

NCCC-134

APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

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Suggested citation format:

Nelson, R. D. 1985. "Systematic Risk and Volatility in Commodity Markets." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [<http://www.farmdoc.uiuc.edu/nccc134>].

Systematic Risk and Volatility in Commodity Markets

Ray D. Nelson*

In order to manage price uncertainty, a decision maker must first devise an operational way of measuring risk. Among the most commonly used measures are statistics based on variances and covariances. Although variance definitions dominate the commodity market literature which treats price risk management, the covariance alternative shows promising potential. The use of covariances already finds widespread implementation for defining and managing risk in securities markets. Portfolio managers often gauge the risk of an individual security by comparing its volatility to that of a stock market index. Similar applications to commodity markets could prove equally valuable.

The increasing notoriety of commodity indexes may make application of covariance based risk definitions more feasible. The proposal by the New York Futures Exchange to trade a contract based on the Commodity Research Bureau's Futures Index gives evidence of the increasing importance of commodity indexes. Even if such indexes fail to achieve prominence, commodity price risk can still be measured relative to custom constructed market aggregates.

The present paper illustrates the utility of measuring commodity risk by comparing individual price changes to fluctuations in a market index. This explanation begins with an overview of systematic and unsystematic risk as defined in the context of Sharpe's (1963) market

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model. Next a discussion of price index construction methodology reveals the possibility of successfully adapting the single index model to commodity markets. After a review of the estimation challenges inherent in the market model, two applications demonstrate the potential insights the market model offers managers making decisions in an environment of uncertain commodity prices.

Systematic Risk in Markets

Markets often contain a constellation of prices which move in concert. Scholars studying such market phenomenon usually refer to this positive covariation among prices as systematic risk. Well known indexes published by Standard and Poors, Dow Jones, and others summarize the daily changes in thousands of security prices and serve as proxies for systematic risk in financial markets. Given the success of these indexes in capturing the joint changes in security prices, is it possible that similar constructs could represent systematic risk in commodity markets?

A brief overview of Sharpe's formalization of his risk definitions initiates the investigation of the compatibility of commodity markets with the single index model. Then a short review of the literature summarizes previous attempts to apply single index models to commodity markets. Because applications to commodity markets require price indexes which appropriately measure the systematic risk in these markets, this part of the investigation concludes with an evaluation of

the suitability of various indexes as potential proxies for systematic commodity price risk.

The Market or Single Index Model

Fama (1976) develops the market model by using a framework very familiar to those who share a common background in econometrics. He begins his presentation by assuming that security returns $\{R_{1t}, \dots, R_{nt}\}$ form a multivariate normal distribution. This assumption leads to the important conclusion that R_{mt} and R_{it} , any two linear combinations of $\{R_{1t}, \dots, R_{nt}\}$, share a bivariate normal distribution.

One possible bivariate normal distribution rates especially significant because of its role in the market model. One of the random variables R_{mt} of this joint distribution is an equally weighted linear combination of all possible returns. The second random variable corresponds to any of the individual security returns R_{it} from the multivariate distribution. The conditional mean of R_{it} given a value for R_{mt} is:

$$(1) \quad E(R_{it}|R_{mt}) = \alpha_i + \beta_i \cdot R_{mt}$$

where

$$(2) \quad \beta_i = \frac{\text{cov}(R_{it}, R_{mt})}{\text{var}(R_{mt})}$$

$$(3) \quad \alpha_i = E(R_{it}) - \beta_i \cdot E(R_{mt})$$

Defining e_{it} as the deviation of R_{it} from the above conditional mean gives:

$$(4) \quad R_{it} = \alpha_i + \beta_i \cdot R_{mt} + e_{it}$$

The market model interprets (4) to mean that the return on security i contains two parts. The first, commonly called the systematic portion, results from the product $\beta \cdot R_{mt}$. The β coefficient, or the volatility measure, represents a proportionality constant which relates changes in the return on the market composite to the return of an individual security. When $\beta_i > 1$, a 1 percent increase or decrease in the value of the market composite on average translates into a profit or loss which exceeds 1 percent for security i . When $\beta_i < 1$, then the security's return on the average rates smaller than that of a similar 1 percent change in the market composite.

The first and third terms in (4) represent the second part or unsystematic component of the return to security i . The sum $\alpha_i + e_{it}$ constitutes that part of the return which is independent and therefore not proportional to the return of the market composite. The α_i corresponds to a constant, riskless component. The e_{it} denotes the random part of the unsystematic risk.

Regression analysis using similar notation to (4) decomposes the total variance of the dependent variable into explained and unexplained categories. Since the form of (4) corresponds exactly to that of a simple regression equation, an analogous procedure classifies risk as either systematic or unsystematic. Determining the variance of an individual security's return using (4) gives:

$$(5) \quad \text{var}(R_{it}) = \beta_i^2 \cdot \text{var}(R_{mt}) + \text{var}(e_{it})$$

The cov (R_{mt}, e_{it}) terms vanish in the above calculation because of the easily demonstrable independence of e_{it} and R_{mt} . Dividing both sides of (5) by $\text{var}(R_{it})$ and substituting

$$(6) \quad \rho_{im}^2 = \beta_i^2 \cdot \frac{\text{var}(R_{mt})}{\text{var}(R_{it})}$$

into the result gives:

$$(7) \quad 1 = \rho_{im}^2 + \frac{\text{var}(e_{it})}{\text{var}(R_{it})}$$

This reiterates the idea that the variation of a given return contains two parts. The first corresponds to systematic risk or that explained by the return on the market portfolio. The second or unsystematic part depends only on the residual or unexplained variation.

The market model also facilitates combining individual securities into portfolios. If x_i represents the proportion of assets allocated to security i , then the return on the portfolio R_p corresponds to:

$$(8) \quad R_{pt} = \alpha_p + \beta_p \cdot R_{mt} + e_{pt}$$

where

$$(9) \quad \alpha_p = \sum_i x_i \cdot \alpha_i$$

$$(10) \quad \beta_p = \sum_i x_i \cdot \beta_i$$

$$(11) \quad e_{pt} = \sum_i x_i \cdot e_{it}$$

This means that the portfolio's return and variance equal:

$$(12) \quad E(R_{pt}) = \alpha_p + \beta_p \cdot E(R_{mt})$$

$$(13) \quad \text{var}(R_{pt}) = \beta_p^2 \cdot \text{var}(R_{mt}) + \text{var}(e_{pt})$$

Any manager designing a portfolio can control exposure to systematic market risk by selecting an appropriate β_p . If the unsystematic risk components e_{it} exhibit the independence assumed in the market model, then the $\text{var}(e_{pt})$ rapidly declines towards zero as the number of securities included in the portfolio increases.

The creation of futures contracts based on stock market indexes gives new relevance to the market model. This methodology provides portfolio managers a familiar framework for determining optimal hedging strategies. Figlewski and Kon (1982) demonstrate that stock market futures indexes allow managers to eliminate systematic risk from their portfolios. Managers accomplish this by simply selling β_p units of the stock index for every unit of value represented by the unhedged portfolio. The amount of residual, unsystematic risk remaining in the portfolio depends on the number of included securities.

Applications of the Single Index Model to Commodity Markets

The emphasis during the development and application of the single index model and its equilibrium counterpart, the capital assets pricing model, centers on securities markets. Some efforts do, however, attempt to adapt the single index model to commodity markets. These research

efforts all point to the importance of choosing an index capable of metering systematic movements in commodity markets.

One series of papers employs the CAPM to search for evidence of normal backwardation in commodity futures markets. Dusak (1973) uses an equation similar to (4) to regress returns in the wheat, corn, and soybean futures markets on the S&P 500. Because of insignificant estimates for the coefficients which correspond to unsystematic and systematic returns, she finds no evidence of backwardation. Carter, Rausser, and Schmitz (1983) take issue with the use of the S&P 500 as the appropriate measure of systematic risk. After constructing an index of equally weighted commodity and security returns, they present results which contradict the no backwardation conclusion of Dusak. Finally, Marcus (1984) criticizes the equal weighting given by Carter, Rausser, and Schmitz to futures markets in their hybrid index. Marcus concludes that the index construction methodology causes the resulting favorable evidence of backwardation.

Three other research efforts also analyze returns in commodity markets relative to those in security markets. Holthausen and Hughes (1978), Bodie and Rosansky (1980), and Lee and Leuthold (1983) find little correlation between returns in commodities and securities markets. These results suggest that combining futures and security positions reduces the risk of the resulting portfolio. These studies also show that capital market composites poorly represent joint movements in commodity prices. This emphasizes the necessity of

searching among alternative economic indexes for better measures of systematic risk in commodity markets.

Systematic Risk Measurement Using Commodity Indexes

Some commodity indexes such as those published by the Commodity Research Bureau and the Dow Jones Corporation attempt to measure systematic changes in commodity prices. The Commodity Research Bureau, for example, calculates an unweighted geometric mean of the individual price relatives for 27 commodity futures prices. The ratio of the current price to the annual 1967 average for each commodity gives the price relatives needed to construct the index. The calculations include only those prices which mature within twelve months of the date corresponding to the index.

Even this narrower commodity rubric, however, contains individual products which share little in common with the aggregate. Some of the correlations of commodity subgroups with the CRB Futures Index reported in Table 1 clearly demonstrate this point. The low correlation associated with livestock means the CRB statistics serve as poor measures of systematic price changes in cattle markets.

Since most regularly published commodity indexes aggregate changes in a wide assortment of products, they may not find utility in measuring systematic changes in a given commodity price. To the degree that cash, forward, and futures prices do share systematic structure resulting from common fundamental economic determinants, measurement of joint price movements may require the construction of a customized index. Although customizing an index lacks the accessibility of its periodically

Table 1

Correlation Coefficients
CRB Futures Index with Various
Subindexes
1/1/82 - 12/20/83

Subindex	Correlation
Grains	.898
Oilseeds	.888
Imports	.767
Miscellaneous	.722
Industrials	.372
Metals	.253
Livestock Meat	-.154

Source: Cox, Stephen W. "The CRB Futures Price Index--
A 'Basket of 27 Commodities' That May Soon Be A
Futures Contract." 1984 Commodity Year Book.
Commodity Research Bureau, page 34.

published alternatives, it does allow adaptation to the problem at hand.

Economic Indexes

Even in the case of an individual commodity such as corn, the product price varies because of time, location, variety, and quality characteristics. Because the overall corn price level is not a directly observable market phenomena, measuring the level of a composite group of prices necessitates the construction of an index. Such calculations greatly benefit from the economics literature which details the methodology of price index construction.

Construction of an Appropriate Index

Allen (1975) summarizes the numerous types of economic indexes with potential for measuring systematic risk in commodity markets. Many of the applicable indexes utilize the idea of price relatives. For a given commodity i , the ratio of the price in period t to the corresponding price in the reference period s gives the price relative P_{ist} , i.e.,

$$(14) \quad P_{ist} = \frac{P_{it}}{P_{is}}.$$

Calculating an arithmetic or geometric mean of all appropriate price relatives gives possible price indexes. The geometric mean alternative, for example, corresponds to:

$$(15) \quad GM_{0t} = \exp \left\{ \frac{1}{n} \sum_i \ln (P_{i0t}) \right\}$$

This formula generates the equivalent of computing the n^{th} root of the product of n possible price relatives.

Simple runs of indexes only utilize the information in the current and base years. As discussed by Allen, a chaining procedure allows incorporation of all the information between time 0 and t . Rather than simply using a ratio such as (15), the chaining procedure based on the geometric mean utilizes the following formula:

$$(16) \quad I_{0t} = I_{0(t-1)} \cdot GM_{(t-1)t}$$

This signifies that the price index in period t with reference period 0 results from multiplying the price index in the previous period by the

geometric mean of price relatives calculated from period t and $t-1$ values.

One significant advantage of the geometric mean stems from its ability to allow for changes in the set of price relatives used to calculate the index. This facility rates as extremely important when the index contains quotations for futures contracts. Because a futures contract goes off the board at maturity, replacing it with a price corresponding to the same delivery month but with maturity one year in the future allows continuity in the index. As an illustration, Table 2 gives the prices included in the calculation of $GM(t-1)_t$ on March 20, 1985 and April 1, 1985 for an index based on corn futures with maturity less than 12 months. In the calculation of the

Table 2

Prices Used to Construct
an Index Based on Corn Futures

March 20, 1985	April 1, 1985
March 1985	May 1985
May 1985	July 1985
July 1985	Sept 1985
Sept 1985	Dec 1985
Dec 1985	March 1986

geometric mean for April 1, the price relative for March 1986 simply takes the place of the March 1985 contract. This chaining and substitution process finds use in some of the best known commodity indexes.

Even after making a selection such as the chained geometric mean as the best statistic for customizing a price index, a decision still remains regarding the combination of prices appropriate for index construction. In the case of a commodity such as corn, candidate prices for calculating the index exist in the cash, forward, and futures markets. Choosing cash prices introduces known seasonality into the index. Inconsistent reporting of forward prices discourages their inclusion. For these reasons, the customized index used in the present analysis utilizes only futures prices with maturity not exceeding 12 months.

Volatility Estimation

The construction of an appropriate commodity index allows the estimation of the market model. The commodity return calculations used in the regression procedure follow the definitions of Holthausen and Hughes. They define the return to a position in the commodity market as:

$$(17) \quad R_{it} = \ln p_{it} - \ln p_{i(t-1)}.$$

In regressing R_{it} on the return in the corn market R_{mt} using an equation such as (4), the question of biasedness in estimators arises because R_{it} itself can constitute a component of R_{mt} . The possibility of biasedness results from the obvious simultaneous relationship between dependent and independent variables.

Cerchi and Havenner (1983) discuss many of the problems inherent in estimating the market model. They demonstrate that ordinary least squares estimation can yield unbiased estimators despite the simultaneous equation problem. Achieving this unbiasedness, however, can cause the loss of efficiency.

The ordinary least squares estimates reported in Table 3 result from regressing Stockton, California cash and Chicago Board of Trade corn futures prices on a custom constructed index. A chained geometric mean of futures prices for the years 1973-1982 constitutes the index used in the estimation. The estimates show very significant levels of systematic risk. Of course, in this case the systematic components refer to common movements in corn prices rather than high positive correlation with changes in security markets. These estimates also allow further discussion of the risk management possibilities of combining cash and futures prices into a portfolio.

Limits of Diversification

Elton and Gruber (1977) derive analytical expressions which demonstrate the effect of portfolio size on risk reduction. They show that increasing the number of included securities quickly reduces portfolio risk to the level of systematic variation in the market. Futures contracts based on stock market indexes permit further risk reduction through elimination of even the systematic risk inherent in security markets. The following discussion follows a similar line of logic for commodity markets.

Table 3
 OLS Estimates for Corn Market Model
 California Cash and CBT Futures
 1973-1982

Price	$\hat{\alpha}_1$	$\hat{\beta}_1$	Mean Squared Error	R-Square	Durbin-Watson
Cash	.0015 (.0009)	.599 (.031)	.0003322	.50	1.98
March CBT	-.0003 (.0003)	.978 (.009)	.0000369	.96	2.16
May CBT	.0004 (.0003)	.972 (.009)	.0000373	.96	1.92
July CBT	.0003 (.0003)	.986 (.010)	.0000441	.95	2.02
September CBT	-.0005 (.0003)	1.038 (.011)	.0000570	.95	2.34
December CBT	-.0002 (.0004)	1.024 (.012)	.0000740	.94	2.22

Note: Parenthesized values report the standard errors of the estimated parameters.

Hedging a Commodity Portfolio Using an Index

Assume that a corn trader in Stockton, California can hold combinations of long cash and short futures positions. Equation (8) represents the return on such a portfolio. In this case, x_1 denotes the size of position 1 relative to the absolute magnitude of the corn position X_c . The value x_c equals one because the trader has 100 percent of the corn asset in the cash market position. This fact combined with

the information in Table 3 means .59 represents the β of an unhedged corn portfolio. The return to a portfolio composed of only a cash corn position has a systematic component of:

$$(18) \quad .59 \cdot R_{mt}.$$

The unsystematic return of an unhedged cash corn position totals:

$$(19) \quad .0015 + e_{ct}.$$

Table 3 gives the variance estimate of approximately .0003 for the stochastic residual part of the unsystematic return.

By using (13) and remembering that x_c equals one, an expression for the variance of a portfolio which contains short futures contracts combined with the long cash position becomes:

$$(20) \quad \text{var}(R_{pt}) = \beta_p^2 \cdot \text{var}(R_{mt}) + \text{var}(e_{ct})$$

$$+ \sum_{i \neq c} [x_i \cdot \text{var}(e_{it})]$$

The trader can minimize the risk of the cash corn position by selling futures contracts so that the combination of cash and futures positions yields a portfolio whose β value equals zero. Because $x_i < 0$ for a short futures position, this means

$$(21) \quad \beta_c = -\sum_{i \neq c} x_i \cdot \beta_i$$

With a zero β portfolio of cash and futures positions, only unsystematic risk remains. The variance of such a portfolio is given by the following:

$$(22) \text{ var } R_{pt} = \text{var}(e_{ct}) + \sum_{i \neq c} [x_i^2 \cdot \text{var}(e_{it})]$$

The minimum risk portfolio results from choosing the x_i which minimize (22) and which satisfy the constraint that the β of the resulting portfolio equals zero. Table 4 reports the risks involved in such hedges where the corn trader utilizes varying numbers of futures contracts to hedge. This analysis assumes that the trader rolls the hedges forward into the succeeding crop year as futures contracts mature. The statistics in Table 4 show that a corn trader in California can reduce the risk level of an open cash position by an estimated 45.8 percent by selling a risk minimizing quantity in a single futures contract maturity. Through hedging, the trader avoids all of the systematic risk inherent in a cash corn position. In exchange, however, the trader assumes the additional unsystematic risk of the futures contract. If the trader uses futures contracts with differing maturities, the unsystematic risk lessens as the diversity resulting from the larger number of elements in the portfolio increases.

Comparisons of Volatility

In addition to augmenting the understanding of the limits of diversification, the market model shows promise as a means of ascertaining the effect of contract maturity on futures price

Table 4

Limits of Diversification
California Corn Portfolio

Number of Futures Contracts Used to Hedge	Systematic	Unsystematic Corn	Futures	Total	Percentage of Risk Diversified
0	.0003322	.0003322	.0000000	.0006644	0
1	.0000000	.0003322	.0000277	.0003599	45.8
2	.0000000	.0003322	.0000140	.0003462	47.9
3	.0000000	.0003322	.0000098	.0003420	48.5
4	.0000000	.0003322	.0000078	.0003400	48.8
5	.0000000	.0003322	.0000068	.0003390	49.0

volatility. Samuelson (1965) utilizes a theoretical model to suggest that volatility should increase as futures contracts mature. Rutledge (1976) finds evidence which neither supports nor strongly discredits the idea of increasing volatility. Samuelson (1976) argues further in support of his previous statements. He also proposes a reservation of final judgment about his conclusions until the performance of more powerful statistical tests. More recently, Anderson and Danthine (1983) develop a model which shows that volatility depends on the rate of flow of information into the market. If revelation of information increases during the maturity month, then Anderson and Danthine agree with Samuelson.

Empirical tests encounter a possible problem if cash, forward, and futures prices differ in variability from month to month. Under these circumstances, simply comparing the variance of nonmaturity and maturity month prices, as done in previous studies, could lead to erroneous

conclusions. If all December corn prices were to vary less than all other months, for example, then simple comparisons would indicate a smaller variance for the December contract during its delivery month.

The market model offers methodology capable of circumventing the possible problem of prices with differing monthly variances. As mentioned previously, β measures the volatility of individual price changes relative to those in the relevant commodity market. Even though the variance of all prices might change, the volatility of a given price when measured relative to the index can remain constant. Assume in expression (5), for example, that both the variance of R_{1t} and R_{mt} increase while the residual variance stays the same. As long as the relative magnitudes of fluctuations in the individual and index prices remain steady, the β coefficient remains unchanged.

The decomposition of price risk into systematic and unsystematic components causes the comparison of monthly variances to occur in two steps. The first constitutes a simple t-test based on the β coefficient of the market model. These t-tests result from reformulating expression (4) by adding intercept and slope dummy variables D_{11t} and D_{21t} . These variables indicate whether or not a given observation corresponds to the delivery month of a futures contract. This gives the following regression equation:

$$(23) \quad R_{1t} = \alpha_1 + \gamma_{11} \cdot D_{11t} + \beta_1 \cdot R_{mt} + \gamma_{21} \cdot D_{21t} + e_{1t}$$

Table 5 reports the regression results needed to test the null hypothesis that γ_{21} equals zero. Rejection of this hypothesis gives

Table 5

Tests of Differing Volatility
in Futures Contract Maturity Months

Contract	α_1	$\hat{\gamma}_{11}$	$\hat{\beta}_1$	$\hat{\gamma}_{21}$	R-Squared	Durbin-Watson
March	-.0004 (.0003)	.0016 (.0012)	.979 (.009)	-.046 (.059)	.96	2.17
May	.0001 (.0003)	-.0067 (.0011)	.963 (.009)	.184 (.040)	.96	1.93
July	.0003 (.0003)	-.0007 (.0014)	.975 (.010)	.122 (.036)	.95	2.02
September	-.0003 (.0004)	-.0026 (.0015)	1.031 (.011)	.119 (.045)	.95	2.34
December	-.0003 (.0004)	.0022 (.0018)	1.023 (.013)	.013 (.064)	.94	2.21

Note: Parenthesized values report the standard errors of the estimated parameters.

evidence indicating unequal price volatility during the maturity month. Large t-statistics for for May, July, and September suggest the possibility that the volatility in the delivery month for these contracts exceeds that of the nondelivery months. Remember, however, that these comparisons of volatility only consider systematic risk.

The second step in the comparison between maturity and nonmaturity monthly variation focuses on the unsystematic risk. The null hypothesis in this case postulates equal variances for maturity and nonmaturity months. For each futures contract, the test first requires segmenting the residuals estimated using expression (23) into two sets. Set

assignment for a given residual depends on whether or not it corresponds to a nonmaturity or maturity month. The comparison for each contract then proceeds by constructing an appropriate F-ratio from the two sets of observations. Table 6 summarizes the statistics needed to complete the test of equal unsystematic risk. The F-ratios indicate larger unsystematic risk in the maturity months for March, May, July, and September contracts. The reverse holds true for the December futures.

Table 6
Differences Between Residual Variances
for Maturity and Nonmaturity Months

Contract	Maturity Month		Nonmaturity Month		F-ratio*
	Mean Squared Residual	Degrees of Freedom	Mean Squared Residual	Degrees of Freedom	
March	.00012	26	.000032	447	3.87
May	.00010	25	.000029	450	3.52
July	.00014	26	.000038	448	3.61
September	.00014	25	.000052	451	2.63
December	.00003	25	.000076	454	2.35

*The F-statistic represents the ratio of the larger mean squared residual divided by the smaller.

Since both systematic and unsystematic variation corresponding to the maturity months for the May, July, and September futures exceeds the variation of nonmaturity months, this evidence partially supports expectations consistent with Samuelson's theoretical work. Just the opposite, however, seems true for the December contract. These empirical findings invite further investigation regarding the validity

of Anderson and Danthine's conclusion that volatility depends on the rate of flow of information into the market.

Summary and Conclusions

The importance of the market model in studies of financial markets makes attempts to apply the same concepts to commodity markets a natural extension of this valuable analytical methodology. Certain characteristics of commodity markets do, however, complicate its straightforward adaptation. Commodity prices in general do not all share the strong positive correlation structure found in security markets. This subverts the use of a general index to distill systematic variation from a constellation of commodity price changes. The broad spectrum of products included in most popular indexes also compromises their utility as measures of systematic risk in commodity markets. The methodology detailed in the economics literature which prescribes proper index construction techniques, however, does foster custom calculation of statistics capable of measuring systematic movements in specific commodity markets.

Two applications of the single index model show its analytical potential in studies of commodity markets. The first demonstrates that hedging a California cash corn position using Chicago Board of Trade futures halves price risk. The second application reveals that some futures contracts seem to exhibit increased price variation during their maturity months.

Future applications of single index concepts to commodity markets can continue to benefit from the substantial academic investment which develops the foundation, estimation, and relevance of the market model. Should commodity index futures achieve even a degree of the success enjoyed by their security market counterparts, the importance of the single index model to commodity markets could indeed multiply in significance.

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