A Note on Composite Forecasting Technique: The Case of Multiple-Step-Ahead

by

Donald J. Liu and Terry L. Roe

A NOTE ON COMPOSITE FORECASTING TECHNIQUE: The Case of Multiple-Step-Ahead

Donald J. Liu and Terry L. Roe*

It is quite typical that, preceeding an economic decision, several forecasts of a future event are available to the agent. The question arises as to whether a single or some composite forecast is to be chosen. It has been argued that since each alternative forecasts almost always contain a unique component of information, it might be beneficial to incorporate different forecasts into an overall combined one. Moreover, given the already available single forecasts, the agent should always be willing to formulate a composite forecast with little additional cost.

Numerous studies have focused on this point. Bates and Granger's combining method involves minimizing the mean squared forecasting error of the resulting composite forecast. Nelson proposed a "regression technique" to find the optimal weights assigned to individual forecasts. Bessler and Brandt used the approach developed by Bates and Granger to combine forecasts from expert's opinion, an econometric model, and a time series model, for selected commodity prices. Their results favored composite forecasts and they recommend the use of a composite forecast whenever possible.

The above studies, However, only deal with methods of combining individual one-step-ahead forecasts. Since economic decisions, often time, are based on the agent's subjective probability distribution of the event which is

---

*Donald Liu and Terry Roe are a graduate student and a professor, respectively, in the Department of Agricultural and Applied Economics at the University of Minnesota. The authors wish to thank Maureen Lahiff and Paul Fackler for their helpful comments and suggestions.

to occur in the more distant future, the study of multiple-step-ahead compsite forecasting methods is potentially rewarding.

The purpose of this note is to present a method to combine different multiple-step-ahead forecasts derived from different forecasting models. To provide motivations for the method to be proposed, Bates and Granger's minimum mean squared forecasting error (MMSFE) approach of combining individual one-step-ahead forecasts is first reviewed. Their method is shown to be identical to Nelson's "regression technique" (RT) in section II. Then, the multiple-step-ahead composite forecasting method is presented. The empirical results of applying the proposed method to the monthly slaughter steer price and concluding remarks are presented in section III and IV, respectively.

I. THE MMSFE APPROACH

For ease of exposition, consider only two forecasting models. Let p_1 and p_2 be the two unbiased one-step-ahead forecasts derived from the two models. Denote the weights assigned to p_1 and p_2 as w_1 and w_2, respectively. Then, the composite of the above two individual forecasts can be expressed as

$$p_c = w_1 p_1 + w_2 p_2$$

To preserve the property of unbiasedness of the two individual forecasts, Bates and Granger consider the case where the composite weights sum up to one. Subject to a unit total-weight constraint, they combine the two individual forecasts in such a way that the mean squared forecasting error of the resulting composite forecast is minimized. Let e_i be the forecasting error associated with the i-th forecast (i = 1,2). Then, subject to $w_1 + w_2 = 1$, the optimal weights can be found by minimizing

$$MSFE_c = E[w_1 e_1 + w_2 e_2]^2 \qquad (1)$$

$$\equiv w_1^2 MSFE_1 + w_2^2 MSFE_2 + 2 w_1 w_2 E[e_1 e_2]$$

where $MSFE_j = E[e_j]^2$ is the mean squared forecasting error of the j-th forecast (j = 1,2,c), and E is the expectation operator.

Differentiating (1) with respect to w_1 and using the unit total-weight constraint, the optimal weight assigned to the first forecast is

$$w_1^* = \{MSFE_2 - E[e_1e_2]\} \;/\; \{MSFE_1 + NSFE_2 - 2\,E[e_1e_2]\} \tag{2}$$

Bates and Granger showed that the mean squared forecasting error of the composite forecast obtained in the above fashion can not be greater than those of the individual forecasts. Hence, there is a potential for one to reduce the mean squared forecasting error through the employment of their combining method.

Since (2) involves the covariance $E[e_1e_2]$ between forecasting errors of the two individual forecasts, it has no practical use in specifying the optimal weights. To cope with this problem, Bates and Granger assumed that e_1 and e_2 are binormally distributed and derived the maximum likelihood estimate of the optimal weights as

$$w_1^* = \frac{\sum_\tau \{e_{2,\tau}^2 - e_{1,\tau}e_{2,\tau}\}}{\sum_\tau \{e_{1,\tau}^2 + e_{2,\tau}^2 - 2\,e_{1,\tau}e_{2,\tau}\}} \tag{3}$$

where $e_{i,\tau}$ is period τ in-sample forecasting error from model i; i = 1,2.

II. THE RT APPROACH

Given the two individual forecasting models, Nelson proposed to obtain two sets of in-sample forecasts against which the observed series can be regressed. The estimated coefficients associated with the two regressors are then assigned as the composite weights for the corresponding individual forecasts of the future event. That is, the composite weights are obtained by applying ordinary least squares to the following

$$p_\tau = w_1p_{1,\tau} + (1-w_1)p_{2,\tau} + u_\tau \tag{4}$$

where $p_{i,\tau}$ is defined as the one-step-ahead in-sample forecast of p_τ, obtained from model i; i = 1,2.

The advantage of Nelson's approach of combining different one-step-ahead forecasts is its ease of implementation and, hence, it facilitates generalization to the case of multiple-step-ahead. Before generalizing his method to admit the case of multiple steps, a justification of his method is provided in the following. In particular, it will be shown that the RT approach is equivalent to the MMSFE approach of Bates and Granger. Hence, the optimality of the former is demonstrated.

To see this, note that (4) is equivalent to:

$$(p_\tau - p_{2,\tau}) = w_1 (p_{1,\tau} - p_{2,\tau}) + u_\tau$$

Now, since

$p_\tau - p_{2,\tau} \equiv e_{2,\tau}$ and

$p_{1,\tau} - p_{2,\tau} \equiv e_{2,\tau} - e_{1,\tau}$,

the regression in (4) is equivalent to regressing $e_{2,\tau}$ on $(e_{2,\tau} - e_{1,\tau})$, which yields the optimal weights proposed by Bates and Granger (i.e. (3)).

Furthermore, since p_τ is identically equal to $w_1 {*} p_{1,\tau} + (1 - w_1){*}p_{2,\tau} + (w_1 {*} e_{1,\tau} + (1 - w_1){*}e_{2,\tau})$, the error term of equation (4) has the interpretation of being a linear combination of the two in-sample forecasting errors. Hence, what Nelson amounts suggests is that one chooses the optimal composite weights to be assigned to individual forecasts of the future event in such a way that the linear combination, using the weights, of the individual in-sample forecasts best explains the historical series.

This provides insights into a possible method of combining forecasts with a lead-time more than one-step. In particular, suppose one is interested in the optimal composite weights for two individual h-step-ahead forecasts. Then, two sets of h-step-ahead in-sample forecasts could be simulated from the two individual forecasting models, where the optimal composite weights can be determined by regressing the observed series on the two sets of in-sample forecasts obtained. That is, the regression equation of the following can be employed:

$$p_\tau = w_1 p^h_{1,\tau} + (1-w_1)p^h_{2,\tau} + u_\tau \qquad (5)$$

where $p^h_{i,\tau}$ denotes the h-step-ahead in-sample forecast of p_τ, obtained from model i.

This is essentially the procedure developed and applied in this paper. The optimality of the procedure can be demonstrated by a direct generalization of the argument made previously. The problem of autocorrelation, however, is a possible complication of the above "modified RT" method. This problem arises in the stage of parameter estimation of the individual forecasting models, because the current statistical methods only minimize some "distance" of the in-sample one-step-ahead forecasting errors (i.e. observed residuals). Consequently, the in-smple multiple-step-ahead forecasting errors from the individual forecasting models will not necessarily possess white noise properties. Hence, it cannot be guaranteed that the resulting linear combination of these forecasting errors (ie. u_τ of (5)) will behave like a white noise process.

If autocorrelation poses a problem, it is proposed that one adds a time series model (ARIMA) for the error structure of (5). This not only has the function of coping with the problem of autocorrelation, it also serves as a way to ease the bias of the current statistical estimation techniques against multiple-step-ahead forecasts. Then, in obtaining composite forecast of the future event, the following equation can be employed:

$$p_f = w_1 p^h_{1,f} + (1 - w_1)p^h_{2,f} + \hat{u}_f \qquad (5')$$

where f refers the period of the future event to be forecasted and $\hat{u}_f$ is the forecast of u_f.

III. AN APPLICATION

Both econometric (reduced-form) and time series analyses are used to obtain forecasting models for the monthly price (measured in dollars per hundred weight) of slaughter steers at Omaha (Choice Grade, 900-1,100 pounds). The time series extends the period from the first month of 1970 (70,1) through the twelfth month of 1983 (83,12).

Only forecasts up to three-steps-ahead are considered. The forecast commences with the twelfth month of 1981. Using the sample data ranges from 70,1 to 81,12, all the first three-steps-ahead forecasts are obtained from the econometric and from the time series models (i.e. forecasts are obtained from both models for the periods of 82,1, 82,2, 82,3). Then, the forecast is advanced one period to the subsequent month. With the newly realized data of 82,1 on the relevant variables being incorporated into the forecasting models (i.e. values of parameter updated), the forecasts of the next three months (82,2, 82,3, 82,4) are obtained. This procedure is iterated until data up to the eleventh month of 1983 is incorporated into the two forecasting models and forecasts of the subsequent month are obtained.

Econometric Model: Estimation

The econometric reduced-form involves explanatory variables of number of cattle on feed (CANOF), pig crop (PCROP), both in thousands of head, personal disposable income (DPI) in billions of 1972 dollars, the ratio of the price of cull cow to price of milk received by farmers (RATIO); both measured in dollars per hundred weight, and a dummy variable (with a dividing point of 79,1) to capture the effect of recent high inflation and interest rates. Since the estimated coefficients and relevant statistics appear to be stable over the evaluation periods (i.e. 81,12 to 83,11), only those obtained at 81,12 are reported here.

$$PS_t = \underset{(3.1)}{15.15} + \underset{(18.4)}{23.27}\ DUMMY - \underset{(1.9)}{0.0009624}\ CANOF_{t-4} \tag{8}$$

$$- \underset{(2.0)}{0.000770}\ PCROP_{t-4} + \underset{(6.0)}{5.689}\ RATIO_{t-1} + \underset{(6.4)}{0.01895}\ DPI_t$$

$$+ \underset{(7.5)}{0.6746}\ e_{t-1} + \underset{(1.8)}{0.1745}\ e_{t-12} - \underset{(3.0)}{0.2920}\ e_{t-13}$$

where $RBAR^2 = 0.95$, SEE = 2.6, D.W. = 1.74, and Q(27) = 16.5.

The above fitted model indicates that all the estimated coefficients are significant and their signs remain intact. The adjusted R^2 increases slightly over that of (6), and the problem of autocorrelation corrected.

Time Series Model: Estimation

Since the time plot of the prices indicates nonstationarity of the series, first-difference is taken. Through Box and Jenkins's three-step procedure, the following ARIMA model was obtained:

$$(1-B)\ PS_t = \underset{(1.2)}{0.2013} + (1 + \underset{(3.5)}{0.2279}\ B + \underset{(2.6)}{0.1600}\ B^{12})\ a_t \tag{9}$$

where SEE = 2.74, D.W. = 1.90, Q(27) = 19.06, a_t is a white-noise disturbance, and B is the lag operator.

Again, the Durbin-Watson and Q statistics indicate the appropriateness of the model. Furthermore, the coefficients associated with the two moving-average terms suggest invertibility of the process. The reported standard error estimated is rather high for the first-differenced series. Some seasonal-differenced (i.e. twelve months) models also are tried and the resulting estimated standard errors decrease drastically. These models, however, poses the problems of either noninvertibility or nonstationarity.

Individual Model Forecasts

After the estimations of (8) and (9), forecasts of the future prices can be obtained. Ideally, one needs to obtain forecasting models and make relevant forecasts of regressors in order to make use of (8). To limit the scope of the study, naive forecasts of no change are employed for the regressors of (8). However, forecasts of e_{t-1}, e_{t-12} and e_{t-13} of (8) are obtained from (7). All the three steps forecasts with forecasting origin ranging from 81,12 to 83,11 are presented in Table 1.

Composite Forecasts

From (8) and (9), in-sample forecasts with different lead time are obtained for the most recent seventy sample points. From these, the "modified RT" method of (5) is used to find the optimal composite weights. All the weights assigned to the econometric forecasts are reported in Table 2. Not surprisingly, the econometric forecast receives a heavier weight as the step of forecast increases. This is due to the fact that time series analysis is an extrapolation and, hence, there is a component of cumulative error associated with its multiple-step-ahead forecasts.

Performance Evaluation

The criterion used to evaluate the performance of forecasts is the root mean squared forecasting error (RMSFE). The evaluation for the naive forecasts of no change is presented in column 2 of Table 3 and it is there to serve as a benchmark of the performance of other models (see Theil U statistics). Columns 3 and 4 report the performance of econometric and time series models, respectively. Three types of composite forecasts are evaluated. The first two are the one with the "modified RT" composite weighting scheme and the one with equal weights. Their performance are presented in columns 4 and 5, respectively. The last column reports the third composite method which assigns the optimal one-step composite weights to forecasts with a lead time more than one-step. Its purpose is to provide insights into whether the trouble of finding the optimal composite weights for multiple-step-ahead forecasts is useful relative to just arbitrarily using those weights associated with one-step-ahead forecasts.

As indicated by all the U statistics' reported in Table 3, the naive forecasting method clearly is not desirable. Between the econometric and time series forecasting models, the former outperforms the latter. Furthermore, as the lead time of the forecast increases, the superiority of the econometric model to the time series one increases. Respectively, for forecasts with a lead time ranges from one step to three steps, the RMSFE of the former is 7 percent, 11 percent, and 15 percent less than that of the latter. As mentioned earlier, this result is not surprising because of the extrapolative nature of the time series forecasting model.

The "modified RT" method of combining different forecasts does not seem to perform well in the analysis; its RMSFE is 3 percent, 2 percent and 4 percent more than those associated with the econometric forecasts. Recall that, in deriving the composite weights by "modified RT" method, the in-sample forecasts of the most recent seventy periods are chosen arbitrarily. A possible explanation of the unsatisfactory result of the RT method is that the relative performance of the two individual forecasting methods is not stationary over time. This suggests that an application of a weight system that can adapt fairly quickly through time to the in-sample forecasts when (5) is used to obtain the composite weights is in order (see Granger and Newbold).

The use of equal weighting scheme to combine different forecasts appears to be satisfactory for one-step-ahead forecast; its RMSFE is slightly less than that associated with econometric forecast. For forecasts with a lead time more than one-step-ahead, however, it is outperformed by the econometric method.

Finally, in comparing the fifth and the seventh columns of Table 3, one finds that it is not desirable to just assign the optimal one-step-ahead composite weights to forecasts with a lead time more than one.

IV. CONCLUSIONS

The note extends previous studies on combining different one-step-ahead forecasts derived from different models to the problem of multiple-step-ahead. The "regression" method of combining different forecasts is shown to have its justification and ease of implementation. To demonstrate the feasibility of the method, multiple-step-ahead forecasts of slaughter steer price are obtained. The composite forecasts are found to be slightly outperformed by the best of the individual forecasts, which suggests a possible nonstationary relative performance of the individual forecasting models considered. Since the additional cost of formulating composite forecasts is relatively low, however, the employment of the method may prove to be rewarding in other applications.

REFERENCES

Bates, J. and Granger, C. "The Combination of Forecasts." Oper. Res. Q. 20(1969): 451-468.

Box, G. and Jenkins, G. Time Series Analysis: Forecasting and Control. San Francisco: Holden-Day, 1967.

Bessler, D. and Brandt, J. "Composite Forecasting of Livestock Prices: An Analysis of Combining Alternative Forecasting Methods." Purdue University Agricultural Experiment Station Bulletin 265,1979.

Granger, C. and Newbold, P. Forecasting Economic Time Series. New York: Academic Press, 1977.

Nelson, C. Applied Time Series Analysis: For Managerial Forecasting. San Francisco: Holden-Day, 1973.

Table 1. Steer Price Forecasts from Individual Models

		one-step-ahead		two-step-ahead		three-step-ahead	
period	actual price	econo-metric	time series	econo-metric	time series	econo-metric	time series
82, 1	60.75	60.0899	59.1990				
82, 2	63.54	60.8514	61.0333	60.4920	59.1112		
82, 3	65.80	63.4880	64.4223	60.8653	61.3160	60.6263	59.3760
82, 4	69.11	67.1258	66.9703	6 .6112	65.2540	62.4667	62.1478
82, 5	72.10	68.4140	69.9660	66.7593	67.3066	64.6913	65.5758
82, 6	70.18	71.1622	73.0780	67.8940	70.3982	66.4278	67.7052
82, 7	66.18	67.5106	69.5643	68.9896	73.1713	66.4178	70.4665
82, 8	65.14	65.0830	65.3404	66.3938	69.5958	67.5343	73.2369
82, 9	61.25	64.8129	65.3137	65.1771	65.5654	66.0479	69.8553
82,10	58.78	61.3140	59.7868	64.6992	64.9067	65.0144	65.1264
82,11	58.91	59.8389	58.6520	62.8651	59.9112	65.4575	65.0739
82,12	58.92	59.9189	59.0651	61.7346	58.7398	64.2365	60.0158
83, 1	59.33	59.6239	59.3438	61.0887	59.5267	62.7205	59.1997
83, 2	61.20	61.3230	59.9469	60.7916	59.9640	62.0617	60.1479
83, 3	64.03	62.5375	61.9499	61.2881	60.3663	60.7270	60.3832
83, 4	67.70	64.9998	65.1531	63.2749	62.5197	61.9896	60.9176
83, 5	67.51	67.4864	69.0158	65.3250	65.7728	63.7810	63.1118
83, 6	65.90	66.0482	66.8452	66.3225	68.7346	64.5918	65.4855
83, 7	62.22	63.2050	65.2962	64.0353	66.4984	64.1441	68.3943
83, 8	61.27	62.6502	61.6088	63.6296	65.4992	64.2719	66.7107
83, 9	59.19	60.8087	60.5736	62.3817	61.0026	63.1579	64.9731
83,10	59.58	60.6943	58.8573	62.3296	60.6017	63.6540	61.0343
83,11	59.41	61.1057	59.9884	62.0641	59.0755	63.4564	60.8340
83,12	62.85	59.9075	59.4165	61.4361	60.1471	63.1887	59.2292

Table 2. Composite Weights Assigned to Econometric Forecasts

period	one-step-ahead	two-step-ahead	three-step-ahead
82, 1	0.2096044		
82, 2	0.2095381	0.4521422	
82, 3	0.2101498	0.4397176	0.5679715
82, 4	0.2079955	0.4418042	0.5524254
82, 5	0.1588654	0.4416731	0.5539182
82, 6	0.1456151	0.4258887	0.5591999
82, 7	0.1586575	0.4053025	0.5428721
82, 8	0.1535861	0.4078091	0.5121775
82, 9	0.1540877	0.4176141	0.5147772
82,10	0.1554655	0.4254993	0.5197411
82,11	0.1434905	0.4308922	0.5334644
82,12	0.1420869	0.4266716	0.5514723
83, 1	0.1392764	0.4243005	0.5440822
83, 2	0.1400958	0.4244024	0.5457541
83, 3	0.1403157	0.4245012	0.4229179
83, 4	0.1412271	0.4253902	0.5430313
83, 5	0.1349672	0.4292292	0.5428363
83, 6	0.1362963	0.4177608	0.5398930
83, 7	0.1345732	0.4204099	0.5279173
83, 8	0.1303911	0.4207001	0.5288744
83, 9	0.1306409	0.4212860	0.5257015
83,10	0.1260006	0.4296717	0.5332190
83,11	0.1234597	0.4222939	0.5394271
83,12	0.1160999	0.4212961	0.5409675

Table 3. Forecasting Performance Evaluations

	Individual Forecasts			Composite Forecasts		
evaluations	naive	econometric	time series	modified RT method	equal weight	Bates & Granger's weight
one-step-ahead						
RMSFE	2.38	1.86	2.01	1.92	1.84	
U(1)		0.78	0.85	0.80	0.77	
two-step-ahead						
RMSFE	4.19	3.24	3.64	3.31	3.29	3.51
U		0.77	0.87	0.79	0.78	0.84
three-step-ahead						
RMSFE	5.74	4.33	5.11	4.50	4.54	4.91
U		0.75	0.89	0.78	0.79	0.86

(1): Theil U is the ratio of the RMSFE of the forecasting method in question to the RMSFE of the naive forecast of no change.