Relationships Between Quarterly Grain Prices and Stocks

by

Paul C. Wescott and David B. Hull

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Introduction

Corn and wheat are two of the largest and most important sectors within the agricultural economy. Over the last 5 years, cash receipts from corn and wheat have averaged nearly a third of crop receipts and almost a sixth of total cash receipts (2). Further, corn and wheat play important roles in the linkages within the agriculture sector among the various crops and between crops and livestock. Consequently, events which affect corn and wheat are usually carefully watched by other sectors within agriculture.

This paper examines the relationships between ending stocks and prices for corn and wheat in a quarterly framework. Van Meir (5) studied the effects of year-ending stocks on annual season average corn prices. Because stocks summarize the effects of both supply and demand factors, annual prices were found to be highly correlated with stocks. Green (1) and Westcott, Hull, and Green (7, 8) employed a hyperbolic functional form relating ending quarterly stocks to quarterly prices for various grains. Consistent with the results from the annual framework, higher ending stocks in any particular quarter result in lower farm-level prices. The effect of stocks on prices, however, differs through the marketing year, largely reflecting the annual nature of crop production. Early in the marketing year large levels of stocks are necessary to meet demand until the next harvest. As the marketing year progresses and the next

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harvest approaches, lower stocks are sufficient to meet use requirements. Consequently, a given level of stocks later in a marketing year results in lower prices than the same level of stocks earlier in the marketing year. For this paper, the Westcott, Hull, and Green studies for corn and for wheat are re-examined using a somewhat different formulation of the model for empirical implementation.

Two of the estimated price equations are then incorporated into a quarterly econometric model of the agriculture sector in order to examine properties of some nonlinear simulation techniques typically used for implementing large-scale forecasting models. Simulations are performed using Jacobi and Gauss-Seidel simulation methods with simulation results and speed of convergence of the model compared. Convergence problems related to the hyperbolic functional form and variable definitions used for the corn and wheat price equations are addressed by using damping factors in the simulations. Implications for implementing forecasting models using these techniques are drawn in a concluding section.

The Model

The general framework used here relating quarterly prices to ending stocks derives from a disequilibrium model where ending stocks clear the market as a residual. In a quarterly framework, a disequilibrium model is more appropriate than an equilibrium model because, with shorter time periods, the market is more likely to be observed in adjustment rather than approximating equilibrium.

The functional form used derives from the general hyperbolic function $(P-a)(S-d) = c$ where P represents quarterly prices, S denotes quarterly ending stocks, and a, c, and d are parameters. To avoid nonlinearities in estimation the parameter "d" is assumed to equal 0. Solving for price, $P = a + c\ S^{-1}$. To represent the different effects of stocks through the year, a separate "c" parameter is assumed for each quarter. Also, S is measured relative to the "scale of activity" in the corn and wheat industries, represented here by utilization (U). This is necessary because of the growth in those industries over the last 15 years. Further, lagged price is included to reflect "stickiness" of prices in a quarterly framework, largely due to the lag structures in underlying supply and demand functions. Inclusion of lagged price also allows the analysis to be conducted using nominal prices, thereby circumventing the issue of choosing an appropriate price deflator. These adjustments result in the following equation.

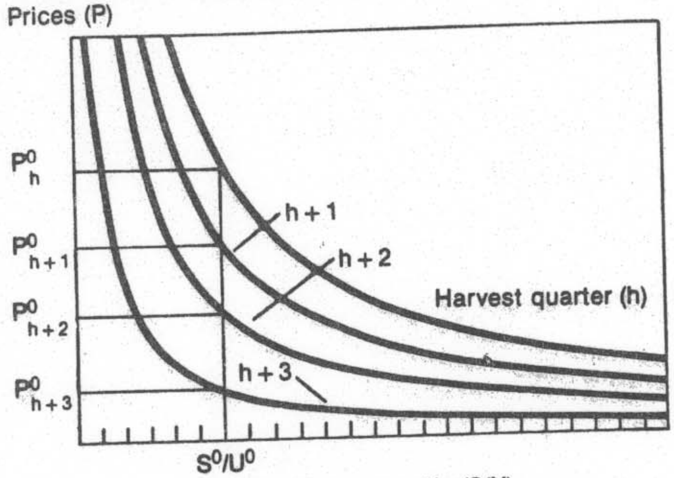
$$(1) \qquad P = a + b\ \mathrm{lag}(P) + \sum_{i=1}^{4} c_i\ D_i\ (S/U)^{-1}$$

D_i are quarterly dummy variables (equal to 1 in the i-th quarter, 0, elsewhere), lag(P) is the 1-quarter lag of P, and a, b, and c_i are parameters to be estimated. The subscripts "i" denote quarters, where i=1 is the January-March quarter, i=2 is the April-May quarter, i=3 is the June-September quarter, and i=4 is the October-December quarter. All other variables are as defined before.

The inclusion of four $c_i\ D_i\ (S/U)^{-1}$ terms allows a different effect of stocks on prices in each quarter. Each c_i is expected to be positive, with the largest coefficient occurring in the harvest quarter and successively smaller coefficients occurring in the 3 following quarters. Thus, equation 1 is expected to yield a family of 4 hyperbolic curves such as in figure 1, which shows prices related to the stocks-to-use

Figure 1

Hyperbolic Family of Curves Relating Quarterly Prices to the Stocks-to-use Ratio



ratio.[1] As the stocks-to-use ratio increases in any particular quarter, price falls, indicated by a move along that quarter's curve. Also, for any given stocks-to-use ratio (such as S^0/U^0), the resulting prices (P^0_h, P^0_{h+1}, P^0_{h+2}, P^0_{h+3}) are smaller later in the marketing year, indicated by a move from one curve to the next.

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1 While the hyperbolae being estimated can be expressed to show a direct relationship between prices and the stocks-to-use ratio (S/U) as shown in figure 1, the inverse of that ratio, $(S/U)^{-1}$, is the appropriate explanatory variable for use in estimating equation 1. Therefore, the inverse of the stocks-to-use ratio will be referred to in the discussion of estimation results, but the stocks-to-use ratio will be referred to in discussing implications drawn from those parameter estimates.

Data--Definitions and Sources

The data used to estimate equation 1 are published by USDA. The farm prices for corn and wheat are monthly series published in *Agricultural Prices*. Quarterly prices were derived by averaging the monthly prices from each quarter. Use and total stocks data are from supply and disappearance tables for corn and wheat published in the *Feed Outlook and Situation* and in the *Wheat Outlook and Situation* which are based on data from the Statistical Reporting Service.[2] Data for the categories that comprise total stocks are from the Agricultural Stabilization and Conservation Service.

Three alternative stocks definitions were used in estimation--total stocks and two alternative free-stock definitions. Total stocks include stocks that are privately held, owned by the Commodity Credit Corporation (CCC), under outstanding CCC loans, and in the farmer-owned reserve (FOR). The first free stock definition is total stocks less CCC-owned stocks, less FOR stocks. The second free stock definition further subtracts outstanding CCC loans from total stocks. The latter free stock definition represents removal of all government program stocks, while the former free stock definition includes outstanding CCC loans which can be redeemed at any time without penalty. Units for stocks and use categories are million bushels, while units for prices are dollars per bushel.

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[2] To avoid problems related to the uneven quarters of the corn and wheat marketing years, a four-quarter moving average of utilization was used as the measure of industry scale.

Model Estimation

Equation 1 was estimated over 1971-1981 (44 observations) for both corn and wheat using each of the three stock definitions discussed above. Table 1 summarizes the results.

For corn, equation C2 which uses the first free stocks definition appears to be the best. Over 88 percent of quarterly corn price variation is explained by equation C2. The mean absolute error over the estimation period of 17.6 cents per bushel represents a 7.9 percent error relative to the average price of $2.22 per bushel over the estimation period.

All coefficients in equation C2 are significant at the 5 percent level. As expected, the coefficients of the inverse stocks-to-use ratios are positive. The largest coefficient occurs in the harvest quarter (October-December) and successive quarters' coefficients diminish in size. Lagged price also plays an important role.

In equations C1 and C3, the R^2s are lower than in equation C2. Further, fewer of the coefficients of the inverse stocks-to-use ratios are significant at the 5 percent level. Also, in equation C1 the coefficient of the inverse stocks-to-use ratio in the January-March quarter exceeds that from the October-December quarter, contrary to expectations of the largest coefficient occurring in the harvest quarter.

For wheat, equations W1 and W2 have nearly identical within sample statistical properties and are both superior to equation W3. Nearly 88 percent of quarterly wheat price variation is explained by equations W1 and W2. The mean absolute errors over the estimation period of 27.5 and 27.9 cents per bushel represent 9.0 and 9.1 percent errors, respectively, relative to the average price of $3.06 per bushel over the estimation period.

Table 1--Estimated Equations Relating Prices to Ending Stocks, Corn and Wheat

Equation	Stocks Definition	Estimated Coefficients for: Intercept	Lagged Price	Inverse Ending Stocks-to-Use Ratio in: Jan.-Mar. quarter	April-May quarter	June-Sept. quarter	Oct.-Dec. quarter	R^2	Mean Absolute Errors: Within Sample 1971-81	Beyond Sample 1982	Beyond Sample 1983
									$/bu	$/bu	$/bu
Corn											
C1	Total	0.099 (0.56)	0.855 (12.54)	0.633 (1.47) [0.13]	0.335 (1.09) [0.10]	0.160 (1.89) [0.14]	0.546 (0.80) [0.07]	0.852	0.192	0.123	0.403
C2	Free-1	-0.207 (1.13)	0.711 (9.24)	1.857 (3.47) [0.41]	1.202 (3.20) [0.38]	0.376 (3.76) [0.41]	2.580 (2.97) [0.36]	0.881	0.176	0.223	0.140
C3	Free-2	-0.038 (0.12)	0.885 (13.65)	0.648 (1.12) [0.16]	0.353 (0.90) [0.13]	0.120 (1.22) [0.17]	0.672 (0.69) [0.10]	0.844	0.193	0.308	0.175
Wheat											
W1	Total	0.118 (0.58)	0.841 (12.46)	0.834 (1.75) [0.13]	0.094 (0.29) [0.02]	2.326 (2.41) [0.18]	2.053 (2.89) [0.20]	0.876	0.275	0.202	0.300
W2	Free-1	0.111 (0.55)	0.852 (13.21)	0.629 (1.87) [0.12]	0.033 (0.17) [0.01]	1.934 (2.44) [0.17]	1.641 (2.99) [0.19]	0.876	0.279	0.308	0.305
W3	Free-2	0.021 (0.07)	0.905 (14.78)	0.371 (1.20) [0.10]	0.011 (0.07) [0.01]	1.416 (1.69) [0.14]	1.194 (2.19) [0.16]	0.857	0.286	0.255	0.290

Numbers in parentheses are t-statistics. Numbers in brackets are price flexibilities evaluated at quarter-specific variable means.

All coefficients in equations W1 and W2 are significant at the 5 percent level except for the coefficients of the inverse stocks-to-use ratios in the April-May quarter. The coefficients of the inverse stocks-to-use ratios are all positive, with the largest coefficient occurring in the harvest quarter (June-September) and successive quarters' coefficients diminishing in size. Lagged price again plays an important role.

In equation W3, the R^2 is lower than in equations W1 and W2, and fewer of the inverse stocks-to-use ratio coefficients are significant at the 5 percent level. However, the largest coefficient still occurs in the harvest quarter with successive quarters' coefficients diminishing in size through the marketing year.

Model Estimates for 1982 and 1983

To assess the performance of the estimated equations, each was used to estimate quarterly prices for 1982 and 1983, two years beyond the estimation period. In each quarter, actual exogenous and lagged endogenous data were used. For 1983, an adjustment was made in the stocks data for the third and fourth quarters to reflect the effects of the Payment-in-Kind (PIK) program. Unpaid PIK entitlement stocks in the farmer-owned reserve or in CCC inventories (stock positions not normally considered free) were included as free stocks after the PIK participants' 5-month entitlement period had begun. In the third quarter, the entitlement period had begun for only a small amount of PIK payment corn (estimated to be 70 million bushels representing participants in Florida, Louisiana, and much of Texas). However, for the fourth quarter of 1983, all PIK payment corn had begun that 5-month period, so all remaining PIK payment corn in the FOR or owned by the CCC was assumed to be free (estimated to be 1418 million bushels). For wheat, the 5-month entitlement period began for all participants in the June-September quarter, so all PIK wheat in the FOR or owned by the CCC was assumed to be free. This amount was estimated to be 365 million bushels for the third quarter and 310 million bushels for the fourth quarter. These adjustments affect equations C2, C3, W2, and W3 in the third and fourth quarters of 1983, but do not affect equations C1 and W1 which use the total stocks definition.

The last two columns of table 1 show beyond sample (1982 and 1983) mean absolute errors for each equation. For corn, equation C1's mean absolute error is the lowest for 1982, but is considerably higher than either of the other two equations' mean absolute errors for 1983. For both years, the mean absolute errors for equation C2 are lower than those for equation C3. The 22.3 cent per bushel mean absolute error for equation C2 in 1982 represents a 9.3 percent error relative to the average 1982 corn price, only slightly greater than that attained over the estimation period. The 14.0 cent per bushel mean absolute error for equation C2 in 1983 represents a 4.7 percent error relative to the average 1983 corn price, considerably less than that attained over the estimation period.

For wheat, equation W1 has the lowest mean absolute error for 1982 while nearly identical mean absolute errors are attained by each of the wheat equations for 1983. In both years, each of the three equations has a smaller percent error relative to the respective annual wheat price than attained in the estimation period. Equations W1's 20.2 cent per bushel mean absolute error for 1982 and 30.0 cent per bushel mean absolute error for 1983 represent 5.7 and 8.3 percent errors, respectively, compared with the 9.0 percent error over the estimation period.

Plots

Figures 2 and 3 show plots of the quarterly hyperbolic curves that result from estimated equations C2 and W1. The figures are intended to show the relative positions of the estimated hyperbolae relating price to the stocks-to-use ratio, holding other things constant. Therefore, mean values for lagged prices over the estimation period of $2.22 per bushel for corn and $3.06 per bushel for wheat were assumed for the plots.

Each figure shows four hyperbolae from the same "family of curves." This is a consequence of each estimated equation being restricted to have the same intercept and the same lagged price coefficient across quarters. Consistent with expectations, in each figure higher stocks relative to use within any quarter give lower prices. Both figures also show the harvest quarters' curves (quarter 4 for corn; quarter 3 for wheat) the highest. Further, any particular level of stocks relative to use later in the marketing year gives lower prices than the same level does earlier in the marketing year, as indicated by the relative positions of the curves for successive quarters in each marketing year.

Convergence Properties of Nonlinear Simulation Techniques

We now examine some properties of nonlinear simulation techniques typically used for implementing large-scale forecasting models. Of particular interest is how the hyperbolic functional forms for the corn and wheat price equations affect convergence properties of large-scale models using various nonlinear simulation methods. To do this, corn equation C2 and wheat equation W1 were incorporated into a quarterly econometric forecasting model for U.S. agriculture currently being used in ERS to make short-term forecasts. This model, discussed in Westcott

Figure 2--Plot of Estimated Equation C2

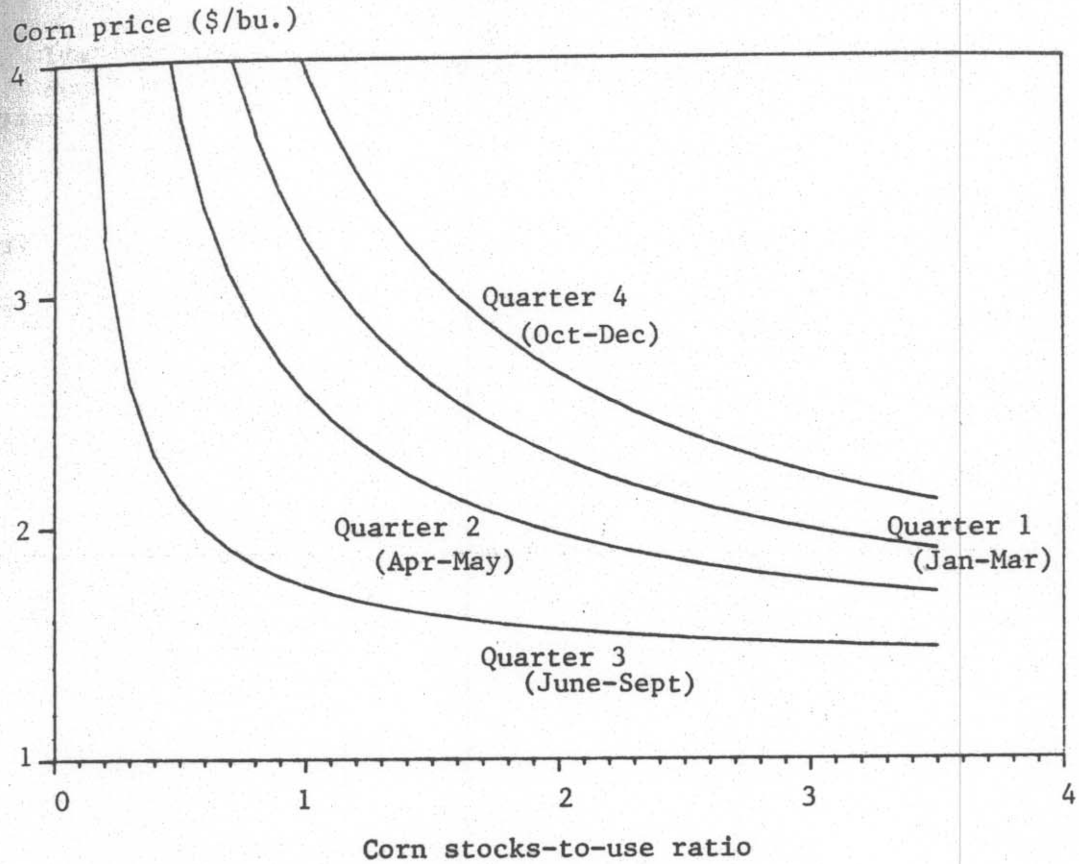
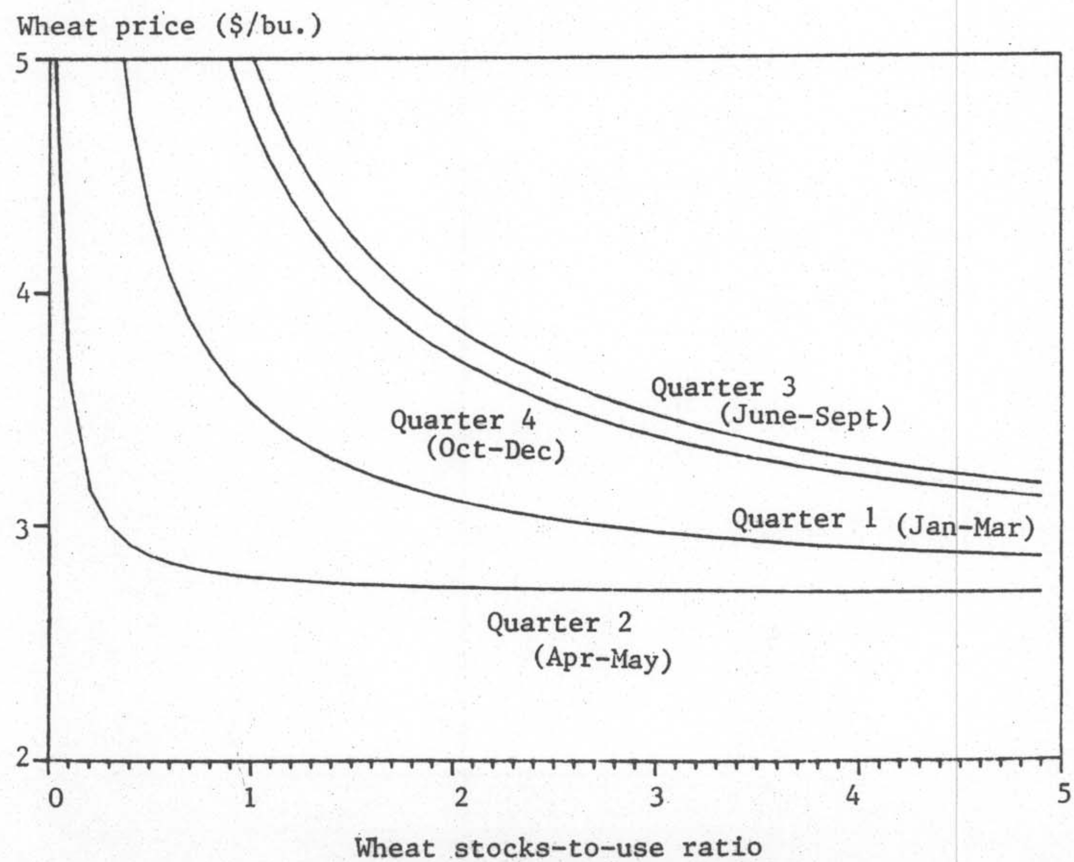


Figure 3--Plot of Estimated Equation W1



and Hull (6), consists of approximately 120 equations (about half, stochastic and half, identities) and covers corn, wheat, soybeans, cattle, hogs, and poultry.

Nonlinear simulations of the model were performed for each year from 1975-1984 with equations C2 and W1 used in place of the model's own corn and wheat price equations. All simulations were performed using the Statistical Analysis System (SAS) nonlinear simulation package SIMNLIN (3). Simulation techniques initially used were Jacobi and Gauss-Seidel (4). The convergence criterion was set at 0.001 for the simulations--for any particular simulation period, the model was assumed to converge on the i^{th} iteration if for all endogenous variables, y,

$$\frac{|y_i - y_{i-1}|}{|y_i| + 10} < 0.001.$$

All variables converged to essentially identical levels in these simulations but, as table 2 indicates, the Jacobi simulations took longer to converge in every period. Further, table 3 indicates that with both techniques convergence required considerably more iterations to converge in the third quarter of each year. This was primarily a consequence of 2 factors:

- The inverse stocks-to-use ratio in the corn price equation tends to be more volatile between simulation iterations when free stocks are relatively low at the end of the corn marketing year.

- The hyperbolic functional form used for the corn price equation tends to push corn prices into the steeper ranges of the hyperbola when free stocks are relatively low at the end of the corn marketing year.

Table 2. Iterations Required for Model Convergence Using Selected Nonlinear Simulation Techniques, 1975-1984

	Nonlinear Simulation Technique			
Date	Jacobi	Gauss-Seidel	Damped Jacobi	Damped Gauss-Seidel
1975-1	20	4	20	6
-2	20	4	18	7
-3	135	29	62	6
-4	13	4	17	6
1976-1	16	3	18	5
-2	16	3	16	7
-3	60	11	38	5
-4	14	3	19	5
1977-1	16	3	18	5
-2	16	4	18	6
-3	33	6	27	6
-4	14	3	15	7
1978-1	15	3	18	5
-2	15	4	17	8
-3	185	45	62	5
-4	14	3	17	8
1979-1	16	3	18	4
-2	16	3	18	7
-3	50	10	34	5
-4	14	3	15	4
1980-1	15	3	18	3
-2	16	3	16	5
-3	51	10	40	4
-4	14	3	18	7
1981-1	19	3	18	3
-2	20	3	18	4
-3	57	13	38	3
-4	13	3	19	7
1982-1	16	3	18	4
-2	16	3	16	6
-3	140	42	58	4
-4	14	3	19	8
1983-1	18	3	18	4
-2	20	4	19	7
-3	48	11	31	5
-4	13	3	19	9
1984-1	45	11	34	5
-2	26	5	21	7
-3	525	140	75	11
-4	15	4	23	10

As a result, small changes between iterations in corn stocks and use cause relatively large changes in the inverse stocks-to-use ratio which result in relatively large corn price changes in the steep parts of the third quarter corn price hyperbola.

To address this, a damping factor was used on the corn price equation for both simulation techniques to limit the change between iterations--for any particular simulation period, the damped endogenous variable on the i^{th} iteration is defined as

$$y^*_i = d\ y_i + (1 - d)\ y^*_{i-1}$$

with y_i representing the undamped endogenous variable for iteration i, y^*_i and y^*_{i-1} representing the damped endogenous variable for iterations i and i-1, respectively, and d representing a damping factor between 0 and 1 (4).[3] All variables in these simulations converged to essentially the same values as in the undamped simulations, so damping did not affect the model's solution. However, computational efficiency was improved. Table 2 shows the iterations required using the damped Jacobi and damped Gauss-Seidel simulation techniques and indicates that all third quarters converged more quickly except for the Gauss-Seidel simulation for 1977-3 which required the same number of iterations. Although most of the other quarters required slightly more iterations to converge, the average number of iterations over all simulation periods was sharply reduced (see table 3). This suggests that when implementing large-scale forecasting models, damping can improve computational efficiency if particular equations in the model can be identified that cause a relatively large number of iterations to be required for the model to converge.

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[3] The damping factor used in these simulations was 0.5. It was chosen for purposes of illustration and may differ from the optimal damping factor.

Table 3. Summary Statistics of Iterations Required for Model Convergence

Summary Statistic	Nonlinear Simulation Technique: Jacobi	Gauss-Seidel	Damped Jacobi	Damped Gauss-Seidel
All simulation periods:				
Minimum	13	3	15	3
Maximum	525	140	75	11
Mean	45.0	10.6	25.5	5.8
First quarters:				
Minimum	15	3	18	3
Maximum	45	11	34	6
Mean	19.6	3.9	19.8	4.4
Second quarters:				
Minimum	15	3	16	4
Maximum	26	5	21	8
Mean	18.1	3.6	17.7	6.4
Third quarters:				
Minimum	33	6	27	3
Maximum	525	140	75	11
Mean	128.4	31.7	46.5	5.4
Fourth quarters:				
Minimum	13	3	15	4
Maximum	15	4	23	10
Mean	13.8	3.2	18.1	7.1

Summary and Conclusions

Quarterly hyperbolic equations have been estimated relating corn and wheat prices to ending levels of their respective stocks. Three different definitions for stocks were considered--total stocks and two alternative free stocks concepts. For corn, the first free stocks definition, which subtracts CCC-owned stocks and farmer-owned reserve stocks from total stocks, was generally preferred. For wheat, the total stocks definition was marginally preferred to the first free stocks definition. Results are consistent with expectations that higher stocks relative to use in any particular quarter give lower prices. Any particular level of stocks gives lower prices later in the marketing year than the same level of stocks does earlier in the marketing year. Beyond sample estimates for 1982 and 1983 indicate reasonably good performance for the estimated equations.

Incorporating the estimated price equations into a large-scale econometric forecasting model for U.S. agriculture results in simulations requiring a relatively large number of iterations to converge in the third quarter each year using either the Jacobi or the Gauss-Seidel simulation methods. This is primarily a consequence of the free stocks definition used in the corn price equation along with the hyperbolic functional form employed which made the inverse stocks-to-use ratio sensitive to changes in stocks and use between iterations and which tended to push the iterations into the steeper ranges of the third quarter corn price hyperbola when free stocks are relatively low at the end of the corn marketing year. Using damping factors on the corn price equation to limit the changes between iterations leads to faster model convergence. This illustrates how damping factors can be used to improve computational efficiency of nonlinear simulation techniques and suggests that this approach can be used more generally when implementing forecasting models if particular equations in a system cause a large number of iterations to be needed for convergence.

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