NCCC-134
APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

Estimating the Effects of Risk on the Supply of Storage

by

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Introduction

The literature on storage decision-making under uncertainty assumes the firm consider only this period's price and the expected price in the subsequent period (Keynes; Williams; Working 1948, 1952; Brennan; Peck; Kohn; and Just and Rausser). More recently, Chavas presented a model of competitive speculation for a storage firm operating in a cash market under uncertainty and a multi-period planning horizon. A key finding of his study is that the risk-premium is a function of the expected inventory holdings in future periods. Glauber and Powers have extended this analysis to include inventory and hedging decision-making in the context of a mean-variance utility function.

This study incorporates these recent theoretical developments in an empirical model that seeks to estimate the effect of risk on the supply of storage relationship. The study extends previous empirical work on the supply of storage (e.g., Working; Brennan; Weymar; Paul; Peck) to a consideration of whether risk affects the costs speculators must bear to store the commodity.

A model is developed to explain the level of the carrying charge between adjacent futures contract prices (the price spread). Explanatory variables include the current level of private and public stocks, the interest rate, the variability of the futures price, and a covariance term that captures the effect of future inventory holding decisions. The formulation of the model draws upon the recent literature on risk and uncertainty as well as recent research findings in the literature on commodity storage (Gardner 1979, 1983; Wright and Williams 1982, 1983; Plato and Gordon 1983, 1984; Helmberger and Akinyosoye; Lowry, Glauber, Miranda and Helmberger).

The model is estimated over the sample period 1961-1985 with quarterly data for corn, wheat and soybeans. Using daily futures price quotes variance and covariance terms are constructed to provide <u>ex ante</u> measures of price risk. Results indicate that inventory holders for corn, wheat and soybeans share common risk preferences and that these preferences have a signicant effect on the size and sign of the carrying charge.

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Inventory Holding Under Price Uncertainty

At the beginning of each period the firm can purchase the storable commodity at a given price, P_t, and sell it at a price, P_{t+k}, k periods later. The firm's revenue in period t is given by $P_t(x_{t-1}-x_t)$, where x_t is the level of inventory carried out of period t. If x_t is greater than x_{t-1}, then the firm has purchased inventory and this appears as a cost. Likewise, if x_t is less than x_{t-1} the firm has sold inventory and this appears as revenue. A storage cost, $C(x_t)$, is incurred to store the commodity from one period to another. The cost function is assumed to be continuous and twice differentiable with respect to x_t such that $C'(x_t)$ is greater than zero and $C''(x_t)$ is nonnegative. The net proceeds earned for the storage service in period t is thus:

(1) $$PR_t^c = P_t\,(x_{t-1}-x_t) - C(x_t)$$

We assume the firm maximizes the certainty equivalence of the present value of an additive stream of discounted net proceeds by choosing the level of carryouts from period t to period T -- the planning horizon of the inventory holder:

(2) $$\max_{x_i} L_t = E_t(Z) - R^h/2(V_t(Z))$$

$$i = t,\ t+1,\ \ldots,\ T-1,\ T;$$

$$\text{s.t. } x_i \geq 0, \text{ for all } i; \qquad L_t \geq 0;$$

$$Z = \sum_{i=t}^{T} B^{T-i} PR_i^c;$$

where E and V denote the mean and variance, Z is profit, R^h is the risk-aversion coefficient of the inventory holder, and B is one plus the rate of interest.

The firm is assumed to make an inventory decision, x_t, at the beginning of each period to maximize (1). The restrictions on $C(x_t)$ assure the existence of optimal x_t's for a risk-neutral or risk-averse firm. We assume that $C'(x_i)$ and B are nonstochastic and that the risk preferences of the inventory holder are unchanged over the planning horizon. It is also assumed that the inventory holder is not a consumer of the commodity and hence the convenience yield of storage--the so-called demand for pipeline stocks--is zero (Working 1948).

The price at t, P_t, is exogenous and the prices for future periods are randomly distributed about a known distribution. The distribution of prices is conditional on the market information available to the firm at period t.

The optimal inventory level for each period can be solved using dynamic programming (Bellman). Starting at the end of the planning horizon, T, and maximizing (2) with respect to (w.r.t.) x_T the following first-order-conditions (f.o.c.) are obtained:

$$(3)\qquad \begin{aligned} &[E_T(-P_T-C'(x_t))] \leq 0 \\ &x_T[E_T(-P_T-C'(x_t))] = 0 \,. \end{aligned}$$

Since $C'(x_t)$ and P_T are greater than zero, the optimal value for x_T (x_T^*) is zero. There is no economic incentive to carry out stocks beyond the planning horizon.

Working backwards to period T-1, (2) is maximized w.r.t. x_{T-1} conditional on the optimal value of carryout for period T ($E_t(x_T^*) = 0$). Maximizing (2) w.r.t. x_{T-1} yields the following f.o.c.:

$$(4)\qquad \begin{aligned} &[E_{T-1}(P_T) - (P_{T-1}+C'(x_{T-1}))B - R^h(V_{T-1}(P_T))x_{T-1}] \leq 0 \\ &x_{T-1}[E_{T-1}(P_T) - (P_{T-1}+C'(x_{T-1}))B - R^h(V_{T-1}(P_T))x_{T-1}] = 0. \end{aligned}$$

Assuming risk-averse firms and that carryout in period T-1 is positive, the marginal risk-premium $R^h(V_{T-1}(P_T))x_{T-1}$ is positive and increasing in x_{T-1}. The expected price must exceed the cost of purchasing and storing the commodity for the firm to store inventories from one period to another (Sandmo; Leland; Feder et al.; Holthausen).

Of interest are the special cases where the firm is risk-neutral ($R^h=0$) or where the expected price is known with certainty ($V_{T-1}(P_T)=0$). In these cases, the firm will hold inventory if and only if the expected price equals the current price plus storage costs (Gustafson; Samuelson; Gardner).

Proceeding backwards we solve for x_i in periods i = T-2, T-3, T-4, etc., and arrive at the current period (i=t). In general, the intertemporal arbitrage conditions for period t are:

$$(5)\qquad \begin{aligned} &[B^{-1}E_t(P_{t+1})-(P_t+C'(x_t))-R^h(V_t(P_{t+1})(x_t-E_t(x_{t+1}^*)) + \\ &\qquad \sum_{i=2}^{T-t} B^{1-i}COV_t(P_{t+1},P_{t+i})E_t(x_{t+i-1}^*-x_{t+i}^*))] \leq 0 \\ &x_t[\quad " \quad " \quad " \quad " \quad " \quad] = 0 \end{aligned}$$

where $E_t(x_{t+i}^*)$ is the expected optimal carryout level in period t+i and $COV_t(P_{t+1},P_{t+i})$ is the covariance between price in the next period and price in subsequent periods.

As in (4), (5) includes a marginal risk-premium. However, in the latter equation the sign of the marginal risk-premium depends on whether there will be a net sale or purchase of inventory in future periods. If we assume that the covariance between P_{t+1} and prices in subsequent periods is negligible (e.g., between the old and new crop years), the marginal risk premium reduces to $R^h(V_t(P_{t+1})(x_t - E_t(x^*_{t+1})))$. If there is an expected net sale, the marginal risk-premium is positive. Since the price in period t+1 is uncertain, inventory holders who expect to sell their inventory will require a premium above the physical costs of storage to induce them to hold grain. Conversely, if there is an expected net purchase, the marginal risk-premium is negative. The risk-averse firm is willing to pay more for stocks purchased in the current period at a known price, P_t, than it would to purchase all of its inventory in period t+1 at an uncertain price, P_{t+1}. Lastly, when the expected net sale is zero, the marginal risk-premium is zero and the expected price equals the current price plus the carrying charge (Chavas).

The arbitrage conditions in (5) reflect the effect of subsequent storage decisions on the storage decision made in the current period. The decision to store is based indirectly on expected prices and variances in all periods. This result contrasts with the view that risk-averse firms have no reason to consider prices beyond the following period (Kohn).

At the aggregate level these results suggest that there may exist a positive risk premium (i.e., $E_t(P_{t+1})$ is greater than $(P_t + C'(x_t))B$) whenever stock levels are expected to decline, and a negative premium when stocks are expected to increase. Thus we would expect that for discontinuously stored commodities such as potatoes and onions a positive risk premium would exist throughout most of the storage season. For continuously stored commodities such as corn and soybeans, we have shown that is possible to have positive carryout of the old crop into the new crop year despite prices that show an inverse carrying charge between the old and new crop year. Thus, speculative as well as pipeline stocks may be carried forward when carrying charges are less than full (Working).

An Empirical Application

From the theoretical model developed above an empirical model can be estimated that measures the effect of risk on the supply of storage. Assuming an interior solution, equation (5) is rewritten to obtain:

$$(6)\quad B^{-1}E_t(P_{t+1}) - P_t = C'(x_t)) + R^h(V_t(P_{t+1})(x_t - E_t(x^*_{t+1})) + \sum_{i=2}^{T-t} B^{1-i}COV_t(P_{t+1}, P_{t+i})E_t(x^*_{t+i-1} - x^*_{t+i}))$$

Equation (6) represents the supply of storage relationship for a representative firm. If we assume that there are n identical firms in the industry then it can be easily shown that R^h represents the coefficient of absolute risk aversion for the industry.

In order to simplify the estimation of (6) we assume that the covariance between price in period t+1 and prices beyond period t+2 have a negligible effect on the marginal risk premium and can be ignored.

Given a quarterly model and the effect of the discounting factor on future periods this may be a reasonable assumption. The model becomes:

(7) $\text{Charge}_t = b_0 + b_1 X_t + b_2 \text{Risk}_t + e_t$

where:

$$\text{Charge}_t = B^{-1}E_t(P_{t+1}) - P_t$$

$$\text{Risk}_t = V_t(P_{t+1})(x_t - E_t(x^*_{t+1})) + B^{-1}\text{COV}_t(P_{t+1}, P_{t+2})E_t(x^*_{t+1} - x^*_{t+2})$$

$$e_t \sim N(0, \sigma^2)$$

If the marginal physical costs of storage are increasing or constant in x_t we would expect b_1 to be greater than or equal to zero. Note that b_2 is a measure of the coefficient of absolute risk aversion. If inventory holders are risk-averse (preferring) we would expect b_2 to be greater (less) than zero. Failure to reject the null hypothesis that b_2 is equal to zero would suggest that inventory holders are risk-neutral.

To approximate the carrying charge between quarters the spread between the relevent futures market contracts is calculated (Brennan; Paul; Peck). For example, to construct the carrying charge between the second and third quarters of the marketing year for soybeans, an average is taken of the daily closing spreads between the March and May futures contracts during the month of February:

$$\text{Charge}_2 = (1+r/6)^{-1}(FP^5_2 - FP^3_2)$$

where FP^j_i is the futures price for delivery in contract month j taken in month i and r is the annual rate of interest. To avoid liquidation bias (Paul 1985), data is drawn from the month prior to the maturity month of the nearby contract.

Stock data is available from USDA on a quarterly basis with reporting dates on January 1, April 1, June 1 and October 1 for corn and wheat, and for soybeans, on January 1, April 1, June 1 and September 1. To account for the unequal lengths of the quarters the spreads are divided by the number of months in the spread and then multiplied by the number of months in the relevent quarter.

Two problems arise in measuring the risk variable. The first problem consists of how to quantify the variance and covariance terms. Previous researchers have used lagged prices or lagged deviations from expected prices as proxies for risk variables (Just; Lin; Traill), but one must question whether such ex post measures are consistent within the framework of rational expectations models. A more desirable measure of ariance would be an ex ante measure of price variance which would easure the variance of future prices rather than the variability of past rices.

As an alternative to lagged prices, variances and covariances are alculated using daily closing prices for various futures prices. If one ssumes that market information remains relatively constant over a given eriod then changes in daily futures prices within that period could eflect the underlying price distributions, or, at least, provide a easonable proxy for that distribution. While admittedly restrictive, he assumption allows the construction of ex ante measures for price ariances and covariances. For example, to construct the variance of rice in the second quarter, $V_1(P_2)$, a variance is calculated for the arch futures contract from daily closing prices during the preceding onth of October. Likewise, one can calculate $COV_1(P_2, P_3)$ from aily closing quotes for the March and May contracts over the same period.

A second problem arises in deriving a measure for the expected arryout in future periods $(E_t(X^*_{t+1}), E_t(X^*_{t+2}))$. Following ardner and Wright a relationship is estimated that relates carryout to otal available supply (the optimal carryout rule). This relationship is xpressed thusly:

3) $$X_t = a_0 + a_1 \, Supply_t + a_2 \, D_4{*}Supply_t + a_3 \, Rint_t + a_4 \, Gov_t + a_5 \, D_4{*}Gov_t$$

iere $Supply_t$ is the amount of supply available for consumption in eriod t (production plus carryin), $Rint_t$ is the real rate of interest he nominal rate of interest minus the inflation rate), Gov_t is the nount of publicly-held stocks (CCC plus FOR) in period t, and D_4 is a inary variable that is equal to one for the fourth quarter of the rketing year and zero otherwise.

The coefficient a_1 represents the marginal propensity to store iich we hypothesize to be positive and less than one. From the work of lmberger and Akinyosoye and Lowry et al. we would expect a_3 to be ss than zero; inventory holders would hold less if the interest rate re to increase. The coefficient a_4 measures the marginal effect of a it increase in public stocks on total stocks. Previous research ardner; Wright) suggests that this coefficient is less than one because the substitution effect between public and private stocks. Lastly, e inclusion of binary variables for the supply and government stocks riables (coefficients a_2 and a_5) allows us to test whether the fects of interyear storage are significantly different from intrayear orage.

The data for corn, wheat and soybeans are drawn for quarterly servations for the period 1961 to 1984. Quarters refer to the quarters the marketing year. Thus, for corn and soybeans, the marketing year gins in September, while for wheat it begins in June. All prices are flated by the GNP implicit price deflator (1972=100).

Results

Estimates of the expected carryout relationship (equation (8)) for corn, wheat and soybeans are presented in Table I. The marginal propensity to store for intrayear storage ranges from .905 for corn to .789 for soybeans. For the fourth quarter, the marginal propensity to store falls off for all crops, reflecting the tendency to consume available supplies with the coming of the new crop. The coefficients for the real interest rate are negative for corn and wheat but not statistically significant.

The effect of public stocks on total carryout is most pronounced in the fourth quarter. This reflects the fact that most nine month non-recourse loans are forfeited to the government during this quarter. (Since only a small percentage of soybean stocks have been placed in the CCC over the sample period, these variables were omitted for the soybean equation.) Substitution effects of public stocks for private stocks for the fourth quarter are .298 for corn and .382 for wheat. These results suggest that over sixty percent of the grain placed in public stocks is grain which normally would be held by private inventory holders.

The two stage least squares estimates for the supply of storage equation (equation (7)) for corn, wheat and soybeans are presented in Table II. There are several results that merit comment. The coefficient of carrryout (the marginal cost of storage) is positive and of similar magnitudes for all crops as expected (Paul), and statistically significant at the 99 percent level for corn and wheat. The coefficients of absolute risk-aversion are positive and significant at the 90 percent level for all crops. This suggests that inventory holders are risk-averse and that their risk preferences affect the size of the carrying charge. Note that the coefficients of absolute risk-aversion for corn and wheat are of similar magnitude and lower than that for soybeans. This may reflect the fact that a larger proportion of these crops are held in public stocks and not by risk-averse private inventory holders.

Table III measures the monthly marginal risk premiums for corn, wheat and soybeans. Note that the level of the risk premiums declines throughout the storage season. Soybeans, for example, show a decline from .183 cents in quarter one to .153 cents in quarter three. In quarter four, however, the carrying charge is -.456 cents indicating that inventory holders are willing to hold grain even if the carrying charge is less than the physical costs of storage because they are more willing to purchase grain at a known price in quarter four, ceterus paribus, then to purchase all of the grain at an uncertain price in the new crop year.

Note that the size of the marginal risk premiums are larger for the post-1972 subsample. This supports the fact that grain markets have been more volatile since the change in world markets in the early 1970's. Chow tests were constructed to test whether or not there was a statistical difference between the risk parameters for the pre and post-1972 subsamples. Results indicate that one could not reject the null hypothesis that the risk parameters were equal for the two sample periods. Thus, increased risk premiums reflect increased price volatility, not increases in risk preferences.

Conclusions

In the preceding analysis a theoretical model of risk-averse storage behavior has been presented and estimated econometrically for corn, wheat and soybeans. Results indicate that inventory holders for these crops share common risk preferences, and that these preferences, along with the levels of price variance and expected net change in inventory, determine the sign and magnitude of the marginal risk premium.

Several extensions of the model could be considered. First, the model considered here has focused primarily on U.S. stockholding behavior, ignoring how world supplies, particularly production in the southern hemisphere, affect the risk premium. For example, it is not unusual for the carrying charge for soybeans to weaken in the Spring when the Brazilian and Argentine crops are being harvested and marketed. Broadening the study to include world stocks may provide better parameter estimates for the supply of storage equation.

Secondly, contingent claims markets such as futures and options markets, have been omitted from this analysis. Glauber and Powers show how hedging can potentially reduce the size of the risk premium. Thus, failure to include hedging activity in the econometric model may have resulted in a downward biased estimate of the coefficient of absolute risk-aversion. A proper econometric model of hedging activity merits future research.

Lastly, in the model presented above it was shown how inverse carrying charges can exist without presuming the existence of pipeline stocks. Nonetheless, there is a demand for working stocks which has been ignored here. The issue of how to model pipeline stocks has been the subject of much research including Working, Peck and most recently, Williams. While beyond the scope of the present study, including pipeline stocks in an econometric model of the supply of storage would provide researchers with a better understanding of what determines the sign and magnitude of the carrying charge.

Table I. Expected carryout equation for corn, wheat and soybeans, 1961-1984a/

Crop	Explanatory Variables						R^2	d.w.
	Constant	Supply	D4*SUP	Rint	Govt	D4*GOV		
Corn	-1651.753 (-1.118)	.905 (42.048)	-.269 (-2.820)	-4.377 (-.724)	.021 (.640)	.277 (2.219)	.994	1.625
Wheat	-45.350 (-.750)	.797 (25.604)	-.164 (-2.325)	7.473 (.990)	-.145 (-1.373)	.527 (2.171)	.935	1.967
Soybeans	-161.528 (-3.495)	.789 (55.955)	-.097 (-3.185)	-1.577 (-1.005)	---	---	.994	1.967

a/ T-ratios are in parentheses.

Table II. Estimated carrying charge equation for corn, wheat and soybeans, 1961-1984a/

Crop	Explanatory Variables			R^2	d.w.
	Constant	Carryout	Risk		
Corn	-3.717 (-3.252)	.00133 (4.740)	.00000162 (4.239)	.470	1.961
Wheat	-4.3004 (-1.871)	.00123 (2.930)	.00000112 (1.868)	.360	2.174
Soybeans	-14.370 (-1.716)	.000961 (.955)	.00000773 (4.435)	.278	2.082

a/ T-ratios are in parentheses.

Table III. Monthly marginal risk premiums for corn, wheat and soybeansa/

Quarter	Corn		Wheat		Soybeans	
	1965-83	1973-83	1965-83	1973-83	1962-83	1973-83
1	.015	.025	.019	.028	.183	.320
2	.014	.024	.009	.014	.143	.299
3	.013	.015	-.024	-.039	.153	.281
4	-.050	-.070	-.014	-.022	-.456	-.846

a/ Marginal risk premiums expressed in cents per bushel per month (1972$)

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