Random Walk Priors, Multiple Time Series and the Forecast

by

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The use of Bayesian procedures for constructing forecasts of multiple conomic time series has recently attracted considerable attention [Doan, itterman, and Sims (1984), Kling and Bessler (1985), Litterman (1986), and lcNeese (1986)]. The impetus for these efforts has been the development of a onvenient prior by Litterman (1979). Noting a general property of many economic time series -- that they are often well approximated by random walks - Litterman proposes mixed estimation of the parameters of a vector autoreression with prior information centering on a random walk.

Experience with Litterman's suggestion has generally been favorable, lthough not conclusive. Forecast performance, relative to unrestricted ARs, has been very good; while performance relative to univariate represenations has been modest (see Litterman (1986, table 3) or Kling and Bessler 1985)). Perhaps one reason for this is that the random walk prior holds pproximately (see Litterman, 1986, p. 29). Forecast performance might be mproved by finding a closer match between the prior and the underlying ecoomic time series.

The efficient market hypothesis suggests that daily price series on comodities traded in auction-type markets will be near random walks. That is o say, if daily price differences show systematic (nonrandom) patterns, nen market traders could make arbitrage profits from buying (selling) and nen selling (buying) the commodity on successive trading days. This is not ne case for some of the series studied in earlier applications of the Bayeian VAR. For instance, Kling and Bessler (1985) studied quarterly observaions on hog slaughter, sows farrowing and disposable income. There did not ppear to be *a priori* reason to expect any of these series to follow a random alk process. And, while the random walk prior may yet be preferred to an uninformed prior in certain cases, it is probably best to view such applicaons as misspecifications of the relevant prior. The present paper begins th what is probably a better match between the researcher's prior and the tterman model.

Of course, a Bayesian (who didn't have to actually solve the problem) ould probably not tolerate discussion on whether or not the prior is constent with the model. He would suggest that one build the model using the tual prior. Methods which attempt to force prior information into convennt forms (conjugate forms) are viewed by such critics with considerable ispicion (Berger, 1980, p. 97). Our answer to such criticism is that fullown Bayesian specifications require more information and processing time an we are prepared to offer at this time. If convenient forms (models) are monstrated not to work well (that is, if they forecast poorly) the commitnt of additional time and money to analysis of actual elicited priors will justified.

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This paper is presented in three additional sections. First, we briefly review the Litterman model. Second, we present empirical forecasting results with the random walk prior. Finally, we discuss our results and make suggestions for further research.

The Bayesian Vector Autoregression

The general multivariate time series model is given in its autoregressive form in equation (1):

$$(1)\quad X_t = \sum_{s=1}^{\infty} \phi(s) X_{t-s} + \epsilon_t,$$

where X_t is a (k x 1) vector of variables measured at period t, $\phi(s)$ is a (k x k) matrix of autoregressive coefficients, which relate X_t to X_{t-s} and ϵ_t is a (k x 1) vector of white-noise disturbances. Under usual methods, equation (1) is approximated by fitting a m^{th}-order autoregression (s = m), using classical least squares regression, equation-by-equation.

Litterman proposes a prior on the coefficients of equation (1) characterized as follows:

- the ϕ's are jointly normally distributed;
- the mean of $\phi_{ij}(t)$ is zero except that on $\phi_{ii}(1)$, which has a mean of one (here $\phi_{ij}(t)$ is the i, j element of the autoregressive matrix at lag t);
- the $\phi_{ij}(t)$'s are independent across all i, j, and t.

In addition to the above information on the center of the prior distribution, the following tightness information is specified:

- λ, a constant standard deviation on the coefficient associated wi the first lag of the dependent variable (overall tightness);
- the standard deviation of all coefficients in the lag distributio are decreased in a harmonic manner, according to the parameter (decay tightness);
- standard deviations on other variables in the system can be ma tighter than own lag distributions according to the parameter (interaction tightness).

Given the above parameters (λ, γ_1, and γ_2), the standard deviation of co ficient i, j at lag ℓ will be given as:

$$(2)\quad \delta^{\ell}_{ij} = \begin{cases} \dfrac{\lambda}{\ell^{\gamma_1}} & \text{if } i = j \\[2ex] \dfrac{\lambda\gamma_2\hat{\sigma}_i}{\ell^{\gamma_1}\hat{\sigma}_j} & \text{if } i \neq j, \end{cases}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are the standard deviations on innovations from univariate autoregressions for equations i and j, respectively. Their purpose is to scale the prior in equation (2) for relative size of the original series.

The prior summarized above treats each series symmetrically. That is, γ_2, the tightness parameter associated with coefficients of variable i in equation j, is the same for all i and j. Often, one has prior information which suggests that the above may not be reasonable. For example, one might expect one variable to be nearly exogenous in a system -- and thus may wish to put a very tight prior around zero on coefficients on other variables in its equation. On other equations, the researcher may be quite uncertain on coefficients of lagged variables -- and thus may wish to impose a rather loose prior. Accordingly, equation (2) may be modified using equation specific interaction tightness; $\gamma_2(i,j)$ reflects tightness information on coefficients of variable j in the i^{th} equation of the vector autoregression. Values of $\gamma_2(i, j)$ between (1, 0) will reflect more (1) or less (0) series interactions.

Below we consider forecasting performance under various specifications on δ^{ℓ}_{ij}. In particular, we elicit experts' opinions on $\gamma_2(i,j)$, the tightness of the prior on coefficients of variable j in the i^{th} equation of the vector autoregression. Recall for $i \neq j$, the prior mean is zero, so higher elicited values of $\gamma_2(i,j)$ will suggest the expert is willing to allow the data to have more influence than lower values of γ_2. Conditional upon the expert assessments of γ_2, we set alternative values of λ and γ_1 -- as we find experts' opinions on these parameters are much more difficult to elicit.

Application to Daily Price Series

In this paper we study seven economic time series for which the random walk prior is judged to be very good. The series are measured daily over the period 1980-1984 and relate to the grain/livestock market. The series are cash grain sorghum prices at Houston port, Chicago cash corn prices, Illinois cash soybean prices, Omaha cash live cattle prices, Oklahoma City cash feeder cattle prices, the U.S. dollar/Japanese yen exchange rate, and U.S. 90-day T-bill rates. Table 1 gives the estimated autocorrelation and partial autocorrelation functions on the first differences of these data over years 1980-1981. Recall, for these series to be random walks, both the autocorrelation and partial autocorrelations should be not significantly different from zero, at all lags. There are several deviations from this rule -- notably, the milo, corn and and yen series. As an alternative to the random walk, one might model these series as first order moving average processes.

While a random walk is not a first order moving average process, it is within the same family of first differenced processes, with a coefficient equal to zero. Accordingly, a random walk prior should not perform too bad. Prior tightness information on the interaction of each series in the autoregressive representation of each other series was elicited from commodity experts (grain and livestock extension economists in the Department of Agricultural Economics at Texas A&M University). That is, we elicited values of $\delta_2(i,j)$ between zero (i.e., lagged values of series j have no influence on the current value of variable i) and one (i.e., lagged values of series j have the same relative effect on series i as do lagged values of series i). Two experts were consulted -- one providing opinions on the mechanism generating feed prices (milo, corn and soybean prices), the other providing opinions on the process generating feeder and live cattle prices. The authors provided the subjective settings on the lagged process which generated yen and T-bills. These settings are given in table 2.

Levels on overall tightness (λ) and the rate of decay on past lags (γ_1) were set at $\lambda = .300$ and $\gamma_1 = 1.50$. These settings were based on our previous experience with agricultural data (in part these were based on findings described in Kling and Bessler, 1985).

The seven commodities were studied in a five lag VAR. We estimated and forecasted VARs on three separate data periods. Starting in January 1980, VARs were estimated on the next 300 data points. Then the subsequent 100 data points were forecast using the fitted VAR (a Kalman filter was used to update the VAR during the forecast period). This sequence of estimation and forecasting was continued with two additional time intervals up to December 1984 (the forecast period was always the last 100 trading days of successive 400-day intervals starting in January 1980). Tables 3 and 4 give Theil U-statistics for the Bayesian VARs and unrestricted VARs, respectively.[1]

The forecast performance of the Bayesian VAR is not particularly good relative to the random walk (Table 3). We expected that at longer horizons, forecast performance would be improved (i.e., we expected to see Theil U-statistics below unity at horizons longer than one period). Such were found for corn, soybeans and feeder cattle over the first forecast period, but results are mixed with no generalizable pattern emerging. Over the first forecast interval corn, soybeans and feeder price forecasts improve relative to the random walk forecast; while over the third period corn and soybean price forecasts (show more) deteriorate relative to the random walk forecast. Milo forecasts over all three periods are not good relative to the random walk forecast. Live cattle forecasts also show no improvement relative to the random walk.

From table 4 one can see considerable improvement from using the Bayesian VAR, relative to the unrestricted VAR. The Bayesian VAR offers very impressive reductions in Theil-statistics. Of the 105 forecast evaluations in either table 3 or 4, the unrestricted VAR dominates the Bayesian VAR 13 times. Interestingly, when the unrestricted VAR does dominate the Bayesian

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[1]We choose to use the Theil statistic because our prior is centered on a random walk. As the Theil statistic measures forecast performance relative to the random walk, it provides a natural measure for our purposes.

VAR, it is usually at long forecast horizons (20 or 40 period lead times). The unrestricted VAR does quite poorly at low forecast horizons.

These results generally support those found in previous studies on multivariate forecasts -- the Bayesian VAR gives forecasts which outperform the unrestricted VAR. The optimal univariate model (in our case an approximate random walk) is difficult to beat -- at least, difficult to beat consistently.

Discussion

This paper applies the recently developed Bayesian VAR to daily forecasts of seven (interrelated) price series. The motivation for modeling daily prices with the random walk is explored. We suggest that earlier applications of the Bayesian VAR have probably been misspecifications of the researcher's prior -- although the random walk specification may even then be useful as an approximation. In the problem studied in this paper -- forecasting daily prices -- we have strong *a priori* and empirical support for the random walk model.

The forecasting results show the Bayesian VAR as an improvement over an unrestricted VAR. In over 80% of the forecast comparisons, the BVAR outperformed the unrestricted VAR. On the other hand, the BVAR did not outperform the simple random walk forecast -- even at long horizons. That is, our prior seems to be quite reasonable. Little improvement is found by incorporating additional series and lags (beyond last period's price) for forecasting each of the seven series. Perhaps we've provided one more set of observations consistent with the "efficient market hypothesis."

Table 1. Estimated Autocorrelation and Partial Autocorrelation on the First Differences of Daily Milo, Corn, Soybean, Live Cattle, Feeder Cattle Prices, Yen/U.S. Dollar Exchange Rate and T-Bill Rates, 1980-1981.

Lag	Milo		Corn		Soybeans		Live Cattle		Feeder Cattle		Yen		T-Bills	
	A	P	A	P	A	P	A	P	A	P	A	P	A	P
1	-.15*	-.15*	-.36*	-.36*	-.05	-.05	-.04	-.04	-.02	-.02	-.12*	-.12*	.04	.04
2	-.04	-.06	.02	-.12*	-.05	-.05	-.02	-.02	.00	.00	-.12*	-.14*	.10	.10
3	.04	.02	-.02	-.07	.07	.07	-.08	-.08	.02	.02	.05	.02	-.00	-.01
4	.04	.05	.02	-.01	.04	.05	.04	.04	-.08	-.07	.10	.10	.18*	.17*
5	-.05	-.03	.00	.01	.02	.03	-.08	-.08	-.02	-.02	-.08	-.04	-.02	-.04
6	.02	-.07	-.10	-.11	-.07	-.07	.13*	.12*	.09	.09	-.04	-.03	.08	.05
7	-.02	-.00	.20*	.14*	.03	-.01	.03	.03	-.11	-.10	-.05	-.08	.02	.02
8	-.00	-.02	-.09	.04	-.05	-.06	-.02	-.02	.02	.01	.09	.06	-.03	-.02
9	-.01	-.00	-.02	-.03	.00	.00	.03	.06	.02	.01	.04	.06	-.06	-.05
10	.14*	-.01	-.04	-.07	-.02	-.02	.02	.00	-.04	-.03	.04	.08	.05	.03

*An asterisk indicates the estimate is significantly different from zero at the five percent level or lower.

Table 2. Subjective settings on relative interactions among lagged values on each series.[a]

Lagged Variables	Milo	Corn	Soybeans	Feeder Cattle	Live Cattle	Yen	T-Bills
	Dependent Variable						
Milo	1.00	.80	.01	.50	.30	.01	.01
Corn	.50	1.00	.50	.50	.80	.01	.01
Soybeans	.01	.50	1.00	.50	.80	.10	.01
Feeder Cattle	.01	.01	.30	1.00	.01	.01	.01
Live Cattle	.30	.50	.30	.80	1.00	.01	.01
Yen	.30	.10	.30	.01	.01	1.00	.10
T-Bills	.01	.30	1.00	.30	.80	.50	1.00

[a]Entries in the table reflect the relative degree of series interaction permitted in a non-symmetric Bayesian prior. A value of 1.0 is full interaction; values close to zero imply little series interaction.

Table 3. VARs by Commodity and Forecast Horizons.[a]

Horizon (Steps Ahead)	Milo	Corn	Soybeans	Feeder Cattle	Live Cattle	Yen	T-Bills
			(1^{st} Forecast Period)				
1	1.03	1.01	1.01	1.00	1.01	1.01	1.01
5	1.08	1.01	1.00	.99	1.02	.99	1.01
10	1.15	.93	.97	.97	1.02	.99	1.00
20	1.30	.83	.92	.91	1.03	1.01	1.03
40	1.33	.83	.99	.93	1.05	1.01	1.12
			(2^{nd} Forecast Period)				
1	1.02	1.02	1.01	1.04	1.16	.98	1.04
5	1.08	1.00	1.07	1.10	1.07	.98	1.13
10	1.15	.94	1.06	1.08	1.10	.98	1.25
20	1.09	.86	.91	1.00	1.12	.96	1.90
40	.88	.79	.74	.89	1.16	.92	3.10
			(3^{rd} Forecast Period)				
1	1.02	1.03	1.02	.99	1.02	2.62	1.16
5	1.11	1.08	1.04	.95	1.01	2.36	1.08
10	1.23	1.12	1.14	.90	1.01	1.92	1.12
20	1.31	1.13	1.27	.75	1.01	1.64	1.17
40	1.55	1.14	1.61	.73	.98	1.81	1.31

[a]Forecast periods are as follows: 1^{st} (April 1 - August 24, 1981), 2^{nd} (November 19, 1982 - April 20, 1983), and 3^{rd} (July 10, 1984 - December 5, 1984).

Table 4. Theil U-Statistics on Out-of-Sample Forecasts from Unrestricted VARs by Commodity and Forecast Horizons.[a]

Horizon (Steps Ahead)	Milo	Corn	Soybeans	Feeder Cattle	Live Cattle	Yen	T-Bills
			(1st Forecast Period)				
1	1.05	1.02	1.08	1.08	1.08	1.06	1.09
5	1.11	1.05	1.08	1.07	1.07	1.07	1.08
10	1.22	1.02	1.12	1.03	1.04	1.00	1.13
20	1.44	1.16	1.23	1.03	1.13	.81	1.33
40	1.53	1.32	1.49	1.12	1.67	.84	2.73
			(2nd Forecast Period)				
1	1.06	1.07	1.09	1.19	1.58	1.02	1.29
5	1.14	1.04	1.16	1.36	1.09	1.11	1.35
10	1.16	.96	1.19	1.48	1.07	1.16	1.65
20	1.15	.85	1.13	1.41	1.01	1.23	2.93
40	1.00	.73	.99	1.18	.91	1.86	4.22
			(3rd Forecast Period)				
1	1.21	1.23	1.05	1.06	1.25	5.15	1.37
5	1.20	1.16	1.13	1.02	1.11	2.65	.99
10	1.28	1.16	1.23	.96	1.06	2.19	1.22
20	1.38	1.15	1.32	.83	1.09	1.55	1.17
40	1.42	1.13	1.34	.80	1.03	1.51	1.63

[a]Forecast periods are as follows: 1st (April 1 - August 24, 1981), 2nd (November 19, 1982 - April 20, 1983), and 3rd (July 10, 1984 - December 5, 1984).

References

Berger, J. O. *Statistical Decision Theory* New York: Springer Verlag, 1980.

Doan, T., R. Litterman and C. Sims. "Forecasting and Conditional Projections," *Econometric Reviews* 1(1984):79-89.

Kling, J. L. and D. Bessler. "A Comparison of Multivariate Forecasting Procedures for Economic Time Series," *International Journal of Forecasting* 1(1985):5-24.

Litterman, R. "Forecasting with Bayesian Vector Autoregressions: Five Years of Experience," *Journal of Business and Economic Statistics* 4(1986):25-38.

Litterman, R. *Techniques of Forecasting using Vectorautoregressions* unpublished Ph.D. Thesis, University of Minnesota, 1979.

McNees, S. "Forecasting Accuracy of Alternative Techniques: A Comparison of U.S. Macroeconomic Forecasts," *Journal of Business and Economic Statistics* 4(1986):5-15.