a large variance even though our estimate of equation (1) has a value of R^2 equal to one. If $R^2 = 1$, $V(\hat{Y}_f) = 0$ by equation (4) but equals $b'D(\hat{X}_f)b$ by equation (8).

(B) Expressions (8) and (10) can be used to show how forecasts made from dynamic models deteriorate in quality as the forecast's target date moves farther into the future. To see this, take a simple case where

$$Y_t = \alpha X_t + \beta Y_{t-1} + \varepsilon_t$$

$$E(\varepsilon_t) = 0, \ E(\varepsilon_t, \varepsilon_{t-i}) = \sigma^2 \text{ for } i = 0$$
$$= 0 \text{ for all } i \neq 0$$

ε_t normal

Assume α and β have been estimated by applying OLS to a sufficiently large sample that the estimates are not only consistent and asymptotically efficient, but their finite sample bias is also negligible. Let the last observation in the sample period be the T-th. Forecasts are to be made for periods T+j. Assume all X_{T+j} are known with certainty, and further assume--for simplicity--that $X_{T+j} = X_T$ for all j. Let a and b denote the estimates of α and β. The forecast for period T + 1 is

$$\hat{Y}_{T+1} = aX_{T+1} + bY_T.$$

Its variance is, from equation (4),

$$V(\hat{Y}_{T+1}) = (X_{T+1} \ Y_T) \begin{pmatrix} V(a) & C(a,b) \\ C(a,b) & V(b) \end{pmatrix} \begin{pmatrix} X_{T+1} \\ Y_T \end{pmatrix} + \sigma^2$$

The forecast for period T+j is $\hat{Y}_{T+j} = aX_{T+j} + b\hat{Y}_{T+j-1}$. Assume for convenience that $\hat{Y}_{T+j} = Y_T$ for all j. Then, according to equation (4), $V(\hat{Y}_{T+j}) = V(\hat{Y}_{T+1})$. But by applying equation (8) we obtain

$$(12)\quad V(\hat{Y}_{T+j}) = V(\hat{Y}_{T+1}) + [b^2 + V(b)]\, V(\hat{Y}_{T+j-1})$$

$$= \Sigma_{i=0}^{j-1}\, [b^2 + V(b)]\, V(\hat{Y}_{T+1})$$

Consequently $V(\hat{Y}_{T+1}) < V(\hat{Y}_{T+2}) < V(\hat{Y}_{T+3}) < \ldots.$

(C) This implication concerns criteria for selecting variables for inclusion in forecasting equations. The effect of adding or deleting a variable on value of R^2 may not be a reliable clue to effect of the added variable on reliability of forecasts. Adding a variable, while it increases R^2, may reduce the reliability of forecasts if the added variable must itself be predicted. To see this, take an artificial case in which (a) variable k+1 is added to an equation that contains 1, 2, ..., k, (b) $D(\hat{X}_f)$ is diagonal, i.e., predicted regressors are distributed independently, and (c) the k+1-st variable is orthogonal to variables 1, 2, ..., k. Use V(k) and V(k+1) to denote the variances of forecasts made with k and k+1 predictors. Their estimated difference is

$$(13)\quad V(k+1) - V(k) = [\Sigma_{i,j=1}^{k}\, \hat{X}_{if}\hat{X}_{jf}a^{ij} + \Sigma_{i=2}^{k}\, a^{ii}V(\hat{X}_{if}) + 1](s_{k+1}^2 - s_k^2)$$

$$+ \hat{X}_{k+1,f}^2 V(b_{k+1}) + b_{k+1}^2 V(\hat{X}_{k+1,f})$$

$$+ V(b_{k+1})\, V(\hat{X}_{k+1,f})$$

Here s_{k+1}^2 and s_k^2 are estimates of error variance from the equations with k+1 and k independent variables. In this expression, $(s_{k+1}^2 - s_k^2)$ is negative, the bracketed term multiplying this difference is positive. Hence their product is negative. The last three terms on the RHS of

(13) are positive. Consequently (13) may be positive or negative.

Thus we see that the effect of an added variable on the reliability of a forecasted endogenous variable depends upon the reliability of the forecasts of the added variable. Adding a variable may reduce $V(\hat{Y}_f)$ if the added variable is predicted by one method, but increase $V(\hat{Y}_f)$ if it is predicted by another method.

Expression (13) refers to but one observation. We can (a) estimate sample-period values of exogenous variables using the same method used to forecast future values, (b) use these to estimate all sample-period values of Y_t, (c) apply (8) to each estimate of Y_t, and (d) select the equation that provides the minimum mean square error (MSE).

$$MSE = \sum_{t=1}^{T} V(\hat{Y}_t)/T + (\bar{\bar{Y}} - \bar{Y})^2$$

(D) In validating models to be used in predicting endogenous variables, *ex ante* validation of sample-period performance is preferable to *ex post* validation. In *ex post* validation, known values of exogenous variables are used to estimate endogenous variables. In *ex ante* validation, estimated values of exogenous variables are used. *Ex post* validation tends to overestimate reliability of forecasts of future values of endogenous variables from models that require predictions of future values of exogenous variables.

(E) And finally, the results lead us to propose that we economists reallocate our resources: that we devote fewer of our resources to obtaining the best structural equations (or best equations for predicting endogenous variables) and devote more resources to improving our methods of predicting exogenous variables. We hypothesize that the marginal product of resources devoted to the latter effort will exceed the marginal product of resources devoted to the former effort. A book edited by Hibbs and Fassbender reports on studies of the interdependence between macroeconomic performance, political support, and macroeconomic policy. Rausser discusses relations between government and agriculture. These two publications and the references cited therein provide a number of ideas that may help us to understand and predict exogenous variables. One implication of these studies is that some variables we commonly classify as exogenous are affected by economic forces.

Application

Mo modeled the domestic demand structure for wheat. An objective of his study was to forecast U.S. wheat utilization under different prospective government programs. We used Mo's data and model. The model is a linear dynamic recursive system containing six endogenous variables. Table 1 defines each variable in his model. His structural equations can be summarized briefly as

$$P_t = f_1(P_{st}, K_t P_{0t})$$

$$q_{ht} = f_2(P_t, P_{ct}, G(I_t))$$

$$q_{ft} = f_3(P_t, P_{0t}, L_t, D_t)$$

$$C_{gt} = f_4(P_{st}, \bar{K}_t(1-D_{t-2})\ 0_t, C_{gt-1})$$

$$C_{ct} = f_5(P_t, C_{gt}, C_{ct-1})$$

$$q_{Et} = f_6(q_{ht}, C_{ct-1} + C_{gt-1}, q_{Et-1}).$$

Annual data from 1928 through 1964 was used to estimate each structural equation by ordinary least squares (OLS) and by three stage least squares

Table 1. Definitions of Variables Used in Wheat Sector Model

Exogenous variables

$G(I_t)$	- A non-linear transformation of variable I.[a/]
I_t	- Per capita disposable income at time t (dol. per capita).
L_t	- Grain consuming animal units of livestock fed annually at time t (mil. units).
O_t	- Total U.S. wheat production at time t (mil. bu.).
P_{ct}	- Consumer price index at time t (1957-59 = 100).
P_{Ot}	- Farm price index of other feed grains (corn, oats, barley, and sorghum) at time t (1957-59 = 100).
P_{st}	- Average wheat support price at time t (dol. per bu.).
$D_t = 1$	During WW II
$= 0$	Otherwise
$K_t = 1$	If there is no price support program at time t
$= 0$	Otherwise.
$\bar{K}_t = 1$	If there is a government price support program at time t
$= 0$	Otherwise.

Endogenous variables

C_{ct}	- Commercial wheat inventory at the end of time t (mil. bu.).
C_{gt}	- Government wheat inventory at the end of time t (mil. bu.).
P_t	- Average wheat price received by farmers at time t (dol. per bu.).
q_{Et}	- Total U.S. export of wheat at time t (mil. bu.).
q_{ft}	- Domestic use of wheat for feed at time t (mil. bu.).
q_{ht}	- Domestic per capita use of wheat for food at time t (bu. per capita).

[a/] $G(I_t) = 6e^{-.001I_t} - 5.7468e^{-.002I_t}$

(3SLS). Morris and Cha present complete reports of the OLS and 3SLS studies, respectively.

We assumed that the 1964 wheat price support program continued into 1965 and 1966. The 1965 and 1966 forecasts of the exogenous variables and their variances are presented in Table 2. These forecasts were obtained from OLS estimates of $X_{it} = \alpha_0 + \alpha_1 X_{it-1} + \alpha_2 t$ for each exogenous variable X_{it}. The 1965 and 1966 forecasts differed but little, as did their variances computed from equation (4). The next to the last column illustrates implication A: equation (4) overstates reliability. The last column illustrates implication B: variance of a forecast increases as the target date moves farther into the future.

Tables 3 and 4 present more evidence on implications (A) and (B). To compute these variances, it was necessary to use statistical differentials (see Fuller or Rao) to obtain $V(G(\hat{I}_t))$ from $V(\hat{I}_t)$.

Variances of 1965 forecasts of P, C_g, C_c, and q_E from equations (8) and (10) differed little from variances from equations (4) and (6). But this was not true for 1965 forecasts of q_h and q_f. Neither was it true for 1966 forecasts. The last column in Table 4 illustrates again implication (B): deterioration of quality of forecasts over time.

None of these results are as dramatic as the results obtained in another study. In this study, June 1983 fed cattle price was a function of (a) June 1983 real income per capita, (b) June 1983 federally inspected slaughter per working day, and (c) seasonal dummies.

June 1983 real per capita income was forecasted by assuming disposable income grew at 9 percent per year, consumer prices rose at 6 percent per year, and population grew by 0.2 million per month.

Table 2. Forecasts of Variance of Forecasts of Exogenous Variables

Exogenous Variables	1965 Forecast	Variance of 1965 Forecast Eq. (4)	1966 Forecast	Variance of 1966 Forecast: Using Eq. (4)	Variance of 1966 Forecast: Using Eq. (8)	Variance of 1966 Forecast: Ratio of Eq.(8) to Eq. (4)	Ratio of 1966 to 1965 Variance[a]
P_0	109.15	507.54	115.69	507.25	794.16	1.57	1.56
P_c	111.29	7.08	114.37	7.15	12.24	1.71	1.73
L	169.00	71.07	170.13	71.63	110.25	1.54	1.55
0	1,334.65	24,270.17	1,333.40	24,538.58	29,158.05	1.19	1.20
I	2,334.86	2,777.00	2,401.94	2,816.25	4,684.44	1.66	1.69

a/ Eq. (8) used for 1966 variance.

Table 3. 1965 Forecasts and Variances of Forecasts of Endogenous Variables

Variable	Forecasts		OLS Variances			3SLS Variances		
	OLS	3SLS	Eq.(4)	Eq.(8)	Ratio of Eq.(8)/ Eq.(4)	Eq.(6)	Eq.(10)	Ratio of Eq.(10)/ Eq.(6)
P	1.36	1.36	.0198	.0198	1.00	.0181	.0181	1.00
q_h	2.59	2.59	.0139	.0162	1.17	.0115	.0132	1.14
q_f	149.1	146.4	2,905.4	5,272.9	1.81	3,066.6	4,572.3	1.49
C_g	749.2	704.6	23,565.1	24,069.4	1.02	20,671.4	20,810.4	1.01
C_c	115.6	130.2	3,544.2	3,703.8	1.05	3,039.5	3,039.6	1.00
q_E	691.6	674.2	10,432.7	10,725.4	1.03	9,119.5	9,180.6	1.01

Table 4. OLS Forecasts and Variances of Forecasts of Endogenous Variables for 1966

Variable	1966 Forecast	Forecast Variance Using Eq. (4)	Forecast Variance Using Eq. (8)	Ratio of Eq. (8) to Eq. (4)	Ratio of Eq. (8) for 1966 to Eq. (8) For 1965
P	1.36	0.0198	0.0198	1.00	1.00
q_h	2.56	0.0144	0.0174	1.21	1.07
q_f	162.19	3,046.72	6,416.35	2.11	1.22
C_g	783.55	23,543.12	39,205.46	1.66	1.63
C_c	120.91	3,485.83	4,249.84	1.22	1.15
q_E	674.78	10,229.58	15,979.11	1.56	1.49

June 1983 federally inspected slaughter, was predicted from (d) April 1 feeder cattle inventory, (e) number of 700-900 pound heifers on feed in April, (f) April choice fed steer price, (g) January 1 bull inventory, (h) January beef cow replacements, and (i) seasonal dummies.

The predicted June 1983 fed cattle price was \$62.44. Its standard error computed by formula (4) was \$5.86. But by considering the variances of the predicted values of variables (a) and (b) and by using (8), the standard error of the price forecast increased to \$37.44: an increase of 539 percent.

We have explored implication (C) only briefly. Adding P_{0t} to the equation used to predict P_{ct} increased R^2 from 0.988 to 0.994, a significant increase at the 1 percent level. Its addition changed the 1965 and 1966 forecasts by about 2 percent: from 111.2 to 110.0, and 114.4 to 112.1 respectively. Its addition also reduced the variance of forecasts of P_{ct}: from 7.1 to 5.4 for 1965 and from 12.2 to 6.4 for 1966. In this example, adding a variable improved both the R^2 and variance of forecast. It increased the first and reduced the second.

We also have an example in which dropping a variable affected R^2 and variance of forecast in opposite ways. Removing P_{0t} from the OLS equation for q_{ft} reduced R^2 from 0.780 to 0.720, significant at the 1 percent level. But it also reduced variances of 1965 and 1966 forecasts of q_{ft} from 5,272 to 3,265 and from 6,416 to 3,501. Removing P_{0t} also reduced the 1965 and 1966 forecasts by about one-third.

Conclusions

Analytical and empirical evidence show that textbook formulas for variance of a forecast tend to overstate, sometimes substantially, the reliability of forecasts of future values of endogenous variables. The formulas do so because they ignore one potential source of error: errors of forecasts in exogenous variables. A formula for variance of forecast that allows for errors in independent variables also shows that reliability of forecasts deteriorates as the forecasts' target date moves farther into the future. A set of structural equations that fits the sample data best does not provide better forecasts than a second-best set if the former contains exogenous variables that cannot be predicted accurately and the latter does not.

Footnotes

[1]This assumption is satisfied if elements of X are exogenous. To motivate this assumption, suppose that the system that generates X can be written $X = Z\Gamma + W$ where Z and W are matrices of exogenous variables and random disturbances. Then $\hat{X}_f = Z_f C$ where C is an OLS estimator of Γ, and $E(b-\beta)(\hat{X}_f - X_f)' = (X'X)^{-1} X'E(uw')(Z_f(Z'Z)^{-1}Z_f - I)$. If elements of X are exogenous, $E(uw') = 0$.

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