Basis Risk and Optimal Decision Making for California Fed Cattle

by

Timothy Park and Frances Antonovitz

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Recent work in futures markets has focused both theoretically and empirically on discussions of optimal hedge ratios. Following the work of Johnson (1960) and Stein (1961), Ederington (1979) derived the minimum-risk hedge ratio and a measure of hedging effectiveness. These measures, related to the covariances between spot and futures prices and the variance of futures prices during the hedge period, are easily estimable from a simple OLS regression. In the portfolio approach, the slope coefficient from a regression of spot on futures prices levels yields an estimate of the risk-minimizing hedge ratio. This approach has in turn yielded empirical studies examining the hedging effectiveness of different futures markets. For example, Wilson (1984) compared the **ex post** hedging effectiveness of the three U.S. wheat futures markets.

Theoretical extensions to the concept of the optimal hedge ratio have been concerned with more realistic modelling of the alternatives available to hedgers. In a series of related papers, Anderson and Danthine have discussed extensions such as optimal hedging when production uncertainty is present (1983a), hedging in multiple commodities and cross hedging (1981), along with multiperiod hedges (1983b).

A fundamental limitation of these studies is the exclusive focus on optimal hedge ratios. Clearly, output and hedging decisions are made simultaneously and, as a consequence, are interdependent in any realistic model of decision making by producer/hedgers. The optimal hedge ratio is derived and estimated assuming the producer has already made the optimal production decision. In focusing on optimal hedge ratios, the factors which influence the input and hedging decisions are not explicitly discussed.

This paper formulates a microeconomic model of decision making in futures markets in which basis risk is present. Estimable forms for optimal output and hedging decisions are derived and the interdependent nature of these decisions is highlighted. Useful comparative static results are also generated, suggesting how producers should adjust optimal output and hedging decisions in response to shifts in various parameters.

A main objective of the microeconomic model proposed in this paper is a re-examination of the estimation of the moments of spot and futures prices. The direction of optimal producer responses to shifts in current spot or futures prices along with shifts in expected spot or futures prices will

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*Research Assistant and Assistant Professor, respectively in the Department of Agricultural Economics at the University of California, Davis.

depend on estimates of covariances and variances of these prices. The signs of comparative statics results are shown to depend directly on simple regression relationships between spot and futures prices.

THE THEORETICAL MODEL

For a producer hedging in a futures market in which basis risk is present, the decision making problem can be specified as follows. In the first time period, inputs are purchased at known prices p_1 and the producer hedges f units of output at the known futures price p_1^f. In the second period when production is complete, output is sold at the spot price prevailing in the region. The producer offsets his position in the futures market by buying back f units of output at futures price $\tilde{p}_2^f$. The future spot price $\tilde{p}$ and the second period futures price are random variables when the producer is making output and hedging decisions in the first period. Basis risk arises due to the deviations between the random second period spot and futures prices.

The producer's optimal output and hedging decisions are determined by maximizing expected utility of profit. The objective function of the producer is to

maximize $\quad E\,U(\tilde{\pi}) = E\,U\left[\ \tilde{p}F(X_1) + (p_1^f - \tilde{p}_2^f)f - p_1X_1\ \right]$

where

$\tilde{\pi}$ = profits

$\tilde{p}$ = price of output in period 2

X_1 = vector of input quantities

f = quantity of output hedged in the futures market in period 1

p_1 = vector of input prices

p_1^f = futures market price in period 1 for contracts with maturity in period 2

$\tilde{p}_2^f$ = futures market price in period 2 for contracts with maturity in period 2

$F(X_1)$ = the production function for quantity of output

First-order conditions are

$$\frac{\partial EU(\tilde{\pi})}{\partial X_1} = E\left[\ U'(\tilde{\pi})\ (\ F'(X_1)(\tilde{p} - p_1))\ \right] = 0$$

$$\frac{\partial EU(\tilde{\pi})}{\partial f} = E\left[U'(\tilde{\pi}) \, (p_1^f - \tilde{p}_2^f) \right] = 0$$

where it is assumed that utility increases as profits rise, $U'(\tilde{\pi}) > 0$ and the producer is risk averse, $U''(\tilde{\pi}) < 0$.

Using Stein's Theorem (1981), explicit solutions for optimal output and hedging decisions can be derived. Stein's Theorem, derived independently by Rubinstein (1976), states that if x and y follow a bivariate normal distribution and g(y) is a once-differentiable function of y, then

$$\mathrm{Cov}\,(g'(\tilde{y}), \tilde{x}) = E\, g'(\tilde{y})\, \mathrm{cov}\,(\tilde{y}, \tilde{x}).$$

Stein's Theorem was extended to variables which belong to the continuous exponential family by Hudson (1976). The normal distribution along with the lognormal and gamma distributions are members of this family of distributions. The theorem is especially useful in expected utility models in which covariance terms between marginal utility of profit and random variables in the profit function, such as output price, appear. Applying Stein's Theorem to the first-order conditions leads to the following explicit solutions for output and hedging:

$$F(X_1) = \frac{\sigma_{sf}}{\sigma_s^2} f + \frac{\bar{\pi}_s}{\gamma\, \sigma_s^2}$$

$$f = \frac{\sigma_{sf}}{\sigma_f^2} F(X_1) + \frac{\bar{\pi}_f}{\gamma\, \sigma_f^2}$$

where the following definitions have been used:

$$\bar{\pi}_s \equiv E(\tilde{p}) - \frac{p_1}{F'(X_1)}$$

$$\bar{\pi}_f \equiv p_1^f - E(\tilde{p}_2^f)$$

$$\gamma(\tilde{\pi}) \equiv - \frac{EU''(\tilde{\pi})}{EU'(\tilde{\pi})} .$$

In the above equations $\bar{\pi}_s$ is the expected marginal profit from production and $\bar{\pi}_f$ is the expected profit from hedging. The gamma (γ) term is a measure analogous to the Arrow-Pratt measure of absolute risk aversion and is defined as the expected value of the second derivative of the utility function divided by the expected value of the first derivative of the utility function. If the gamma term is assumed to be constant, the underlying utility function is characterized by constant absolute risk aversion. The Arrow-Pratt measure is equivalent to the gamma measure of risk aversion under this assumption.

As can be seen from the equations, optimal output and hedging decisions are interdependent in a model of decision making with basis risk. Output depends on the amount hedged and, in turn, the optimal hedge is related to the amount produced. Reduced form solutions for output and hedging can be derived.

$$F(X_1) = \left[\bar{\pi}_s + \bar{\pi}_f \beta_f \right] \Big/ D\, \sigma_s^2$$

$$f = \left[\bar{\pi}_f + \bar{\pi}_s \beta_s \right] \Big/ D\, \sigma_f^2$$

where $$\beta_f = \frac{\sigma_{sf}}{\sigma_f^2}, \quad \beta_s = \frac{\sigma_{sf}}{\sigma_s^2}$$

and $$D = \gamma\,(1 - \rho^2), \quad \rho^2 = \beta_s \beta_f$$

These reduced form solutions reveal that both output and hedging decisions will depend on variances and covariances of spot and futures prices, the producer's risk aversion, along with expected returns from production and from the futures market. The β terms have commonly been estimated from the slope coefficients of a linear regressions between spot prices and futures prices. It is important to note that output and hedging depend on the same set of exogenous parameters. In addition, an expression for unhedged output can also be defined.

$$z \equiv F(X_1) - f = \left[\frac{\bar{\pi}_s (1 - \beta_f)}{\sigma_s^2} + \frac{\bar{\pi}_f (1 - \beta_s)}{\sigma_f^2} \right] \Big/ D$$

Because output and hedging may adjust at different rates to shifts in parameters, it is also useful to examine the comparative statics results of unhedged output.

COMPARATIVE STATICS RESULTS

The reduced form solutions for output and hedging can be used to derive comparative statics results for shifts in exogenous parameters. Batlin (1983) derived similar results using a less general approach by assuming a linear mean-variance utility function. In this paper we concentrate our analysis on shifts in spot and futures prices as well as input prices. The objective is to highlight the crucial role that easily estimable covariances and variances play in determining the signs of these comparative static results. Tables 1a and 1b summarize the responses of optimal output, the amount hedged, and unhedged output to changes in input prices and the current futures price. Shifts in expected spot price and the expected futures price are also examined.

Input Prices

As intuition suggests, if input prices rise, the costs of production increase and this leads to an unambiguous decrease in the optimal level of output. The effect of increased input prices on the amount hedged, however, depends on the sign of the covariance between spot and futures prices. If spot and futures prices are positively correlated (β_s , $\beta_f > 0$), higher input prices lead to a decrease in the amount hedged. Unhedged output also responds to changes in input prices. If spot prices are more volatile than futures prices ($\beta_f > 1$), hedging will decline more than output leading to an increase in unhedged output for the short hedger.

Current Futures Price

The impact of higher current period futures prices on optimal producer decisions will again depend on the relative volatilities of spot and futures prices. If spot and futures prices are positively correlated (β_s , $\beta_f > 0$), increases in observed futures prices lead to increased output. The amount hedged is also directly related to changes in the current futures price. Unhedged output will also be directly related to the current futures price but only when futures prices are more volatile than spot prices; that is, β_s exceeds one. When β_s is less than one, the opposite result holds.

Expected Spot Price

Higher expected output prices lead to increased output, a result consistent with models of firm behavior under uncertainty proposed by Sandmo (1971) and Batra and Ullah (1974). Also, if spot and futures prices are positively correlated, the amount hedged will increase as the expected output price increases. Unhedged output will decline when the expected spot price rises only if spot prices have a greater variance than futures prices ($\beta_f > 1$).

In contrast, Batlin has noted that comparative statics results will be different with no basis risk and are no longer dependent on the covariance between spot and futures prices. In this instance the amount hedged will be lower as the producer expects higher spot prices. In turn, unhedged output will increase as the expected spot price increases.

Expected Futures Price

Higher expected futures prices lead to a decrease in output if spot and futures prices are positively correlated (β_s, $\beta_f > 0$). Increases in expected futures prices lower the returns from hedging and the producer will also hedge less. The effects on unhedged output again depend on the magnitude of a beta coefficient. Unhedged output will decline as expected futures prices rise when β_s exceeds one.

ESTIMATION METHODOLOGY

The importance of the β_s and β_f terms, which measure covariances between spot and futures prices is now apparent. The magnitude of these beta terms clearly plays a crucial role in determining, a priori, the signs of the comparative static results. In addition, β_s is useful in comparing the model with basis risk described in this paper to the model without basis risk suggested by Feder, Just and Schmitz (1980) and Holthausen (1979). They have shown that when basis risk is neglected the relationship between output and hedging is determined solely by the expected marginal profit from production. In equation form

$$F(X) \gtrless f \quad \text{as} \quad \tilde{\pi}_s \gtrless 0 \quad .$$

Output exceeds the amount hedged whenever producers expect positive marginal profits from production. By contrast, the producer should hedge an amount that exceeds output when production is expected to yield marginal negative profits.

In contrast, for the model with basis risk, the relationship between output and hedging can be written as

$$F(X) \gtrless f \quad \text{as} \quad \tilde{\pi}_s \gtrless \psi \sigma^2 (1 - \beta_s) f \quad .$$

Clearly, β_s plays a crucial role in distinguishing the model of basis risk from the Feder, Just, and Schmitz and Holthausen models. If β_s equals one, the two models provide equivalent prescriptions for the relationship between optimal levels of output and hedging. In addition, the relationship between output and hedging is independent of risk aversion if this condition holds. However, if futures prices and spot prices are not equally volatile, β_s will not equal one. In this case, the relationship between production and hedging will depend on the expected marginal profits from hedging along with the producer's risk aversion. The important point to note is that β_s can easily be estimated from data on spot and futures prices.

Estimates of betas derived from regression relationships of spot and futures prices are common in the analysis of both agricultural and financial futures. Two types of approaches have been proposed to estimate the beta coefficient: the price levels model and the price changes model.

In the price levels model, β_f is estimated by regressing spot prices against futures prices:

$$P_t = \alpha + \beta_f P_t^f + \epsilon_t \quad .$$

Brown (1986) pointed out that such an approach overlooks that fact that hedging is designed to reduce the risk of **price changes**. Econometrically, the procedure will violate assumptions of the OLS model if the distributions of the prices are not stationary. Although spot and futures price levels may be highly correlated, the hedger will in reality be interested in correlations between spot and futures price changes.

Researchers have become aware of the deficiencies of the price levels model and in response have shifted to the use of price changes in the regressions. In the portfolio model of hedging, the futures position is combined with the cash position with the objective of minimizing losses in value of the cash position. Hill and Schneeweiss (1981) stated that "effective hedging depends on the amount of covariance between **value changes** of the cash security and the futures" (emphasis in original).

The price changes model has been formulated two separate ways. First, Working (1953) proposed regressing first differences of spot prices on the first differences of futures prices:

$$(P_{t+1} - P_t) = \alpha + \beta_f (P^f_{t+1} - P^f_t) + \epsilon_t \quad .$$

However, this model is only appropriate if spot and futures prices follow martingales. That is,

$$E(\tilde{P}_{t+1}) = P_t \qquad \text{and} \quad E(\tilde{P}^f_{t+1}) = P^f_t \quad .$$

A somewhat different interpretation of the Working model can be obtained from the work of Peck (1975). She noted that risk should be measured as deviations from expectations and not simply as deviations from the mean of past outcomes. The only relevant price variability is that which makes the producer's forecast differ from actual output price. As Peck argues, "the crucial variance remaining, however, is that which surrounds that accuracy of the producers' forecasts." Hence, the beta coefficient should be estimated using data based on unanticipated changes in spot and futures prices.

Grant and Eaker (1984), following Peck's reasoning, suggested a variant of Working's approach by specifying more precisely the process of expectations formation. The authors assumed that current futures prices are unbiased predictors of both spot prices and futures prices in later time periods. Under these assumptions, the beta coefficient is estimated by an OLS regression of the unexpected changes in spot prices on the unexpected changes in futures prices:

$$(P_{t+1} - P^f_t) = \alpha + \beta_f (P^f_{t+1} - P^f_t) + \epsilon_t \quad .$$

This method implicitly accepts Peck's argument that estimation of the beta coefficient should measure the unexpected price changes as perceived by producers.

Although Grant and Eaker recognized the importance of a correct specification of expectations, in the presence of basis risk, futures prices would not generally be unbiased predictors of future spot prices. Hence, a model with basis risk must also specify the process producers use to form expectations of future spot prices. However, because producer's subjective forecasts of the mean and other moments of the distribution of the output price for fed cattle are unobservable, these forecasts must be estimated. Numerous methods have been suggested such as adaptive, extrapolative, naive, and rational expectations models. In recent work, Antonovitz and Roe (1986) found that an ARIMA model gave reasonable estimates of fed cattle producer's forecasts of the mean of output price.

Estimation of the beta coefficient in this paper assumes that producers form expectations of future spot prices as if the underlying time series representation of spot prices was known. Unexpected spot price changes are the deviations between expected and observed spot prices. Unexpected futures price changes are modelled following Grant and Eaker by assuming that the futures price in period 1 is an unbiased predictor of futures price in period 2. This assumption, consistent with finance theory, argues that futures positions have no initial or investment value and, hence, do not provide returns on an investment. Thus, the expected rate of return on a futures contract should be zero. The unexpected changes model is specified in the following form:

$$(P_{t+1} - E\bar{P}_{t+1}) = \alpha + \beta_f(P^f_{t+1} - P^f_t) + \epsilon_t$$

EMPIRICAL RESULTS

The beta coefficients (β_f and β_s) specific to the California fed cattle industry were estimated. We suggest a new model using ex ante price forecasts based on an ARIMA model. These results are compared to estimations based on Working's price changes model and the unbiased expectations model proposed by Grant and Eaker.

The estimation procedure used prices readily available to feedlot operators in California for 1970-85. Slaughter steer prices are reported weekly by the Bureau of Market News in California for three locations: the Imperial Valley, Southern San Joaquin Valley and Northern San Joaquin Valley. The empirical models assumed that the feedlot operator made monthly decisions on feeding of cattle with a feeding period extending for 120 days.

Following Peck's methodology, the feedlot operator is assumed to make ex ante forecasts of expected output prices. The forecasts are based on a knowledge of the underlying time series for choice steers over the full 16 year period. A moving ARIMA (0,2,1) model based on 10 years of data was used to estimate, four months in advance, the mean of the expected output price of fed cattle. Four-month-ahead forecasts for the period January 1980 to April 1986 were generated.

Beta coefficients for the three different regions were estimated for the period January 1980 to April 1986 using monthly spot and futures prices. Spot prices are monthly averages of weekly cash prices reported from the California Feeders Report. Futures prices are for the live cattle futures contracts from the Chicago Mercantile Exchange. The futures prices are for the nearby contract prior to the delivery month, assuming a 120 feeding period.

The estimated beta coefficients, standard errors of the beta coefficients, coefficients of determination, and Durbin-Watson statistics are shown in Table 2. The results for the ARIMA model we propose are compared with the estimates obtained from the Working price changes model and the Grant unbiased expectations models for each location. Clearly, comparative statics results discussed in this paper depend on the signs and magnitudes of the beta coefficients.

The coefficient of determination from the β_f model measures the proportion of the variance of unexpected spot price changes which is explained by futures prices changes. The R^2 for our models does not exceed 0.18 for the any of the three California locations. A study by Carter and

Loyns (1985) for hedging Canadian cattle resulted in a similar R^2 value of 0.12. More importantly, the results reported here are ex ante measures of hedging effectiveness which use information available to producers at the time output and hedging decisions are made.

Input Prices

A positive beta coefficient obtained in the three estimation methods indicated that higher input prices lead to decreases in the amount hedged. This result is intuitively reasonable since higher input prices also lead to reduced production. Both output and hedging decisions adjust to shifts in input prices. The beta coefficient, β_f, was not significantly different from one, indicating that output and hedging adjust at similar rates, leaving unhedged output unchanged.

Current Futures Prices

The positive beta coefficients indicated that producers should expand production as the current futures price increases. Higher current period futures prices clearly lead the producer to increase the amount hedged. The response of unhedged output to increases in the current futures price depends on the β_s coefficient. The unexpected changes model suggested that β_s is less than one, indicating that unhedged output and the current futures price are inversely related.

Expected Spot Price

The signs of the beta coefficients determine how producer's optimal decisions adjust to higher expected output prices. The estimated betas are positive suggesting that the producer should not only produce more but also hedge more when output price is expected to rise. This result differs from the Feder, Just, and Schmitz prescription that short hedging should decline when the producer expects higher spot prices. Furthermore, the model with basis risk examined here suggests that producer behavior is more complicated than without basis risk. Expectations of higher spot prices induce the producer to expand output. But the hedging and output decisions are interdependent and hedging also increases. The β_f term was not significantly different than one. The producer should adjust output and hedging at similar rates in response to shifts in the expected output price.

Expected Futures Price

The impact of higher expected futures prices on output and hedging decisions is indicated by the β_s coefficient. The positive beta coefficient leads producers to reduce output when futures prices are expected to rise. The amount hedged will also decline when producers expect higher futures prices. Output and hedging adjust in similar directions to shifts in expected futures prices.

However, output and hedging do not adjust at similar rates. The β_s coefficient less than one indicated that unhedged output is directly related to the expected futures price. The result indicates that hedging adjusts by an amount that is greater than or equal to the change in the amount produced.

SUMMARY

The objective of this paper was to derive and test a set of comparative statics results for optimal output and hedging decisions under basis risk. These results were obtained from a microeconomic model of decision making by producers. Producer response to shifts in both observed and expected spot and futures prices were shown to depend on estimates of covariances and variances of these prices.

Following Peck's analysis, a method was proposed to incorporate the producer's price expectations in the estimation of the beta coefficient. Estimates of the beta coefficient, which measure relative variability of expected spot and futures prices, were also derived using two commonly proposed methods in futures markets literature. The price changes model of Working and the unbiased expectations advocated by Grant and Eaker were also examined.

Some possible extensions of this research are also suggested. Alternative price expectations models for producers are being specified to obtain more precise estimates of unexpected price changes. Recent work by Antonovitz and Green (1986) suggested methods to test for expectations models. The stability of the beta coefficient over the sample period should also be investigated. Producer's expectations about price formation in the fed cattle market may shift over time in response to market conditions. The volatility of futures prices may also change significantly over time. Random coefficients models will be applied to test the stability of the beta coefficient in the three models analyzed here.

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Table 1a

Comparative Statics Results

<u>Input Price</u>

$$\frac{\partial F(X_1)}{\partial p_1} = -\frac{1}{D\,F'(X_1)\,\sigma_s^2} < 0$$

$$\frac{\partial f}{\partial p_1} = -\frac{\rho}{D\,F'(X_1)\,\sigma_s\sigma_f} \lessgtr 0 \quad \text{as} \quad \rho \gtrless 0$$

$$\frac{\partial z}{\partial p_1} = -\frac{(1-\beta_f)}{D\,F'(X_1)\,\sigma_s^2} \gtrless 0 \quad \text{as} \quad \beta_f \gtrless 1$$

<u>Current Futures Price</u>

$$\frac{\partial F(X_1)}{\partial p_1^f} = \frac{\rho}{D\,\sigma_s\sigma_f} \gtrless 0 \quad \text{as} \quad \rho \gtrless 0$$

$$\frac{\partial f}{\partial p_1^f} = \frac{1}{D\,\sigma_f^2} > 0$$

$$\frac{\partial z}{\partial p_1^f} = -\frac{(1-\beta_s)}{D\,\sigma_f^2} \gtrless 0 \quad \text{as} \quad \beta_s \gtrless 1$$

Table 1b

Comparative Statics Results

Expected Output Price

$$\frac{\partial F(X_1)}{\partial E(\tilde{p})} = \frac{1}{D\,\sigma_s^2} > 0$$

$$\frac{\partial f}{\partial E(\tilde{p})} = \frac{\rho}{D\,\sigma_s\sigma_f} \gtrless 0 \quad \text{as} \quad \rho \gtrless 0$$

$$\frac{\partial z}{\partial E(\tilde{p})} = \frac{(1 - \beta_f)}{D\,\sigma_s^2} \gtrless 0 \quad \text{as} \quad \beta_f \lessgtr 1$$

Expected Futures Price

$$\frac{\partial F(X_1)}{\partial E(\tilde{p}_2^f)} = -\frac{\rho}{D\,\sigma_s\sigma_f} \lessgtr 0 \quad \text{as} \quad \rho \gtrless 0$$

$$\frac{\partial f}{\partial E(\tilde{p}_2^f)} = -\frac{1}{D\,\sigma_f^2} < 0$$

$$\frac{\partial z}{\partial E(\tilde{p}_2^f)} = \frac{(1 - \beta_s)}{D\,\sigma_f^2} \gtrless 0 \quad \text{as} \quad \beta_s \lessgtr 1$$

Table 2

Estimates of the β_f and β_s Coefficient

for the Proposed Hedging Models

		Spot Price Changes				
LOCATION		ARIMA MODEL		WORKING MODEL		UNBIASED EXPECTATIONS MODEL	
Southern San Joaquin Valley	β_f	1.167	(.310)	.88	(.095)	1.00	(.055)
	β_s	.144**	(.038)	.63**	(.067)	.82**	(.045)
	R^2	.16		.56		.83	
	D.W.	1.671		.709		1.074	
Northern San Joaquin Valley	β_f	1.09	(.295)	.86	(.097)	1.01	(.053)
	β_s	.15**	(.041)	.61**	(.070)	.83**	(.044)
	R^2	.16		.52		.83	
	D.W.	1.602		.73		1.17	
Imperial Valley	β_f	1.16	(.292)	.91	(.087)	1.04	(.057)
	β_s	.16**	(.040)	.67**	(.064)	.80**	(.044)
	R^2	.18		.61		.82	
	D.W.	1.793		.73		.917	

Comparison of beta coefficients, standard error, explained variances (R^2) and Durbin-Watson statistics (D.-W.) for slaughter steers for three California locations.

**Indicates coefficient significantly different from one at the .05 level.