Changing Variances and Thick-tailed Distributions in Commodity Prices: Estimates and Implications for Price Forecasting

by

J. Douglas Gordon and Richard Heifner

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Most econometricians working on forecasting problems assume that the disturbances in their models are drawn from normal probability distributions with constant variances. These assumptions often are made for analytical convenience with little examination of the actual distributions involved. The evidence that has been assembled, particularly for short-term movements of futures and stock prices, generally indicates that disturbances follow leptokurtic probability distributions, that is distributions with possibly the same mean, but higher peaks, thicker tails, and fewer values in between, than the normal distribution (Figure 1). We believe that these differences between assumptions and observations deserve more attention by forecasters.

Many distributions of actual or percentage changes in prices resemble normal distributions. Only on closer examination are they found to be leptokurtic. To represent such phenomena Mandelbrot proposed the stable (Paretian) distributions, of which the normal distribution is a special case. (Characteristics of stable distributions are described in the Appendix). Except for the normal case, the stable distributions have infinite variances. This makes most of the classical statistical techniques inapplicable. To quote Cootner,

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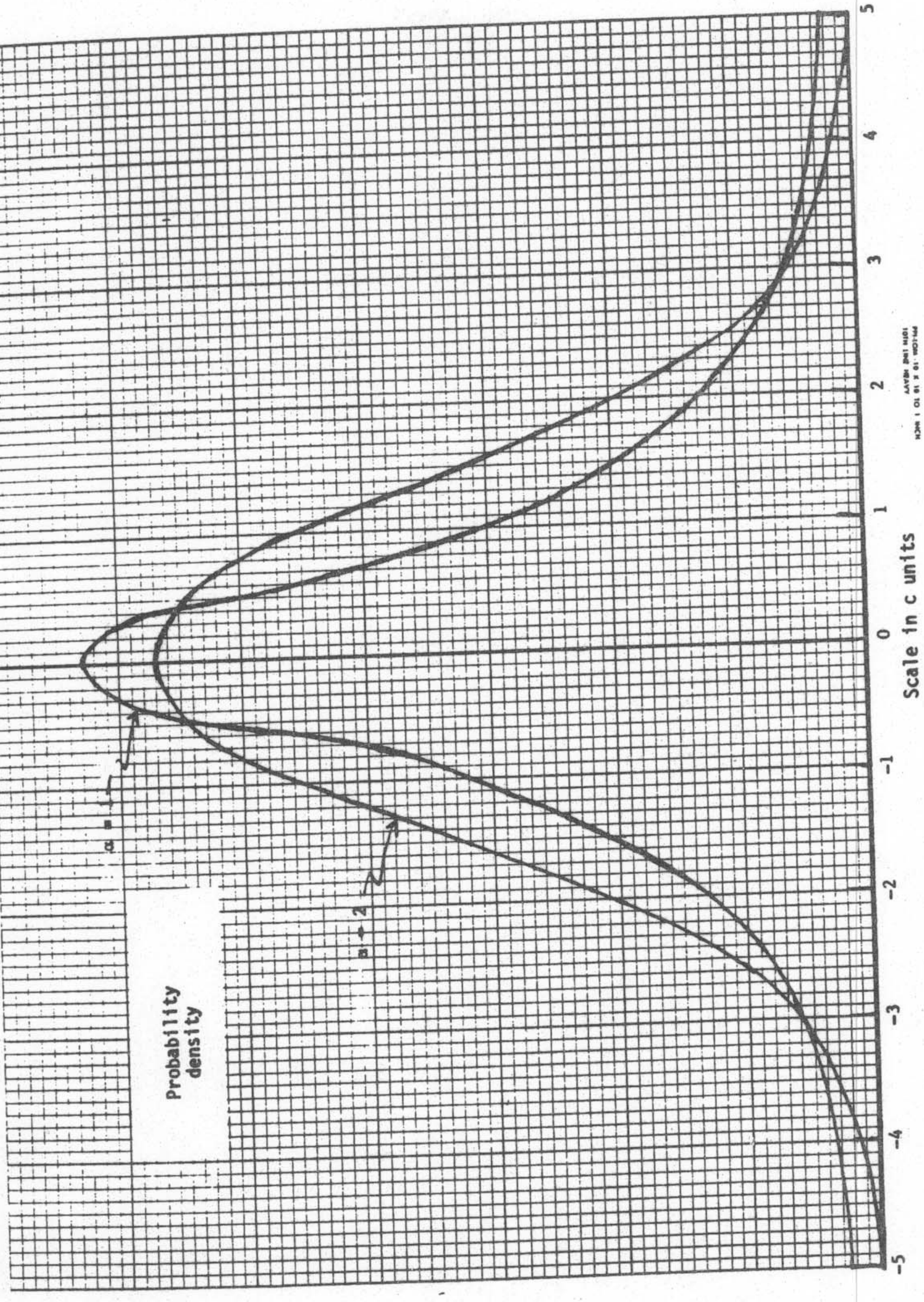


Figure 1 - Example stable distributions

> "In general, the implications of Mandelbrot's model are revolutionary. Linear regression is possible though not by least-squares and is difficult to interpret. Correlation analysis and spectral analysis are meaningless. Because the stable distributions are not generally available in closed form, maximum-likelihood techniques are difficult to apply...."

Thus, the stable distribution hypothesis leads to considerable analytical difficulties, particularly in calculating confidence intervals and performing significance tests.

Mixtures of variables drawn from normal distributions with the same means but different variances can also exhibit thick tails. This offers an alternative explanation for the apparent leptokurtosis observed in price movements. Perhaps the shocks or disturbances are indeed normally distributed with constant mean and finite variances, but the variances change continually. The constant variance assumption also underlies most of the traditional statistical methods. However, relaxing this assumption has less sweeping implications than relaxing the assumption of normality. For example, least squares regression remains useful. It is unbiased, although not efficient. Moreover, statistical tests and confidence interval estimation may be possible either by transforming the data or by using weighted regression.

The variance of futures price changes has taken on new importance with the advent and expansion of commodity options trading. The value of a commodity option depends upon five factors: the strike price, the price of the underlying futures contract, time to maturity, interest rate, and the variance of the underlying futures price (Black). The first three are known and interest rates have relatively small effects on option values. However, futures price variance is a key determinant of the value of a commodity option and the science of forecasting price variances remains in its infancy.

In this paper, we first review previous findings regarding the shape of the probability distribution and the constancy of variance for commodity futures price changes. Second, we summarize some empirical work from Gordon's forthcoming bulletin and present additional results concerning variances of futures prices. Finally, we consider the implications for price forecasting.

Previous Studies

The empirical measurement of departures from normality and changes in variances of market prices is of fairly recent origin. In 1960, Larson observed that daily changes in corn prices during 1922-31 and 1949-58 exhibited more extreme values than expected under the normal distribution. Houthakker drew similar conclusions about cotton prices. Mandelbrot plotted positive and negative tails of cotton prices (cash prices) on a double-log graph to show how they resembled the cumulative density function of a stable distribution. Stevenson and Bear plotted observations of price changes for July corn and July soybeans on normal probability paper, obtaining an S-shaped curve indicating leptokurtosis.

Using methods developed by Fama and Roll, Mann and Heifner fitted members of the symmetric stable family of distributions to futures price changes for the major agricultural commodities. Price changes were found to be best characterized by infinite variance distributions. Estimated values of α, the characteristic exponent in the stable distribution, were mostly well below 2, indicating departures from normality.

Recently, Cornew, Town, and Crowson, using a technique that avoids the a priori assumption of symmetry made by earlier analysts, have found that non-normal stable distributions fit logarithms of futures price relatives better than normal distributions for 16 of 18 contracts. Most estimates of α fell between 1.5 and 1.65.

Changing Variances

Although commodity traders clearly recognize that price volatility changes over time, economists have not successfully explained many of these changes. Samuelson, 1965 and 1976, theorized that the volatility of futures prices would increase as contract maturity approached. Rutledge found only two of four contracts examined followed the Samuelson hypothesis. Miller found a significant trend in the variability of live cattle futures prices over the life of the contract. Anderson used regression and a non-parametric test to explore the effect of season and months to contract maturity on futures price variability for nine commodities. He found strong seasonal effects for cattle and silver as well as crops. The effect of months to maturity was weaker but agreed in sign with the Samuelson hypothesis for all nine commodities.

In a recently published study, Roll found variability of returns on orange juice futures higher during periods when news stories were published about crop forecasts, retail supplier antitrust actions, and international events, than during "no news" periods. As might be expected for orange juice, the highest volatility was during periods when weather news was published.

Empirical Results

The results reported here are based upon daily closing prices for corn, soybean, wheat, cotton, live cattle, hog, and orange juice futures contracts maturing from January 1979 to May 1984. Each contract was analyzed separately over its lifetime, which consisted of between 200 and 350 price changes. The data used in the tests are changes in logarithms of daily closing prices. 1/

1/ The change in the natural logarithm of price approximately equals the percentage price change ÷ 100 for changes between ± 10 percent. Data were provided by the Commodity Futures Trading Commission. Limit price moves were included in the series.

In addition to the tests for normality and constant variance reported here, the larger study included turning point tests for serial independence or efficiency, and difference-sign tests for trend. Based upon the turning point tests the efficient market hypothesis was not rejected for the commodities covered in this paper. Price trends were frequently indicated for live cattle and hogs, but not for the other commodities. Detailed results of the tests for efficiency and trend are reported in Gordon's forthcoming bulletin.

Tests for Normality Over the Life of Contracts

There are several nonparametric tests for normality. We tested the data for normality with the X^2 goodness of fit test. This test enables one to look closely at the tails of the distribution. To apply the test, the observations in each series were grouped into deciles and the number of observations in a given decile was compared with the expected number. The null hypothesis that the distribution of price changes is distributed normally with constant mean and variance was rejected when the test statistic T, exceeded the tabular value of X^2 at the 5 percent level with 7 degrees of freedom. By this criterion normality was rejected for 39 to 82 percent of the contracts for each commodity. Such large percentages in the significant range would occur less than 1 time in 100 if the underlying distributions were all normal.

Estimates of α , the characteristic exponent in the stable distribution, were also calculated using the method described by Fama and Roll. In most contracts, the characteristic exponent was less than 2 as shown Table 1. For many contracts the estimate was less than 1.5. The calculated parameters suggest that changes in logarithms of futures prices may follow infinite variance distributions.

Table 1: Estimates of characteristic exponents over the life of contract 1/

Commodity	Number of contracts tested	$\alpha = 2$	$1.75 \leq \alpha < 2$	$1.5 \leq \alpha < 1.75$	$1.0 \leq \alpha < 1.5$	$\alpha < 1.0$
Corn	27	1	12	11	3	0
Cotton	28	0	1	23	3	0
K.C. wheat	27	0	2	17	8	0
Live cattle	34	6	9	15	4	0
Live hogs	37	0	2	28	7	0
Orange juice	33	0	2	9	22	0
Soybeans	38	1	4	30	3	0

1/ The characteristic exponent, α , of the family of symmetric stable distributions can range from 0 to 2. The only value associated with a finite variance distribution (the normal distribution) is $\alpha = 2$.

Table 2: Estimates characteristic exponents for 2 month intervals 1/

Commodity	Number of contracts tested	$\alpha = 2$	$1.75 \leq \alpha < 2$	$1.5 \leq \alpha < 1.75$	$1.0 \leq \alpha < 1.5$	$\alpha \leq 1.0$
Corn	22	9	5	4	4	0
Cotton	22	5	6	8	3	0
K.C. wheat	22	3	3	8	8	0
Live cattle	20	10	2	5	3	0
Live hogs	20	6	2	8	4	0
Orange juice	22	4	4	5	9	0
Soybeans	22	9	6	4	3	0

1/ The characteristic exponent, α , of the family of symmetric stable distributions can range from 0 to 2. The only value associated with a finite variance distribution (the normal distribution is $\alpha = 2$).

Tests for Normality Over Two Month Intervals

If thick tails in an observed distribution arise not because the underlying distribution is stable, but because the data are from a mixture of normal distributions with different variances, then observed distributions for shorter intervals with presumably more homogeneous variances, should be more like the normal. To test for this possibility X^2 goodness of fit tests were applied and characteristic exponents were estimated for price changes over two-month intervals within the life of each contract.

The results of these X^2 tests for two-month intervals contrast sharply with those applied over the full life of contract. In five of the seven markets the null hypothesis of normality could not be rejected at the 95 percent level. In the two markets which rejected normality, a lower percentage of contracts rejected normality. This suggests that price changes approach normality as the period of observation is shortened.

The corresponding estimates of characteristic exponents are summarized in Table 2. The number of estimates having $\alpha = 2$ or $1.75 \leq \alpha < 2$ was much greater with the two month samples than when whole contracts were tested (table 1). The hypothesis of normality is not as strongly supported by these data as by the results of the X^2 goodness of fit test, yet there are many more characteristic exponents equalling or approaching 2 than when whole contracts were considered. These tests suggest that normality in the percentage price changes of futures contracts may be a reasonable assumption when the periods analyzed occur during the same season and at the same distance to maturity.

Tests for Constant Variance

Constancy of variance was tested directly using the nonparametric squared ranks test described by Conover. The variance of price changes for a two month period in the winter, January and February, was tested against the variance for a two month period in the summer, in most cases July and August. For most crops we expected price variability to be greater in the summer months due to the arrival of weather information bearing upon crop yields. For orange juice, by contrast, winter freeze information has greater price impacts. The livestock futures markets should show less seasonality, because livestock are produced and marketed year-round.

The results from the variance tests are summarized in table 3. The entries in the table are the calculated statistics for the Conover test. A positive entry indicates that the variance was larger in the winter than in the summer; a negative entry indicates the opposite. The preponderance of negative values indicates that corn, wheat, cotton and soybean price variability is generally greater duiing the summer than during the winter. The difference is greatest for corn where for nine of the eleven contracts the difference was statistically significant at the 5 percent level. For the other three field crops about half of the contracts exhibited significantly larger summertime price variances while only 4 out of 33 contracts exhibited positive signs indicating larger wintertime variance, and only one of these was significant at the 5 percent level.

All of the orange juice contracts exhibited greater variability in January-February than July-August. For 8 out of 11 contracts the difference was statistically significant at the 5 percent level. The results for cattle and hogs exhibit no clear pattern of seasonal differences in price variability.

Both fall and spring maturing contracts are shown in table 3 to provide information bearing upon the Samuelson hypothesis of increasing price volatility. If the Samuelson hypothesis held and seasonality were absent,

Table 3: Seasonality of variance: Calculated values of the T_1 statistic for inequality of variance 1/

Contract maturing 2/	Corn	Cotton	KC wheat	Live cattle	Hogs	Orange juice	Soybeans
Fall 1979	-5.33*	-2.71*	-5.16*	-4.41*	-4.88*	0.56	-2.84*
Fall 1980	-4.08*	-2.33*	-1.14*	-0.10	-0.03	2.04*	-4.06*
Fall 1981	-2.74*	-3.04*	2.10*	-1.90	-0.42	3.50*	-0.30
Fall 1982	-1.77	-4.33*	-0.97	-2.08*	-1.82	4.65*	-2.28*
Fall 1983	-5.21*	-4.08*	-0.97	0.78	-4.35*	6.43*	-4.84*
Spring 1979	-4.49*	1.52	-3.33*	-1.37	-0.59	1.24	0.32
Spring 1980	-3.42*	3.35*	-3.36*	-0.95	-0.48	1.40	-1.88
Spring 1981	-2.90*	-1.96*	-0.15	-0.01	2.52*	6.15*	-1.81
Spring 1982	-3.75*	-1.06	-0.44	2.56*	2.35*	2.00*	-3.02*
Spring 1983	-0.55	-1.81	-2.53*	0.70	1.04	5.10*	-1.61
Spring 1984	-4.30*	-1.35	-1.92	—	—	4.95*	-1.67

*Indicates significance at the 5 percent level.

1/ Each statistic measures the inequality in variance for January and February versus July and August except for KC Wheat. The months tested for KC Wheat were January and February versus May and June. The T1 statistics are approximately normally distributed.

2/ The fall contracts were July for KC wheat, November for orange juice and soybeans, and December for corn, cotton, live cattle, and live hogs. The spring contracts were March for KC wheat, April for live cattle and live hogs, and May for corn, cotton, orange juice, and soybeans.

we would expect fall maturing contracts to exhibit larger variances in summer than winter and negative entries in the table. The opposite would apply for spring maturing contracts. Thus, a tendency for the results to be more negative for fall maturing contracts than for spring maturing contracts supports the Samuelson hypothesis. Comparison of the results for fall and spring maturing contracts lends support to this hypothesis for hogs and cotton, but the effects are uncertain for the other commodities.

In summary, the tests for equal variances show important seasonal differences in price variability for the crops. For some commodities a contract maturity effect on price variability is also indicated.

Longer Run Changes in Variance

Since 1972-74, the variability of most crop prices has been substantially larger than during the 1950's and 1960's. This increase in variance should carry over into the variance of daily price changes. Table 4 shows that this is indeed the case. For each of the 3 commodities examined in table 4, variance in the 1973-84 period was significantly greater than in the years 1960-72.2/ For cattle the size of the increase is nearly as great as for corn and soybeans.

In the last two rows of the table the years 1973-74 have been removed from the later period and examined separately. As expected the average variance during those two years was much greater than before or since. The remaining years of the later period, 1975-84, still show variance about double the size of that in the earlier period. These results suggest that

2/ As noted previously, the use of logarithms of price changes is analogous to working with percentage price changes. Thus, results for time periods with different average price levels are comparable.

Table 4: Standard deviations of changes in logarithms of price for selected futures contracts and time periods 1/

Contract & season	1960-84	1960-72	Years 1973-84	1973-74	1975-84
Dec. corn					
Summer	.1895	.1285	.2256	.4171	.2233
Winter	.1183	.0805	.1594	.2625	.1387
May corn					
Summer	.1809	.1196	.2422	.3253	.2271
Winter	.1168	.0784	.1584	.2740	.1353
Nov. soybeans					
Summer	.2117	.1167	.3145	.4282	.2918
Winter	.1270	.0649	.1942	.2990	.1732
May soybeans					
Summer	.1907	.1103	.2711	.1887	.2860
Winter	.1586	.1027	.2192	.2713	.2088
Dec. cattle					
Summer	.1515	.0866	.1948	.2705	.1796
Winter	.1349	.0812	.1662	.2161	.1562
June cattle					
Summer	.1456	.0725	.1681	.1945	.1633
Winter	.1509	.0932	.1894	.2236	.1825

1/ Annualized estimates = (250 x variance of daily price change)$^{1/2}$

the variance of futures price changes is influenced by, and can possibly be predicted from, factors in addition to the season of the year.

Figure 2 and table 5 show average monthly standard deviations of price by month for corn, soybean and cattle futures, over three time periods. For each contract examined, and for each commodity, the variance in the 1975-84 period is much greater than in the 1960-72 period. The years 1973-74 were omitted because of the unusually large movements in price during those years. Had they been added to the 1975-84 period average volatility would have been even higher.

Figure 2 points out another consequence of the change in economic climate and agricultural policy after 1972. The seasonal element of variance is much more pronounced for corn and soybeans than it was in the earlier period. The large grain stocks carried over each year in the 1960's buffered price movements caused by changing expectations about the size of the fall supplies. In the 1970's, uncertainties about foreign demand became an important factor and carryover stocks were relatively low. Low carryover means that a change in expectations about production or demand due to weather or other factors will have a greater effect on prices.

Since seasonality in price variability is inherently less for livestock than for crops, and largely unaffected by stock levels and government programs, we have no reason to expect marked changes in the seasonal pattern for cattle during recent years. The increase in price volatility due to the changes in agricultural policy and in the economic climate should be more evenly distributed and show up as a general increase in variance rather than a sharper increase in one season than another. The volatility of the corn and soybean markets should have a derivative effect on cattle prices though. The December live cattle contract shows a sharper summer variance

Figure 2 - Monthly standard deviations of daily changes in logarithms of prices for selected futures contracts 1/

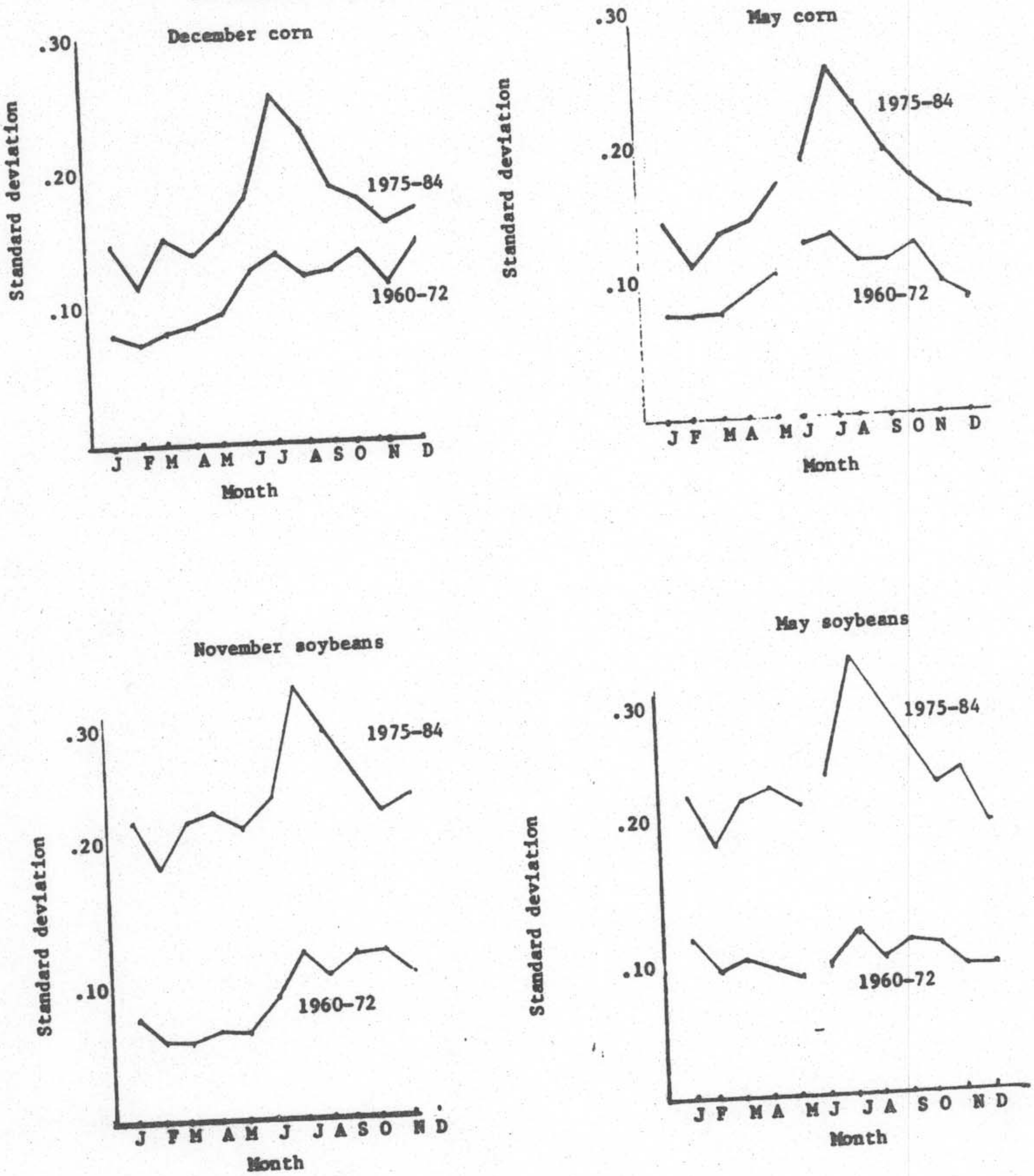
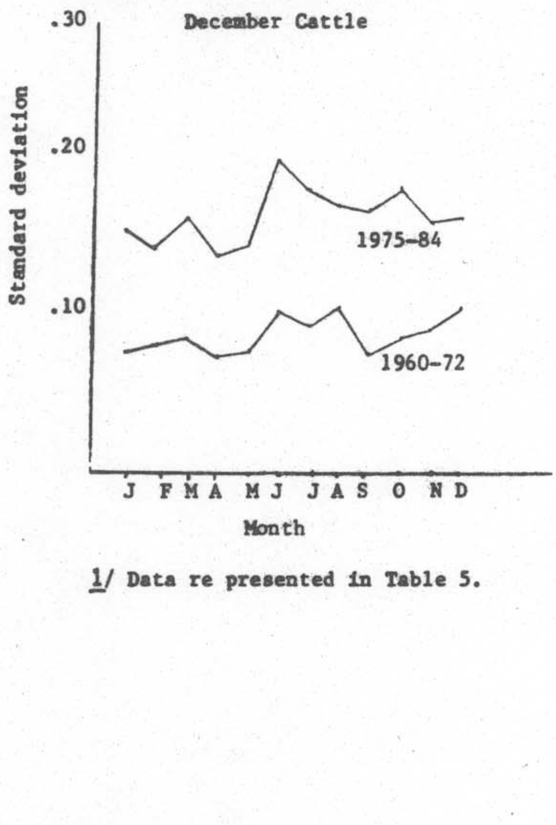
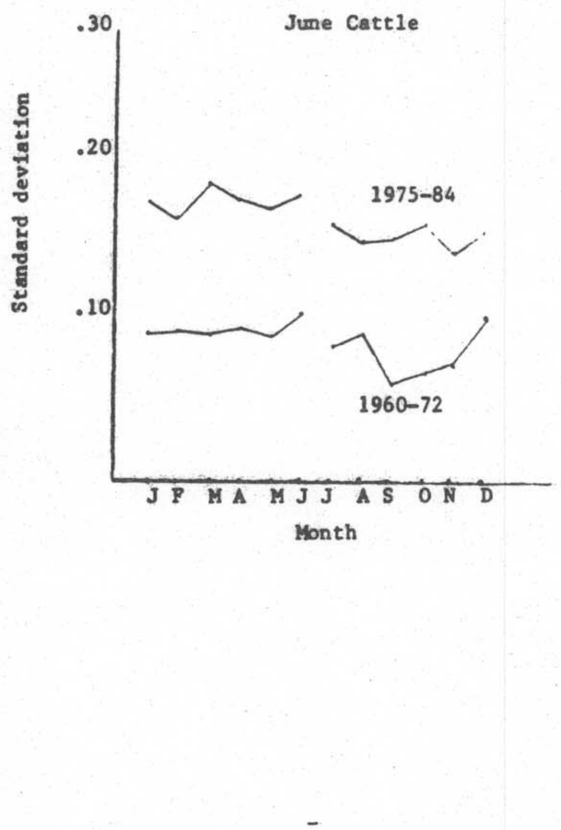


Figure 2 - Continued





1/ Data re presented in Table 5.

Table 5: Monthly standard deviations of daily changes in logarithms of prices for selected futures contracts 1/

Contract	Period	Jan	Feb	Mar	April	May	June	Jul	Aug	Sep	Oct	Nov	Dec
Dec. corn	1960-72	.0806	.0769	.0804	.0892	.0932	.1245	.1394	.1209	.1252	.1393	.1146	.1435
	1975-84	.1476	.1178	.1508	.1402	.1580	.1790	.2520	.2290	.1889	.1777	.1606	.1693
	1960-84	.1204	.1086	.1260	.1278	.1395	.1738	.2058	.1883	.1741	.1702	.1480	.1662
May corn	1960-72	.0781	.0778	.0794	.0917	.1067	.1282	.1332	.1133	.1124	.1252	.0947	.0815
	1975-84	.1451	.1107	.1379	.1428	.1698	.1879	.2570	.2298	.1945	.1782	.1580	.1511
	1960-84	.1184	.1077	.1206	.1294	.1559	.1728	.2001	.1781	.1646	.1588	.1320	.1303
Nov. soybeans	1960-72	.0765	.0596	.0587	.0669	.0653	.0861	.1233	.1063	.1205	.1218	.1070	.0743
	1975-84	.1915	.1505	.1776	.1880	.1599	.2441	.3213	.2888	.2654	.2357	.2450	.1688
	1960-84	.1385	.1158	.1267	.1353	.1261	.1839	.2291	.2104	.1955	.1871	.1731	.1329
May soybeans	1960-72	.1176	.0904	.1001	.0939	.0885	.0932	.1194	.0997	.1119	.1090	.0911	.0930
	1975-84	.2209	.1869	.2187	.2281	.2137	.2366	.3201	.2851	.2529	.2284	.2368	.1974
	1960-84	.1705	.1419	.1635	.1650	.1681	.1872	.2080	.1831	.1810	.1702	.1657	.1544
Dec. cattle	1965-72	.0780	.0801	.0838	.0768	.0773	.1043	.0949	.1075	.0787	.0862	.0907	.1067
	1975-84	.1583	.1449	.1656	.1404	.1486	.2039	.1822	.1738	.1714	.1854	.1625	.1648
	1965-84	.1345	.1241	.1402	.1199	.1234	.1664	.1499	.1545	.1496	.1551	.1463	.1633
June cattle	1965-72	.0932	.0955	.0910	.0983	.0911	.1075	.0860	.0925	.0596	.0679	.0718	.1062
	1975-84	.1811	.1705	.1959	.1911	.1780	.1826	.1683	.1519	.1568	.1649	.1484	.1606
	1965-84	.1501	.1437	.1590	.1559	.1454	.1571	.1474	.1351	.1298	.1346	.1250	.1412

1/ Annualized estimates (250 x variance of daily price changes)$^{1/2}$

increase, as well as a general increase in variance from the 1960-72 period. This difference is not apparent in the June contract where the increase in volatility is spread more evenly over the year.

Summary and Implications

All of the futures markets examined in this study exhibited significant deviations from normality in the daily changes of logarithms of prices over the life of contracts. Daily price changes approached normality when 2 month segments within a specific season were examined. A nonparametric test of seasonality showed that price changes for corn and soybeans tended to be more variable in the summer months than in the winter. Price movements were more variable in the winter than in the summer for frozen concentrated orange juice. Pronounced changes in price variability over longer periods were also observed. Monthly plots of the variance of daily changes in the log of price showed that not only has variance increased in recent years from the 1960's, but that the seasonal effect itself is more pronounced than in the past.

These results have several implications for analysts using futures market prices:

1. Price forecasters should examine their assumptions of normality and constant variance more critically. The mixture of normal distributions hypothesis appears appropriate in many cases. This suggests either transforming data to make variances constant or use of methods such as weighted regression. Tests for constancy of variance should be applied more often and tests and confidence interval estimates of means and regression coefficients need to be interpreted with care.

2. Researchers using models that assume stationary variance, such as autoregressive and spectral analysis models, need to correct for the nonstationary variance of price changes before applying those models to futures market prices.

3. Persons using option pricing models should allow for seasonally changing variance in their formulas. Otherwise the models will tend to underprice options in high variance seasons and overprice options in times with low variance of price changes. Long-term changes in variance should also be taken into account in selecting variance estimates.

4. In general, the results of this study have implications for any work that uses the variances of price changes. Analyses using only the first moment (expectation) of price changes or changes in logarithms of price will not be as greatly affected. However, expectations of actual price changes may be affected in the lognormal case since the mean of a lognormal variable depends on the variance, as well as the mean, of its normally distributed logarithm.

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