Forecasting Livestock Prices Using a Stochastic Coefficient Approach

by

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Introduction

Since the early eighties, econometric forecasting models have systematically overestimated livestock and meat prices, leading to widespread speculation about changes in consumer demand. Indeed, many studies have tried to test for changes in the structure of meat demand, by testing for parameter instability (Nyankori and Miller, 1982, Chavas, 1983, and Dixon and Martin, 1982).

This paper does not attempt to detect, isolate, or identify structural change within the meat complex. Rather, it proposes to use a generalized stochastic coefficient model (Swamy and Tinsley, 1980) as a tool to project the retail prices for beef, pork, and chicken. This empirical model allows a more general focus of determining the influence of the variance of explanatory variables as a result of possible structural change and the factors influencing retail meat prices. This paper uses a given functional form (Stillman, 1985) for a retail price forecasting equation and compares the forecasting ability of the stochastic coefficient model with that of other estimation techniques.

Testing for structural change becomes a test of the main hypothesis (that is, constant structure) against a multiple set of alternative hypotheses (that is, errors in variables, model misspecification, nonlinearities, and so forth). Beyond the problems of defining a proper structure and estimating a functional form are the pragmatic data problems faced by a forecaster. The practical problem is to limit the information going into a forecast to ease operational difficulties. The proposed stochastic coefficient model can compensate for some of these data problems.

The first of this paper's three main sections is the discussion of the theoretical justification and statistical methodology of the Swamy-Tinsley model. Second, results from the estimation of the retail price-dependent forecasting equation (Stillman, 1985) are presented. This section discusses the forecasting performance of the stochastic coefficient model against several alternative estimation procedures. A brief discussion of the patterns of the time varying parameters is also presented. The final section is a summary and conclusions.

Theoretical Justifications of the Stochastic Coefficient Model and Estimation Procedure

The use of stochastic coefficient estimation is a fairly recent econometric development. There are several theoretical and empirical justifications for this type of model. The true coefficients themselves can be seen as generated by a nonstationary or time varying random process.

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Omitted variables which exhibit nonstationary behavior and which are not orthogonal to the included variables can induce variability in the parameters (Duffy, 1969). Econometricians often use proxy variables in the place of unobservable explanatory variables. These variables imperfectly capture the relationship between the true economic variable and the dependent variable. Also, the relationship between the true variable and its proxy may change over time. Aggregation over micro units can induce variation. The assumption of constant weights of micro units over time is very restrictive (Zellner, 1962, 1969). The rationale for imposing the constraints suggested by micro theory on a constant parameter model is typically rather weak. A more general theory of increased aggregation leads naturally to the stochastic coefficients models (Swamy, Barth and Tinsley, 1982).

Coefficient variation may occur as a result of imposing an incorrect functional form on the equation. Rausser, Mundlak and Johnson (1982) noted..."The approximation of highly nonlinear 'true' relationships by simpler functional forms, along with observations outside the narrow sample range, provides perhaps the strongest motivation for varying parameter structure." Lucas (1975) and Lucas and Sargent (1978) note that changes in economic or policy variables will result in a new environment that will, in turn, lead to new optimal decisions and new micro- and macroeconomic structures. The stochastic coefficient modeling approach allows one to deal with instabilities in economic relationships, including constantly occurring ones, without excessive prior information requirements.

Estimation Procedure

The methodology used to estimate the model presented in this paper is a first-order variant of the generalized ARIMA stochastic coefficient process model developed by Swamy and Tinsley (1980). This model represents a generalization of the other stochastic coefficient models, such as the Kalman filter and the Cooley-Prescott procedure.

A general outline of the model and statistical procedure is discussed below. For a more rigorous description of the model and methodology, see Swamy and Tinsley (1980).

The model can be written in vector notation as:

$$y_t = \underline{x}_t' \, \underline{\beta}_t \tag{1}$$

where,

y_t is the value of the dependent variable in period t

$\underline{x}_t$ is a Kx1 vector of the independent variables in period t

$\underline{\beta}_t$ is a Kx1 vector of the parameters in period t

In order to estimate the model one must impose structure on $\underline{\beta}_t$, since there are only T observations. The structure imposed on $\underline{\beta}_t$ in (1) is a stationary stochastic vector e_t, driven around a fixed mean $\underline{\bar{\beta}}$. Therefore,

$$\underline{\beta}_t = \underline{\bar{\beta}} + \underline{e}_t \tag{2}$$

$$\underline{e}_t = \phi \, \underline{e}_{t-1} + \underline{u}_t \tag{3}$$

Where u_t is a vector of white noise innovations.

$$\underline{u}_t \simeq ws(\underline{0}, \Delta_u) \tag{4}$$

The variance-covariance matrix of $\underline{\beta}_t$ is

$$\Gamma = E(\underline{\beta}_t - \overline{\underline{\beta}})(\underline{\beta}_t - \overline{\underline{\beta}}) \tag{5}$$

and the unconditional variance of the dependent variable is

$$var(y_t) = \underline{x}_t' \, \Gamma \, \underline{x}_t \tag{6}$$

where,

$$VEC(\Gamma) = [I - \phi \otimes \phi]^{-1} \; VEC(\Delta_u) \tag{7}$$

Therefore, the conditional expected value and variance of the dependent variables depend on the conditioning variables. If one then allocates the variance of the dependent variable among its contributing factors, the influence of the variance of the dependent variable may be identified (Theil, 1971). Such identification becomes important within the framework of the stochastic coefficient model because it is possible for a variable to affect the variance of the dependent variable without affecting its mean. Within the framework of a fixed coefficient model, Δ_u and ϕ will collapse to scalar characteristics of the intercept coefficients. One may also obtain approximations of t-tests of the individual components by using an asymptotic approximation of the covariance matrix of the estimated column stack, VEC(Δ_u), to test the significance of the uncertainty allocations to the slope coefficients.

One can gain insight into the virtues of the stochastic coefficient model by examining the relationship between the errors generated by the fixed coefficient model and the stochastic coefficient model. Total residual u_t is the weighted sum of the intercept parameters and the time varying parameters $\underline{u}_t \equiv \underline{x}_t \underline{e}_t$. The error associated with the unit vector intercept term is analogous to the additive disturbance in the fixed coefficient model. Tinsley, Swamy, and Garrett (1981) have shown that if ordinary least squares is a consistent estimator of $\underline{\beta}$, the estimates of $\underline{u}_t(t=1,\ldots..,T)$ from the two estimators will converge as T increases.

Another virtue of the stochastic coefficient model is its ability to permit vector serial correlation and heteroskedasticity to exist and correct for them. This can be demonstrated by representing equation (2) in the two-variable case.

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + (e_{0t} + e_{1t} x_t + e_{2t} x_t) \tag{8}$$

Since x_t varies from period to period, equations (2) and (8) allow for the existence of serial correlation and heteroskedasticity. The fixed coefficient model assumes *a priori* that the ϕ_{ij}'s other than ϕ_{00} (the one associated with the intercept term), are 0 and that the u_{it}'s other than u_{0t} are zero with probability of 1.

Estimation and Forecast Results

Price-dependent equations taken from Stillman's (1985) quarterly forecasting model were used as the functional form of the model as represented in equation (9).

$$P_{it} = \beta_{i0} + \beta_{i1}BC_t + \beta_{i2}PC_t + \beta_{i3}CC_t + \beta_{i4}PCE_t + e_t \qquad (9)$$

where P_{it} is the real retail price of beef, pork, or chicken; BC_t is beef per capita consumption, PC_t is pork per capita consumption, CC_t is chicken per capita consumption; PCE_t is real per capita total consumption expenditures; and e_t is an error term. The model was estimated over the period 1964I to 1979IV and projections were made over a 16-period horizon to 1983IV.

The data series are generated by the Economic Research Service from National Agricultural Statistics Service and Bureau of Commerce data. Per capita consumption data is generated from known supplies and other disappearance data. Consumption, therefore, is the residual of all errors within the data measurement system. These errors are small compared with the size of domestic disappearance, but the information on "slippage" within the supply system is not known and should not be assumed constant over time. Prices for these meats are fixed weight aggregations and do not reflect changes in the slaughter mix (that is, increasing cow slaughter increases the supply of hamburger, but does not change the weights of hamburger in the price aggregation scheme). In the case of chicken, the reported price is a whole-bird price and does not reflect the recent movement towards cutup chicken parts available to consumers. The commodity available to consumers has changed in form over the time period the model was estimated. The stochastic coefficient model can correct for these changes in the relationships between the dependent and independent variables over time. The data are not seasonally adjusted. A mean parameter value is generated for each coefficient and the stochastic coefficient model can correct for seasonal patterns.

The procedure used to estimate the parameters is iterative. Swamy and Tinsley (1980) discuss the procedure which starts with arbitrary values of Δ_u and ϕ iterate away from these values. The model utilizes the whole data set and does not require partitioning a data set for starting values and estimation. Estimates of the fixed parameters model were estimated over this same period using ordinary least squares (OLS), Cochrane-Orcutt (C-O), and maximum likelihood (ML) estimators. Table 1 compares results from these estimations with the mean values for the $\underline{\bar{\beta}}$ from the Swamy-Tinsley model.

Further insight into the stochastic coefficient estimation results can be gathered from the coefficient of variation of the parameters (table 2). In examining these coefficients of variation, one should note that the own-price coefficient tends to be the most stable of the parameters, followed by the expenditures variables (with the exception of chicken). The intercept term (omitted variables) tends to have the largest variation of the parameters. This result is expected because it should contain all other influences omitted from the model. Variation in the pork price equation coefficients is very small compared with the other two equations. This stability is similar to result found in other varying parameter research (Nyankori and Miller, 1982 and Chavas. 1983).

Table 1--Comparison of the estimated parameters

Independent variable	Estimation procedures: Varying parameter 1/	OLS	Cochrane-Orcutt	Maximum likelihood
Retail beef price:				
Constant	169.53	122.88	160.75	145.10
	(5.8) 2/	(7.8) 3/	(7.7) 2/	(7.6) 2/
Beef consumption	-4.65	-3.21	-2.67	-2.42
	(-5.2)	(-4.7)	(-4.4)	(-4.0)
Pork consumption	-.86	-.73	-.83	-.83
	(-.6)	(-1.3)	(-1.4)	(-1.4)
Chicken consumption	.74	.94	.66	.99
	(-.6)	(-1.3)	(-1.4)	(-1.4)
Personal expenditures	.17	.26	-.04	-.01
	(1.5)	(3.1)	(-.5)	(-.1)
R^2		.45	.33 4/	.59 4/
DW		.61	2.15 4/	2.11 4/
RHO(initial)			.70	
RHO(final)			.89	.89
Retail pork price:				
Constant	130.99	126.37	129.55	166.24
	(73.9	(9.9)	(7.1)	(7.1)
Beef consumption	-.54	-.26	.65	.90
	(-8.5)	(-.5)	(1.1)	(1.6)
Pork consumption	-5.01	-4.55	-4.04	-4.06
	(-92.6)	(-10.3)	(-7.5)	(-7.5)
Chicken consumption	-4.44	-3.06	-3.60	-3.28
	(-38.1)	(-3.3)	(-2.4)	(-3.8)
Personal expenditures	.42	.31	.14	.17
	(44.6)	(4.5)	(2.4)	(3.0)
R^2		.70	.53	.63
DW		.67	1.61	1.60
RHO(initial)			.66	
RHO(final)			.81	.79
Retail chicken price:				
Constant	95.89	97.00	96.10	94.88
	(8.3)	(13.3)	(9.5)	(10.1)
Beef consumption	-.95	-1.04	.80	-.77
	(-2.4)	(-3.3)	(-2.2)	(-2.2)
Pork consumption	-1.32	-1.70	-1.91	-1.66
	(-4.3)	(-6.6)	(-5.8)	(-5.8)
Chicken consumption	-2.08	-2.03	-1.69	-1.66
	(-3.0)	(-3.8)	(-3.2)	(-3.2)
Personal expenditures	.003	.05	.01	.02
	(.06)	(1.1)	(.4)	(.4)
R^2		.63	.47	.67
DW		.88	2.07	2.06
RHO(initial)			.56	
RHO(final)			.60	.60

1/ Conditioned on iteration estimates of ϕ and Δ_u
2/ Values in parentheses represent asymptotic t-statistics.
3/ Values in parentheses represent t-statistics.
4/ Based on RHO-transformed variables.

Table 2--Estimated coefficients of variations of stochastic coefficients

Equation	Intercept	Beef consumption	Pork consumption	Chicken consumption	Real personal consumption expenditures
			Percent		
Beef retail price	198.30	1.38	39.45	55.50	1.40
Pork retail price	1.13	.25	.09	1.06	.005
Chicken retail price	113.75	18.60	2.39	16.41	140.4

Table 3. Comparison of varying parameter model to standard linear estimation 1980 I to 1983 IV

	Actual value	Ratio to varying parameter error	Actual value	Ratio to varying parameter error	Actual value	Ratio to varying parameter error
Beef retail price:						
Swamy-Tinsley	7.29		6.08		0.32	
OLS	20.16	2.76	22.13	3.64	.75	2.38
Cochrane-Orcutt	14.00	2.76	22.13	3.64	.75	2.38
Maximum likelihood	12.91	1.77	14.02	2.31	.75	2.38
Pork retail price:						
Swamy-Tinsley	5.03		7.28		.32	
OLS	8.09	1.61	12.85	1.77	.38	1.18
Cochrane-Orcutt	2.91	0.58	3.87	0.53	.313	0.98
Maximum likelihood	3.09	0.61	4.13	0.57	.313	0.98
Chicken retail price:						
Swamy-Tinsley	3.78		12.23		.375	
OLS	6.68	1.77	24.01	1.96	.56	1.50
Cochrane-Orcutt	6.34	1.68	21.82	1.78	.50	1.33
Maximum likelihood	6.38	1.69	21.97	1.79	.50	1.33

1/ Turning point errors are calculated by subtracting the predicted from the previous actual and multiplying this value by the change in the actual value and dividing the number of negative values by the number of observations.

Parameter Variation over the Estimation Period

The timepaths of the coefficients of the equations present some interesting results in relation to historical events in the different sectors, figures 1-6. From the 1960 to the midseventies, the beef industry grew as it increased production (and therefore consumption) and expenditures on beef as a percentage of income increased. The model reflects this occurrence with increasing beef own price coefficient over this period. Around 1975, large cattle herds (the highest on record) coupled with high grain prices and adverse weather conditions caused cattle producers to liquidate their herds. The resulting increase in beef production to record levels is reflected in the model by a drop in the own price coefficient, which could be related to a nonlinear reaction to large levels of beef consumption. This nonlinear reaction is further highlighted by the fact that the coefficient increased as supplies of beef declined in the late seventies.

The time profile of the stochastic parameters associated with the expenditure variable shows interesting results. The coefficient is fairly stable over 1964 to 1971. The coefficient then decreases, rebounds, and decreases again. The general economy experienced a deep recession between 1973IV and 1975II. The coefficient actually turns negative during this period. There was vigorous recovery from 1975III to 1978IV; the economy then weakened again in 1979. The expenditure coefficient tends to reflect these movements in the overall general economy, likely reflecting distributional effects not apparent in the aggregated data. The stochastic coefficient model reflects some of the events not explicitly included within the model (omitted variables).

Time profiles for the pork equation also show interesting results. Many analysts believed that there is a kinked nonlinearity in pork demand. The stochastic coefficient model reflects this hypothesis. It has been hypothesized that pork price relationships remain fairly steady at 52-62 pounds per capita. The upper limits were reached in 1970-72 and 1976-80 and the lower levels were reached in 1974-75. The stochastic coefficients model reflects this nonlinear relationship.

The expenditure coefficient for pork exhibits a similar pattern to the beef expenditure coefficient. Variation in this parameter is not as pronounced as the beef coefficient, possibly reflecting the higher relative price of beef and how expenditures may change as budgets are allocated.

Broiler consumption over this period increased as technology allowed producers to offer the product at a declining real price. The relative price of poultry has declined over time, compared with beef and pork. Parameters of the model remain centered around the mean value and show no definite pattern; however, there is considerable variation. The interesting outcome in the chicken price equation is the large spike in 1972-73. The Nixon administration at that time had implemented a wage price freeze which, coupled with high grain prices, caused some chicken producers to destroy chicks rather than feed them to slaughter weight. The model shows a large increase in the coefficient and then a large drop in this period. The model can adjust to changes in data as reflected in the drop in the level of the coefficient in 1978. At this point, there was a change in the procedure for reporting the retail chicken price data. The expenditure coefficient did not show the same patterns as did coefficients for beef and pork. Reflecting the fact that poultry was the least expensive meat and may be

Figure 1.

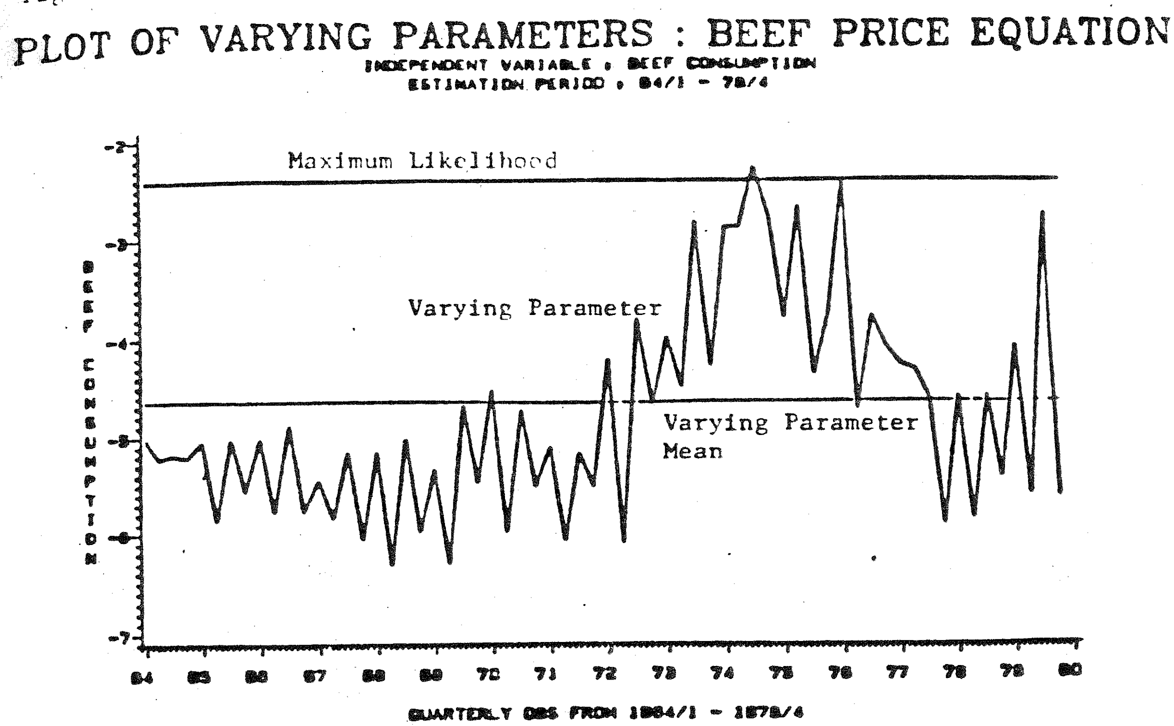


Figure 2.

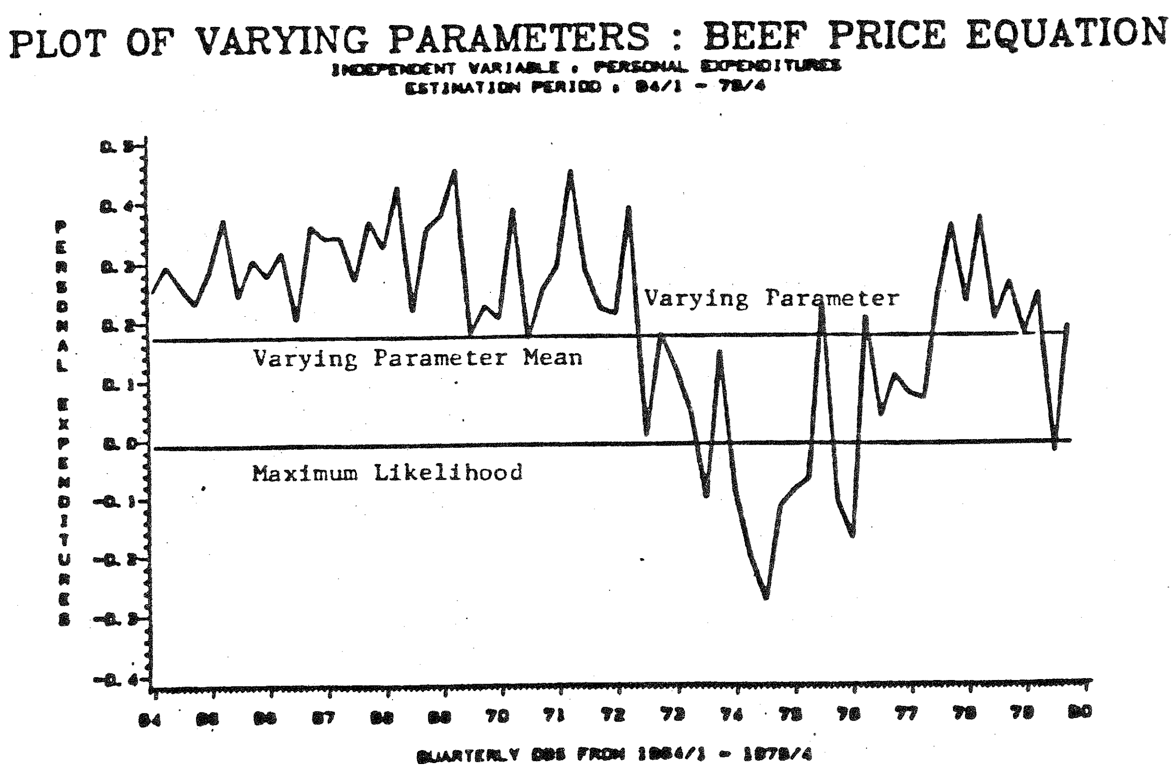


Figure 3.

PLOT OF VARYING PARAMETERS : PORK PRICE EQUATION
INDEPENDENT VARIABLE : PORK CONSUMPTION
ESTIMATION PERIOD : 64/1 - 79/4

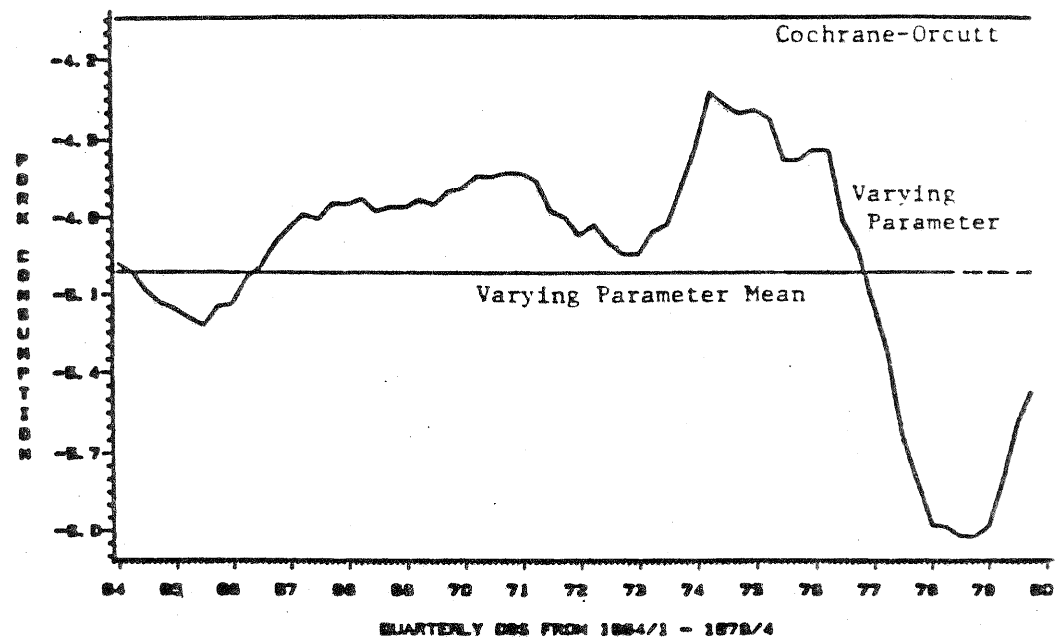


Figure 4.

PLOT OF VARYING PARAMETERS : PORK PRICE EQUATION
INDEPENDENT VARIABLE : PERSONAL EXPENDITURES
ESTIMATION PERIOD : 64/1 - 79/4

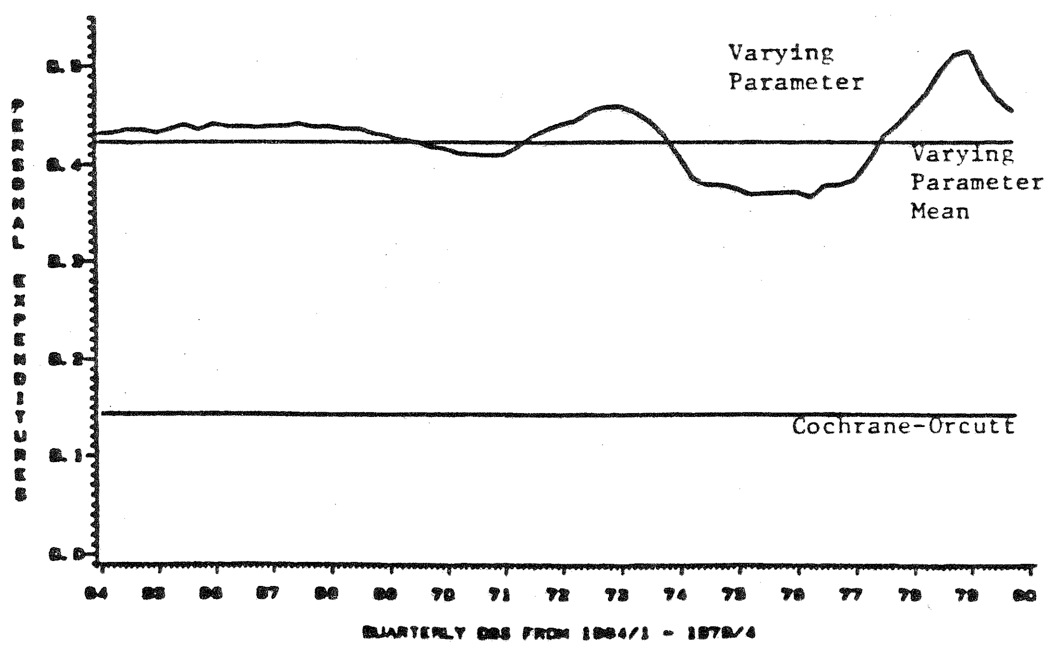


Figure 5.

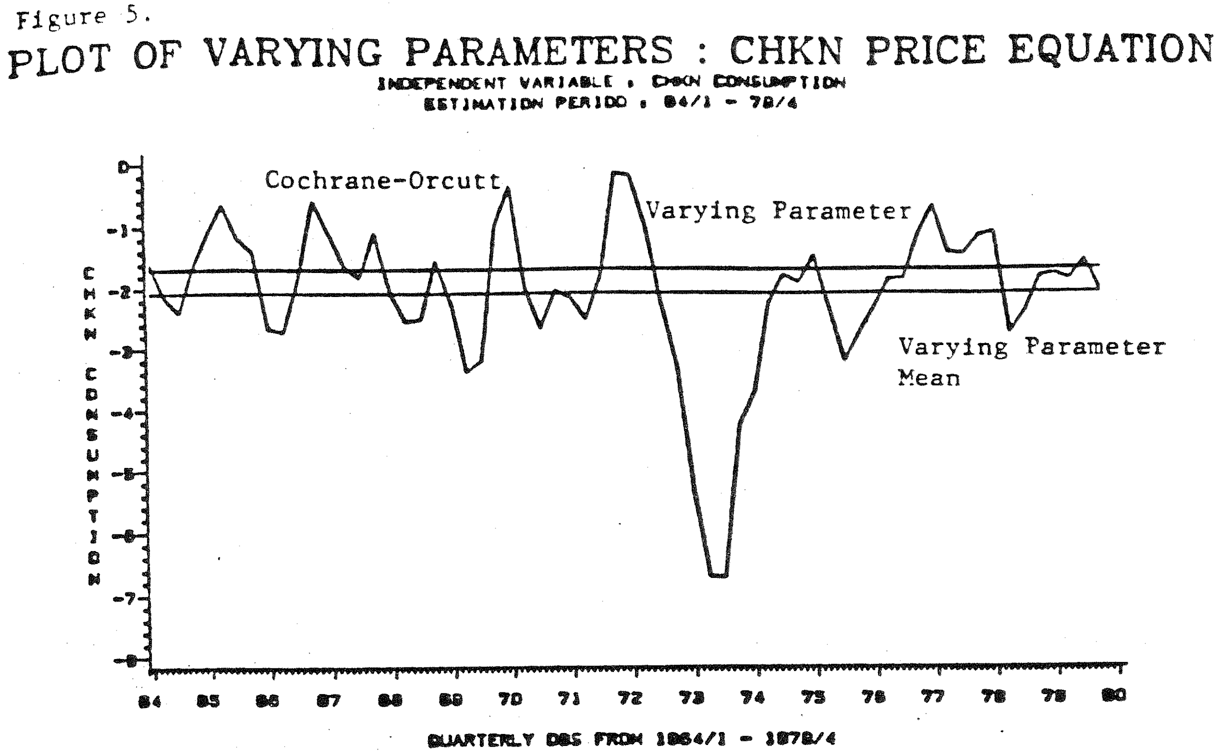
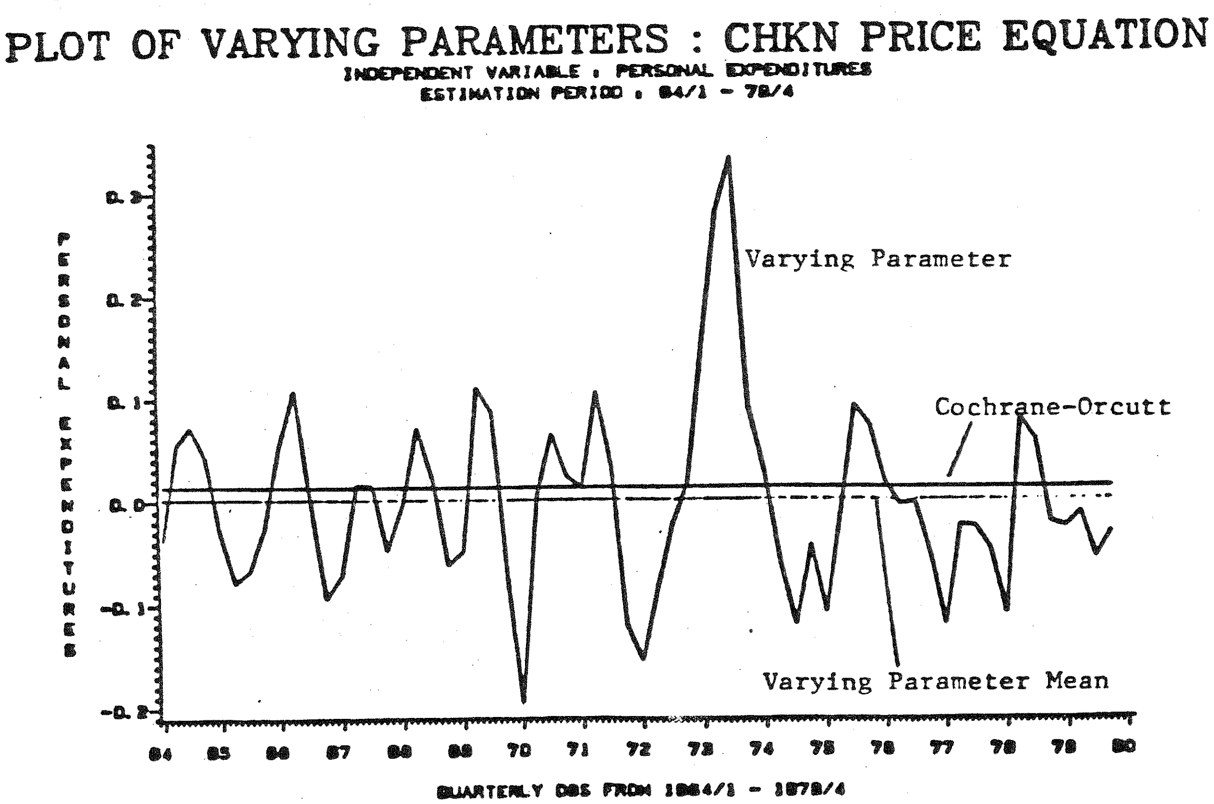


Figure 6.



less sensitive to change in the general economy.

Forecasting Procedure and Results

Multiple step ahead forecasts were obtained for each equation for 1980I to 1983IV (16 periods). Unlike other stochastic coefficient models, the Swamy-Tinsley model does not forecast by periodically updating the coefficients from a sample of dependent and independent variables. Forecasts of the Swamy-Tinsley stochastic coefficient model are generated by deriving an optimal predictor of the coefficients. The coefficients are allowed to vary into the future, allowing for multiple-step ahead forecasts. For each equation, the authors obtained the estimates of $\underline{\beta}$, ϕ, and Δ_u. Swamy and Tinsley (1980) derive the minimum mean square error linear predictor (MMSEL) of y_{T+s}. The optimal prediction formula is stated in equation (10).

$$\hat{y}_{T+s} = \underline{x}_{T+s}'\,\bar{\underline{\beta}} + \underline{x}_{T+s}'\,\phi^s\,\Sigma_{\beta T}'\,D_x'\,\Sigma_y\,(\underline{y} - \underline{x}'\bar{\underline{\beta}}) \tag{10}$$

where x_{T+s} is the value of the 1xK vector of input variables x' in T+s, $\Sigma_{\beta T} = [\phi^{T-1}\Gamma, \phi^{T-2}\Gamma, \ldots, \Gamma]'$ is the matrix made up of the last K columns of the unknown covariance matrix of $[\underline{\beta}_1', \ldots, \underline{\beta}_T']$, $\phi\Gamma\phi + \Delta_u$ represents the unknown covariance matrix of $\underline{\beta}_t$, D_x is the block diagonal matrix with the rows of x as its main diagonal blocks, Σ_y is the unknown covariance matrix of $\underline{y}$, $\underline{y}$ the Tx1 vector of sample observations on y, x is the TxK matrix of sample observations on the input variables in (9).

Because the parameters $\underline{\beta}$, ϕ, and Δ_u are unknown, sample estimates of these parameters are used to evaluate (10). As a result of this substitution, the predictor is no longer MMSEL. Swamy, Kennickell and von zur Muehlen (1986) suggest an improvement is possible by reformulation (10) as

$$\tilde{y}_{T+s} = \underline{x}_{T+s}'\,\hat{\bar{\underline{\beta}}} + c\underline{x}_{T+s}'\,\hat{\phi}^s\,\hat{\Sigma}_{\beta T}'\,D_x'\,\hat{\Sigma}_y\,(\underline{y} - \underline{x}'\hat{\bar{\underline{\beta}}}) \tag{11}$$

where the hats represent sample estimates and c is chosen by the researcher.

Forecast errors were generated for each quarter by subtracting the actual value from the forecasted value from each equation. Table 3 presents the comparison between the stochastic coefficient model and the fixed coefficient model. In evaluating the performance of the stochastic model as compared to the fixed coefficient models, we chose several loss criteria to evaluate the forecasts, root mean square error (RMSE), mean absolute error (MAE), and turning point errors (TP). The stochastic coefficient model performs better than the fixed coefficient model in both the beef and chicken price equations in all of the evaluation criteria. In the pork price equation, the Cochrane-Orcutt and maximum likelihood estimators perform better.

This result does not necessarily condemn the stochastic coefficient model. If you examine the coefficient of variation for the parameters in the equations, you can see that the pork equation coefficients have relatively little variation. This result would agree with the findings of Chavas (1983) and Nyankori and Miller (1982) who found evidence of structural change in both chicken and beef, but not in pork. If there is little evidence of parameter change in the pork equation, it is not surprising that the maximum likelihood estimator performed better than the Swamy-Tinsley model.

Summary and Conclusions

The Swamy-Tinsley stochastic coefficient model proved to be a superior forecasting tool for the beef and chicken price equations. These equations showed considerable variation in the parameters compared with the pork equations. Both the beef and chicken industries substantially changed over the period of estimation. These equations showed change over time; however, the cause of this change cannot be isolated between many alternatives. The important result of this research is that the Swamy-Tinsley model can be used as a tool to aid forecasters.

A secondary result of this research is the examination of patterns in the coefficients that are loosely associated with historical events that may have caused this variation. The own-quantity coefficients in each of these equations would suggest the possibility of fairly stable consumer preferences for meats. Variation in the cross-commodity and intercept coefficients imply that the information contained in the quarterly model is not complete and other factors do influence consumer behavior. The real expenditures coefficients for beef and pork appear to alter their values in line with the business cycle. Macroeconomic conditions appear to have had some effect or are at least correlated with other factors which affect red meat prices.

References

Chavas, Jean-Paul (1983). "Structural Change in the Demand for Meat," American Journal of Agricultural Economics, Vol. 65, pp. 148-153.

Conway, R.K., C.B. Hallahan, R.P. Stillman, P.T. Prentice (1987). Forecasting Livestock Prices: Fixed and Stochastic Coefficient Estimation TB-1725, Econ. Res. Serv., U.S. Dept. of Agr.

Dixon, Bruce L., and Larry L. Martin (1982). "Forecasting U.S. Pork Production Using a Random Coefficient Model," American Journal of Agricultural Economics, Vol. 64, pp 530-538.

Duffy, W.J. (1969). Parameter Variation in a Quarterly Model of the Post-War U.S. Economy, unpublished Ph.D. thesis, University of Pittsburg.

Haidacher, Richard C. (1983). "Structural Change in the Demand for Food Commodities." Southern Journal of Agricultural Economics, 15, pp. 31-37.

Lucas, Robert E. and Thomas J. Sargent (1978). "After Keynesian Macroeconomics," After the Philips Curve: Persistence of High Inflation and High Unemployment, Federal Reserve Bank of Boston Conference Series No. 19, Boston.

Lucas, R.E., Jr. (1975). "Econometric Policy Evaluation: A Critique." The Philips Curve and Labor Markets, Carnegie-Rochester Conference Series, Vol. I, (eds.) Karl Brunner and Allen Meltzer. Supp. to Journal of Monetary Economics.

Nyankori, J.C., and G.H. Miller (1982). "Some Evidence and Implications of Structural Change in Retail Demand for Meats," Southern Journal of Agricultural Economics, Vol. 14, pp. 65-70.

Rausser, Gordon C., Yair Mundlak, and S.R. Johnson (1983). "Structural Change, Updating and Forecasting," in New Directions in Econometric Modeling and Forecasting in U.S. Agriculture, (ed.) Gordon C. Rausser, North-Holland Publishing Co., Amsterdam.

Stillman, Richard P. (1985). A Quarterly Model of the Livestock Industry, TB-1711, Econ. Res. Serv., U.S. Dept. of Agr.

Swamy, P.A.V.B., and J.R. Barth (1985). Random Coefficients, Special Studies Paper, Federal Reserve Board, Washington D.C.

Swamy, P.A.V.B., J.R. Barth, and P.A. Tinsley (1982). "The Rational Expectations Approach to Economic Modelling," Journal of Economic Dynamics and Control, 4, pp.125-147.

Swamy, P.A.V.B., A.B. Kennickell and P. von zur Muehlen (1986). "Forecasting Money Demand with Econometic Models," Forthcoming Special Studies Paper. Federal Reserve Board, Washington, D.C.

Swamy, P.A.V.B., and P.A. Tinsley (1980). "Linear Prediction and Estimation Methods for Regression Models with Stationary Stochastic Coefficients," Journal of Econometrics, 12, pp. 103-142.

Theil, Henri (1971). Principles of Econometrics, John Wiley and Sons, Inc., New York.

Tinsley, Peter A., P.A.V.B. Swamy, and Bonnie B. Garrett (1981). "The Anatomy of Uncertainty in a Money Market Model," Federal Reserve Board Working Paper, Federal Reserve Board, Washington, D.C.

Zellner, Arnold (1962). "An Efficient Method of Estimating Seemingly Unrelated Regression and Tests for Aggregation Bias," Journal of the American Statistical Association, 62, 348-368.