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A STOCHASTIC RISK-RETURN MODEL OF OPTIMAL CASH/FUTURES POSITIONS

Dennis D. DiPietre, Marvin L. Hayenga, and Roger A. Dahlgran*

The variability of agricultural product prices has undergone a tremendous increase in the last decade. Managers at the farm and wholesale level are coming under increasing pressure to formulate effective marketing strategies in order to remain profitable. The selection of effective marketing strategies is a multi-dimensional problem involving the managers loss aversion, target return motives, and anticipated commodity price variability which is often exacerbated by factors beyond management control. Such factors include the biological character of production, the associated effects of weather, government policy changes here and abroad, and the behavior of competitors and consumers.

Within the meat industry, the trend toward boxed beef and the emerging dominance of a few large firms at the processing and wholesale level has introduced additional uncertainties. Some research suggests that the structure of the marketing channel has an influence on the selection of optimal procurement and marketing strategies (Hayenga, 1979).

The growth of futures markets and their use, along with the increased demand for fixed price contracts by large buyers, bears testimony to the desire of managers to mitigate against price risk. On exchanges where meat related futures are traded, contracts for pork

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bellies, live hogs, live cattle and feeder cattle are trading heavily. Although contracts for boneless beef have recently been withdrawn at both the New York and Chicago Merchantile Exchanges, their reintroduction is likely as soon as research indicates the characteristics of contract design which will attract hedging and speculative interest (Gorham, 1982).

Sales officers, inventory managers and trade economists in the meat industry face special difficulties in maintaining profit margins while offering intermediate-term fixed price commitments to large buyers. This is particularly risky when firms offer such commitments without having established their future costs via forward contracting or forward pricing. The same basic problem confronts restaurant managers who usually prefer to fix menu prices for an extended period but cannot purchase perishable products very far in advance.

The basic question which these managers face is how to determine and implement the best cash and/or futures market position to establish a profitable and competitive purchase or sales price. Frequently, sufficient storage is not available to allow purchase-plus-storage pricing in the bidding and/or cost establishment process. Other options include buying hand-to-mouth or using the futures markets to hedge, cross-hedge or establish a forward purchase or sales price. Buying hand-to-mouth generally results in the attainment of an average price over the purchasing horizon which may result in small profits or perhaps losses. Hedging and particularly cross-hedging frequently exposes the firm to a highly variable basis and hence increased price risk (Hayenga and DiPietre, 1981, 1982).

OBJECTIVE

The objective of this research is to develop purchase, pricing and inventory management strategies applicable to meat processing firms in market situations where quantity demanded is known but price distributions are stochastic, transitory and (possibly) nonstationary. A stochastic control formulation will be utilized to select the utility maximizing postitions in both cash and futures markets where utility is dependent on deviations from management determined target objectives.

Such situations are becoming more important in the meat industry. For instance, a wholesale meat processing firm may be asked to submit a bid offering to supply a restaurant chain, dormitory food service, or the military with a large quantity of meat over a six month period at a fixed price. If storage is not available (or suitable), the processing firm exposes itself to tremendous price risk by committing to the fixed price. As another example, a supermarket chain may wish to offer a holiday special on a particular cut of meat. This decision may be made several months in advance with or without the possibility of storage. The procurement manager must decide if current prices are low enough to warrant purchasing part or all of the anticipated needs. If prices are expected to fall, the manager may elect to delay the purchase in which case he will expose the firm to price risks and the potential for loss on an advertised special. Failure to formulate and implement an effective purchasing strategy can mean loss of customers to competitors and decreased profits.

In their 1983 annual report, the Commodity Futures Trading Commission (CFTC) examined the use of meat related futures markets by commercial interests holding large contract commitments. Beef processors were found to carry predominately long futures positions in live cattle contracts as a hedge against widely fluctuating cattle prices. Cattle costs represent about 85 to 90 percent of of the basic cost of production of fresh beef carcasses and fabricated cuts. Both beef and pork processors indicated to the CFTC that the demand for forward pricing meat cuts to hotels, restaurants and institutions (HRI) was increasing. Because the dollar risk of these forward price commitments is so large to meat processors, they have made bids and accepted orders only when they could hedge or crosshedge the sales in a livestock future. Cattle processors were found to preprice as much as 80% of their anticipated cattle needs by using futures markets.

Pork processors were also found to be predominantely long hedgers for the same reason as cattle processors. These positions were normally held in live hog futures but the use of pork belly futures to hedge bacon sales and preprice raw materials was not uncommon.

Nine of the ten large hedgers interviewed indicated that selective hedging as opposed to routine hedging was the most prevalent practice employed by their firm. Furthermore, the size of the selective hedge was typically larger than the routine hedge position. Managers revealed that their selective hedging criteria was a function of their "risk/opportunity" probability in a given situation which is determined both objectively and subjectively. Thus, selective hedging strategies may call for being long futures, short futures, fully hedged, fully exposed or somewhere in between (CFTC, 1983).

Several factors must be considered in the development and implementation of selective hedging strategies. First, the appropriate futures contract(s) offered on relevant exhanges must be identified. Anderson and Danthine have shown that because of differing contract specifications and the potential for cross-hedging and arbitrage among exchanges, it may be desirable to take positions in several contracts on different exchanges in order to achieve profit objectives (Anderson and Danthine, 1980). Second, the futures price today along with current cash prices and storage costs (if storage is available and feasible) must be considered. Third, the probability density functions for cash and futures prices during the time when these positions could be taken should be estimated. The distribution of cash-futures basis, during the time when the futures position would be liquidated must also be considered. Fourth, the loss aversion function and/or target return motives of the decision-maker should influence the determination of an optimal strategy. Lastly, the strategy set should include the possibility of simultaneous cash and futures positions with flexibility to change as new information regarding price movement becomes available over the planning and implementation horizon.

REVIEW OF LITERATURE

Within the economics and finance literature, efforts have been made to extend the theory of investment and inventory management in order to describe the optimal behavior of agents under conditions of imperfect knowledge regarding the future. These developments have proceeded along two distinct yet related lines. The first approach has its roots in the classic mean-variance models of Markowitz and Tobin. These models have been extremely popular because they have intuitive appeal and are easily estimated. The intuitive appeal arises from the notion that portfolios are selected with respect to some expected return on assets (mean) and how likely the return is to be realized (variance).

The basic result of portfolio theory is that if expected returns to assets are not perfectly positively correlated, the utility maximizing, risk averse investor will hold a diversified group of assets rather than a single asset. Investors are assumed to choose this group of assets on the basis of a utility function defined in terms of the mean and standard deviation of the portfolio return. The most popular application of this theory has been to the problem of selecting stocks and bonds.

Within the last twenty years, an increasing application of mean-variance portfolio theory to hedging problems has occured. In these applications, the hedger holds a portfolio of hedged and unhedged stocks of a commodity in order to maximize utility. Johnson (1960) and Stein (1961) were among the first to apply portfolio theory to the problem of selecting optimal cash-futures positions by producers.

McKinnon (1967) examines the problem of a grain producer who faces an uncertain harvest. Since the size of the "investment" is unknown, the willingness to hold hedged and unhedged stocks is affected. McKinnon concludes that in the case of an uncertain harvest, a producer who would normally be a short hedger, may hold a long position to maintain his income if the harvest is expected to be small. Ward and Fletcher (1971) apply mean-variance portfolio theory to cattle producers and marketing firms and conclude that it may be rational for a hedger to hold contract commitments in excess of their expected cash position for speculative purposes. The notion here is that the motivations behind speculation and hedging are points along a single continuum of manager objectives.

Peck (1975) models the hedging behavior of egg producers who use price forecasting models to formulate expected returns. Regardless of the forecast used, the optimal hedge ranged from 75 to 95 percent of production. Using portfolio theory to select the level of hedging versus a total hedged strategy resulted in the total hedged strategy frequently outperforming the portfolio approach. As the author notes however, the optimal hedging schemes are only optimal in an <u>ex ante</u> sense while her evaluation of their result is <u>ex post</u>.

Most recently, Rolfo (1979) and Anderson and Danthine (1980) have applied the mean-variance approach to hedging under both price and quantity uncertainty. Rolfo examines the case of a cocoa producer, modeling both quadratic (mean-variance) and logarithmic utility functions. The result obtained was that limited or no use of futures markets may be superior to a full short hedge of expected output when production variability is high. Anderson and Danthine present a very complete theoretical model which allows the hedger/speculator an opportunity to include several contracts and exchanges along with cross-hedging to extend the "asset" choice from only two (hedged and unhedged stocks) to several (unhedged, hedged on multiple exchanges, cross-hedged in different contracts on multiple exchanges) in the construction of a portfolio.

At least two major theoretical problems limit the usefulness of this approach. First, either a multivariate normal distribution of returns to the assets in the portfolio must be assumed or a quadratic utility function is required to insure conformity with von Neumann-Morgenstern utility theory. Secondly, it is broadly observed that managers exhibit target motives in the selection of risk management strategies. Target motives suggest that risk and return for an individual manager should be measured with respect to deviations from this target return rather than with respect to a parameter such as the mean or variance (Fishburn 1979, Holthausen 1981).

A second set of techniques for use in decision analysis are dominance methods. The dominance methods are sometimes considered when a quadratic representation of utility is not desirable. Stochastic dominance methods are frequently referred to as second-best methods because they describe manager behavior when very little is known regarding the underlying preference structure and thus an estimation of a strict optimum is not possible. These methods are useful when preferences of individual managers are not obtainable and an elaborate mathematical representation of preference is not desirable. The efficient set of strategies or decisions becomes those which are undominated by the criteria employed. The most popular of these methods is stochastic dominance (first, second and third degree: FSD, SSD and TSD). FSD requires only the assumption that the decision maker prefers more to less. More formally, this posits a utility function which is monotonically increasing with a strictly positive first derivative. Higher degrees of stochastic dominance can be employed as more and more restrictive assumptions are assigned to the decision-makers preference structure. This is frequently necessary because relatively few actions can be eliminated by FSD.

SSD adds the assumption that the decision-maker is risk averse. Under the SSD criteria, the decision-makers utility function is not only monotonically increasing but strictly concave. This requires the first derivative to be positive and the second derivative negative throughout. The efficient set of strategies which result are a subset of those which are FSD.

TSD rests on the idea that as a persons wealth increases, they become decreasingly risk averse. This criteria requires the additional assumption of a postitive third derivative of the utility function. It also suggests that there is a predisposition on the part of the decision-maker for positive skewness in the distribution of returns. TSD is relatively expensive to computationally implement. Because of this, and the fact that many empirical price distributions do not appear to possess much skewness, the TSD and SSD efficient sets are frequently found to be nearly identical, leaving the added cost of TSD solutions unjustified (Anderson, Dillon and Hardaker, 1977).

Flood, McCamley and Schneeberger (1983) use stochastic dominance to evaluate whether farmers select crop varieties in a rational fashion on the basis of information supplied by experiment stations' crop variety testing programs. The results published by experiment stations are commonly given in the form of yield/acre. Stochastic dominance was chosen as the evaluation procedure because crop yields are believed to be skewed following the Gamma distribution rather than a normal distribution. Greenhall (1983) compares various corn marketing strategies using stochastic dominance techniques. Here the appeal to stochastic dominance over mean-variance analysis is likewise proposed on the basis of an assumed skewness in corn price distributions where government policy intervention tends to mitigate downside price risk.

Johnson and Burgess (1975) compare the performance of mean-variance and stochastic dominance approaches in analyzing optimal choices out of data drawn from different distributions. In samples drawn from normally distributed distributions, the two techniques were judged equally effective. However, in samples drawn from distributions which were not normally distributed there was reason to believe that stochastic dominance outperformed mean-variance techniques.

Another consideration in the modeling of manager behavior is that managers frequently choose marketing and pricing strategies with respect to a target return. In such a case, managers may not elect to maximize expected return but may wish to minimize the probability of a disastrous level of return. Among the first to recognize these motives was Telser, who modeled hedger behavior in a safety first setting (Telser,1955-56). Developments by Pyle and Turnovsky demonstrated that safety first criteria were not consistent with concave indifference curves in mean-standard deviation space when portfolios were selected in the presence of riskless assets (Turnovsky and Pyle, 1970). The departure from maximizing expected utility through safety first concepts did not provide a suitable correspondence with the well accepted mean-variance approaches.

In an effort to incorporate target return motives in the decision analysis Fishburn (1977) proposed a mean-risk dominance model which measured risk as probability weighted deviations below a specified target. Holthausen (1981) extended the model to a risk-return framework where risk is measured as Fishburn suggests but returns are likewise estimated as probability weighted deviations above the target return specified. This research follows the Holthausen approach. These developments by Fishburn and Holthausen offer the possibility of combining the theoretically attractive elements of the above mentioned techniques (e.g. reasonable computational efficiency, recognition of target motives, etc.) while reconciling the unrealistic elements such as restrictive distributional properties of asset returns and non-conformity with von Neumann-Morgenstern utility theory (Fishburn 1977, Holthausen 1981). This class of models, referred to as α-β-τ models have reasonable computational efficiency, can be compatible with von Neumann-Morganstern utility theory, stochastic dominance and the observed appeal to target motives by investment and inventory managers.

Finally, devising a strategy and implementing it are two related functions. All of the techniques previously mentioned assume a single, stationary asset price distribution. If the distribution realized is markedly different than expected or is nonstationary, the hoped for result may be unrealized. Price distributions for meat and meat products are known to possess both seasonal and cyclical trends. These must be accounted for in the formulation of an effective strategy. Commodity purchasing methods which are dynamic and allow for nonstationary price distributions have been developed in the operations research literature (Kingsman, 1958) but have not been widely employed in agricultural marketing contexts except with respect to government grain stock models. This research combines a dynamic purchasing model with a risk-return model of asset choice in order to formulate initial purchasing and pricing strategies and update them as the purchasing horizon is realized.

THEORETICAL MODEL

The optimal choice of forward priced and/or purchased stocks of a commodity inventory or anticipated inventory is a function of the purchasing strategy and the portfolio selection criteria. In the case of a known demand the problem becomes one of deciding when to purchase and how much to purchase at a given but transitory price offering. In general, the manager desires to purchase the commodity at a low price, even to the point of building inventory if prices are low enough and storage is available. At moderately low to average expected prices the manager may buy nothing or buy only that quantity necessary to supply current needs. Deciding what price is "low" requires some knowledge of the probability density function of future prices.

Future price expectations can be generated by a variey of quantitative and qualitative procedures. However, price forecasting models which only seek to forecast average monthly prices for a few months ahead may not be sufficient to significantly lower long term buying costs. Meat prices in both cash and futures markets are highly volatile and subject to considerable short-run variability. Significant short term price volatility can be observed in these markets in the span of a single day. This phenomena should be accounted for in order to effectively reduce long-run buying costs.

Along with price expectations, the purchase or sales decision may be affected by the agent's target return or cost objectives rather than simple profit maximization. It is now commonly accepted that risk managers frequently make investment decisions with respect to loss aversion or gain seeking motives. These motives may reflect perceived stock holder objectives, upper-level management objectives, or the agent's personal risk management strategies. As such, the decision to buy or sell at a given price will be a highly individualistic choice made with reference to deviations from some subjectively determined target.

While some managers may only seek to beat the average price, others may formulate targets relative to competitors, seeking to implement an input purchasing policy necessary to achieve output prices in line with competitors. Targets may also be conceived as a fixed objective such as a margin between purchase and sales prices or as a fixed rate above or below an acceptable return or cost.

Deviations from target may be perceived with differing degrees of severity. For example, a commercial sausage manufacturer may lose all of his business if the price he offers is a single cent above competitors. On the other hand a branded product with a loyal following may not suffer a significant loss in market share until its price is five to six cents over competition. Further, there is no reason to believe *ex ante* that the significance of deviations will be symmetrical above and below target. Aversions to below target results (for returns) may be much stronger than attractions to deviations above target or vice versa.

A generalized utility maximization model can be constructed to capture the aforementioned behavioral objectives.

We begin by assuming the following:

1. Time is divided into successive periods such as days, weeks, months, etc.

2. A purchase or sales horizon of T periods into the future constitutes the relevant range over which the decisions must be made and the model parameters remain unchanged.

3. Demand and/or sales commitments are nonstochastic for each period in the purchase or sales horizon.

4. At the beginning of each period t the agent must already have in stock, or must purchase in the cash market at this period, sufficient stock to meet period t demands (i.e., no shortages are allowed).

5. Purchase or sales of product are undertaken in the cash market, however, the agent may pre-price known cash commitments by buying or selling on the futures market.

6. Futures positions must be offset before or during period T, the last period in the horizon, such that delivery on a futures contract is not allowed.

7. Delivery of the cash commodity is assumed to be instantaneous.

8. Margins and brokerage fees are ignored.

The general problem is to select purchase quantities and/or pre-price a commodity for eventual purchase so as to maximize the agents utility. Utility is a two-part function in deviations-from-target form. The limits of integration are interdependent and vary in number depending on the number of futures and cash prices which are considered relevant to the purchasing decision.

$$[1a] \quad \max_{\underset{\sim}{X}_t} \sum_{t=1}^{T} \left[\int_{\underset{\sim}{P}_t} \cdots \int (\tau - c_t)^{\alpha} \, h(\underset{\sim}{P}_t) d\underset{\sim}{P}_t + \int_{\underset{\sim}{P}_t} \cdots \int - K(c_t - \tau)^{\beta} h(\underset{\sim}{P}_t) d\underset{\sim}{P}_t \right],$$

$$\alpha, \beta \geq 0$$

s.t.

$$S_{0t} \geq 0$$

$$X_{0t} \geq 0$$

$$S_{0,t+1} = S_{0t} + X_{0t} - d_t$$

$$S_{00} = 0$$

$$\underset{\sim}{S}_{10} = 0$$

$$S_{0T} = 0$$

$$\underset{\sim}{S}_{1T} = 0$$

where

$$[1b] \quad c_t = \left[\underset{\sim}{P}_t' \underset{\sim}{X}_t + g(S_{0t} + X_{0t} - 1/2 \, d_t) \right]$$

τ = target cost

$\underset{\sim}{P}_t$ ~ random

c_t = purchase cost in t

X_{0t} = commodity purchased in t

$\underset{\sim}{X}_{1t}$ = futures purchased in t

S_{0t} = beginning commodity stocks in t

$\underset{\sim}{S}_{1t}$ = futures portfolio in t

g = commodity storage cost function

d_t = "fixed" or known demand of the commodity in t

P_{0t} = cash price in t

$\underset{\sim}{P}_{1t}$ = futures prices in t

$\underset{\sim}{P}'_t = [P_{0t} : \underset{\sim}{P}'_{1t}]$

$\underset{\sim}{X}_t = \left[\frac{X_{0t}}{\underset{\sim}{X}_{1t}} \right]$

$h(\underset{\sim}{P}_t)$ = probability density functions for $\underset{\sim}{P}_t$

The first set of integrals in the maximization formulation measure the "return" or contribution to utility from probability weighted deviations below target cost. The second set of integrals measure the "risk" or disutility from probability weighted deviations above target. α and β weight the importance of deviations below and above target respectively. The resulting utility function can take on a number of different shapes as determined by α, β and K. Figure 1 illustrates some possibilities.

Formulating this in a dynamic programming problem, the cost of an optimal one-period policy for the last period with initial stock S_{0T} is

$$[2] \quad F_T(S_{0T}, \underset{\sim}{P}_T) = \max_{\underset{\sim}{X}_T} \begin{cases} (\tau - c_T)^{\alpha}, \text{ for } c_T \leq T \\ -K(c_T - \tau)^{\beta}, \text{ for } c_T \geq T \end{cases}$$

The cost of an n period purchasing policy with initial stock S_{0n} and price offers $\tilde{P}_n$ is:

$$[3] \quad F_n\left(S_{0n}, \tilde{P}_n\right) = \underset{\tilde{X}n}{\text{Max}} \begin{cases} \left(\tau - c_T\right)^{\alpha}, & \text{for } c_T \leq T \\ -K\left(c_T - \tau\right)^{\beta}, & \text{for } c_T \geq T \end{cases} +$$

$$\int_0^{\infty} \cdots \int_0^{\infty} F_{n+1}\left(S_{0,n+1} + \tilde{X}_{n+1} - d_{n+1}\right) h\left(\tilde{P}_{n+1}\right) d\tilde{P}_{n+1}$$

Decision rules such as given in [3] above are required for each period in the purchasing horizon. Solving [2] and [3] results in recurrence relations indicating optimal inventory accumulation and futures portfolios at each price offering. Figure 2 illustrates this result.

POTENTIAL APPLICATIONS OF THE MODEL

The application of the dynamic programming model just illustrated first requires estimating or assuming pdf's for all relevant futures and cash prices and forecasting these prices over the purchasing horizon. This could be accomplished by a variety of forecasting techniques including fundamental models already available, developing ARIMA models of cash and future price movements, or using seasonal indices and historical probability calculations regarding expected price changes from one month to the next. The model as specified requires a stationary pdf throughout the purchasing horizon. This can be modified to account for nonstationary series with minor modifications to the model. If trend is expected, the magnitude of the rise or fall in price along with variability bounds are required.

Target objectives and the significance of deviations from target are highly individualistic and would be expected to change from manager to manager. Obtaining values for these parameters could be accomplished by phone survey with a number of purchasing officers currently employed in the meat industry. Alternatively, hypothetical targets could be posited and sensitivity analysis performed to determine likely strategies under a variety of target motives and purchase objectives.

The results obtained should provide insights into the purchasing function under a variety of real world contingencies. These insights may provide the basis for generalized strategy recommendations, given known demands and purchasing objectives but uncertain prices. For instance, the model will address how a purchasging strategy might be formulated or change in response to:

a) Cost or return objectives formed with reference to competitors vs. fixed margin objectives.

b) Incremental changes in target objectives.

c) Increased emphasis on, or aversion to negative results compared to the attraction of possible gains.

d) The possibility of arbitrage or spreading when greater marginal utility is gained by better than target results vs. the marginal utility lost by below target deviations.

e) Increases in the forecast variance of cash and/or futures prices (i.e., the effect of forecast method utilized).

f) Anticipated commodity price cycles and trends.

Information and insight into such questions should greatly benefit meat industry purchasing agents and be readily applicable to any market characterized by purchasing decisions in a situation of known demands but stochastic prices.

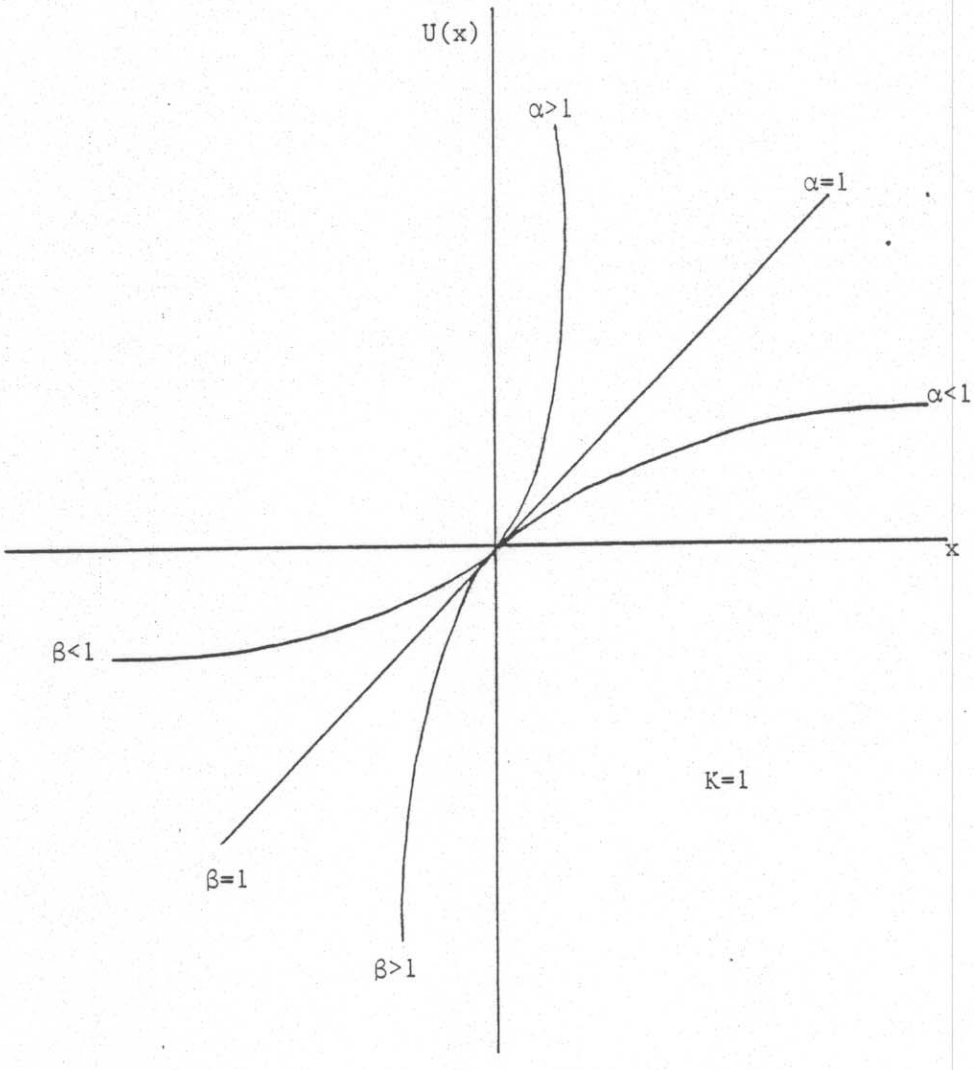

Figure 1. Possible utility function configurations with various values of α, β and K=1

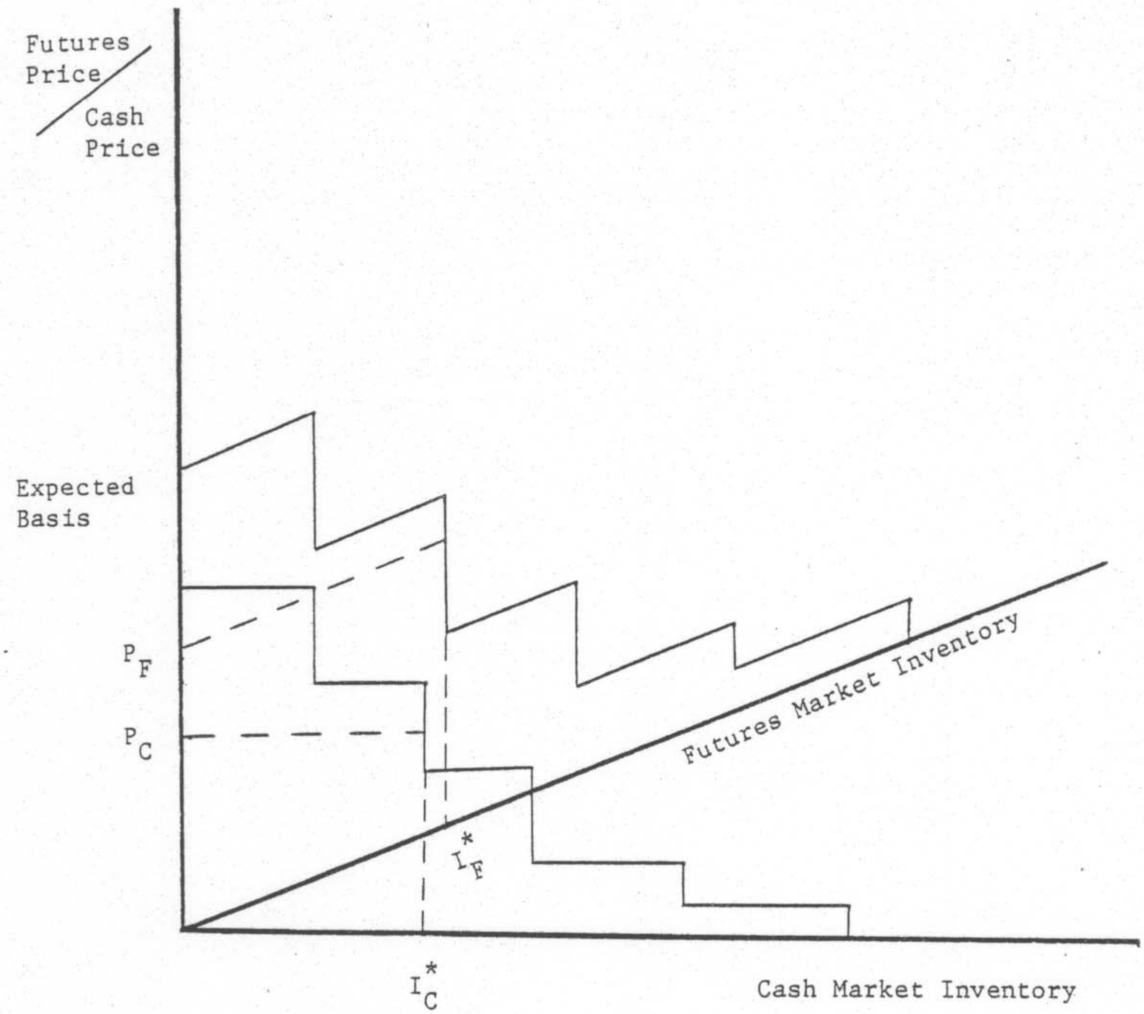

Figure 2. Optimal Cash and Futures Markets Positions as Obtained from the Recurrence Relations

LITERATURE CITED

Anderson, Ronald W. and Jean-Pierre Danthine, "Cross-Hedging", Working Paper Series #CSFM-24, Columbia Center for the Study of Futures Markets, Columbia Business School, New York, N. Y., 1980

Fishburn, Peter C., "Mean-Risk Analysis with Risk Associated with Below-Target Returns", The American Economic Review, Vol 67 No 2, pps. 116-26, March 1977

Flood, Margaret, Francis McCamley and Kenneth Schneeberger, "Stochastic Dominance as an Approach to Evaluate the Rationality of Farmer Variety Choices", Presented Paper, Annual Meeting of The American Agricultural Economics Association, 1983

Greenhall, Larry, "An Application of Ordinary and General Stochastic Dominance Criteria in Ordering Risky Marketing Strategies for Corn Producers", Presented Paper, Annual Meeting of the American Agricultural Economics Association, 1983

Hayenga, Marvin L., "Risk Management in Imperfect Markets: Commodity Procurement Stratey in the Food Manufacturing Sector", American Journal of Agricultural Economics, Vol. 61, No 2, pps. 351-56, 1979

Hayenga, Marvin L. and Dennis D. DiPietre, "Hedging Wholesale Beef Prices: An Analysis of the Basis Risk", Journal of Futures Markets, Vol 2, No. 2, pps. 131-140, 1982

Hayenga, Marvin L. and Dennis D. DiPietre, "Cross-Hedging Wholesale Pork Products", American Journal of Agricultural Economics, Vol 64, No. 4, pps. 747-751, 1982

Holthausen, Duncan M., "A Risk-Return Model with Risk and Return Measured as Deviations from a Target Return", The American Economic Review, Vol 71 No 1, pps. 182-88, March 1981

Johnson, K. H., and R. Burgess, "The Effects of Sample Sizes on the Accuaracy of E, V and SSD Efficiency Criteria", Journal of Financial and Quantitative Analysis, Vol 10, pps. 813-20, 1975

Johnson, Leland, "The Theory of Hedging and Speculation in Commodity Futures", The Review of Economic Studies, Vol 27, pps. 139-51, June 1960

Kingsman, B. G., "Commodity Purchasing", Operational Research Quarterly, Vol 20, pps. 59-79, 1959

Markowitz, H., "Portfolio Selection", Journal of Finance, Vol 7, pps. 77-91, 1952

McKinnon, R. I. "Futures Markets, Buffer Stocks and Income Stability for Primary Producers", The Journal of Political Economy, Vol 75, pps 844-861, December 1967

Rolfo, Jacques, "Optimal Hedging Under Price and Quanity Uncertainty: The Case of the Cocoa Producer", Working Paper Series #CSFM-2, Columbia Center for the Study of Futures Markets, Columbia Business School, New York, N.Y., 1979

Stein, J. L., "The Determination of Spot and Futures Prices", The American Economic Review, Vol 51, pps. 1012-25, December 1961

Telser, L., "Safety-First and Hedging", The Review of Economic Studies, Vol 23, pps. 1-16, 1955-56

Turnovsky, Stephen J. and David H. Pyle, "Safety First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis", The Review of Economics and Statistics, Vol 52, pps. 75-81, 1970

Ward Ronald W. and Lehman B. Fletcher, "From Hedging to Pure Speculation: A Micro Model of Optimal Futures and Cash Market Positions, American Journal of Agricultural Economics, Vol 53, pps. 71-78, February 1971