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by

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## FORWARD CONTRACTING IN FACTOR MARKETS

John J. Haydu, Robert J. Myers and Stanley R. Thompson\*

### I. Introduction

Most research on alternative responses to agricultural production risk has focused on the output side of the production process, particularly when considering forward contracts and futures contracts (e.g., McKinnon, 1967; Chavas and Pope, 1982; Anderson and Danthine, 1983). There has been some attention given to the impact of risk and uncertainty on factors of production. For instance, Batra and Ullah (1974) showed how introducing output price risk into a certainty model altered output levels but left relative input quantities unchanged. Robison and Barry (1987) evaluate input demand under four conditions: (a) output price risk, (b) input price risk, (c) quality of input risk, and (d) production function risk. They also introduce "flexibility" [as have Hartman (1975) and Holthausen (1976)] by allowing the firm to select one input after the uncertainty is revealed. This approach allows the decision maker to respond to new or changing conditions. In each of these cases, however, the research has assumed spot markets only with no forward contracting of inputs. But many farmers forward purchase some of their inputs in order to manage price risk and ensure reliable supplies and quality. This facilitates planning and allows farmers to diversify their input purchases over time.

In this paper the forward contracting of inputs is incorporated into a three-period mean-variance model of farmer decision making. Explicit in the model is the tradeoff between the quantity of input to be purchased in advance (prior to planting) at the forward price, and the remaining portion to be purchased subsequently on the spot market. Empirical results are obtained using fertilizer and corn price data.

In the first section of the paper, a descriptive analysis of the forward contracting problem facing agricultural producers is presented. Of major concern is the decision environment facing the contract participants, the economic incentives that underly the agreement, and the possible tradeoffs involved when operating in an uncertain environment. The second section presents the model and derives decision rules for optimal use of forward contracting of inputs. Finally, an empirical application of the model is illustrated by estimating an "optimal" forward contract ratio. This ratio establishes the quantity of input that should be forward purchased prior to planting.

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## II. Exchange In Forward Contracts For Inputs

An inquiry into input contracting indicates that this form of exchange is usually initiated by the manufacturer. The manufacturer's primary incentive to forward sell is to improve the firm's planning capability. Substantial uncertainty surrounds some of these input markets, particularly for future prices and demand. By establishing a portion of future demand, the manufacturer is able to plan for a minimum production level and cover variable costs. It may be analagous to purchasing an insurance policy to cover operating expenses. The insurance premium is the lower, fixed price necessary to obtain a forward purchase from the farmer. On the other hand, the farmer's incentive to forward purchase may be attributable to the following inducements: (a) a certain price; (b) a certain supply, and (c) a likelihood of cost savings.

The forward contract price is largely a function of manufacturing costs, current input prices and expected input prices. Although contracts often vary across firms, typically they are of short duration (less than one year), have a fixed price, and may require up to a 100 percent advance payment. This financial commitment by the purchaser is compensated by a price discount below the current spot price. A five to ten percent discount is common. Once the contract is consummated, a future increase in the market price implies an ex-post consumer (farmer) gain whereas a price decline implies an ex-post producer (manufacturer) gain. Given the influence of seasonality on input prices, the price usually rises.

Prices, however, do not always dominate farmers' purchasing decisions. Supply assurance is also a highly important consideration for essential inputs (Eversull, 1983). It consists of four individual, yet interrelated dimensions of coordination - quantity, time, form (quality) and place (location). Clearly perfect coordination is not possible, or even desirable, since it could very well be infinitely costly. Nevertheless, these four components of coordination must enter into the decision maker's production equation, at least within some acceptable parameters. These parameters may differ considerably by the type of input, its function, and the biological and agro-climatic constraints imposed on the production process. For instance, to achieve an effective "kill rate" for the pink bollworm, insecticides must be applied at precisely the right developmental stage of the larvae. This may constitute a "window of opportunity" of only a few days, or even less. Under these circumstances, supply certainty takes on a more imposing and urgent function.

To investigate some of these issues further, consider a simple two-period decision environment consisting of a preplant period ( $t=0$ ), a planting period ( $t=1$ ) and a subsequent harvest period ( $t=2$ ) in which all buying decisions are restricted to the first two periods. Assuming the farmer is limited to a single forward purchase, the following choices are available at  $t=0$ : (1) forward contract total input requirements, (2) forward contract no inputs, or (3) forward contract a portion of total input needs.

Each choice is influenced by current ( $t=0$ ) input prices, expected spot price of the input in period 1, and the output price in period 2. Initially, the decision maker must decide whether or not to forward contract in the first

period. Failure to contract indicates the agent will postpone input purchases and, in effect, speculates that prices will turn favorably by planting time.

An opposite approach is to forward purchase all input needs during the preplanting period ( $t=0$ ). This strategy eliminates price uncertainty, although there still remains some likelihood that prices will fall in the next period, thereby making the farmer regret his or her decision, ex-post.

A final option is to spread risk over time by purchasing some input in  $t=0$  and the remainder on the spot market. In this situation, a proportion of total input price is certain, while the balance remains uncertain.

### III. The Model

The following model is based on the decision problem discussed above. Assume there is some predetermined level of output,  $y$ , that the decision maker intends to produce in  $t=2$ . The production technology is defined by a Leontief, fixed proportions production function:

$$y = \min (ax, bz) \quad (1)$$

where  $x$  is the total quantity of the input of interest used in the production process,  $z$  is a vector of other inputs, and  $a$  and  $b$  are input-output coefficients. Input  $x$  can be purchased in two time periods, as a forward contract in  $t=0$  or in the spot market one period later ( $t=1$ ). Input levels required to produce  $y$  with this technology are

$$x = \frac{y}{a} \quad \text{and} \quad z = \frac{y}{b} \quad (2)$$

Furthermore, since  $y$  is fixed (and by definition so are  $x$  and  $z$ ), the farmer's decision problem is reduced to determining the amount of input  $x$  to purchase at  $t=0$  (i.e.  $x_0$ ) and the amount to purchase at  $t=1$  (i.e.  $x_1 = x - x_0$ ). Therefore, the demand for  $x_1$ , which is conditional on  $x_0$  and  $y$ , can be expressed as

$$x_1 = \frac{y}{a} - x_0 \quad (3)$$

and  $z$  remains as defined in (2).

Profit in period 2 can be written

$$\pi = p y - w_1 x_1 - r z - w_0 x_0 \quad (4)$$

where  $r$  is a vector of certain prices for other inputs, ( $z$ ),  $w_1$  is the spot price of  $x$  at  $t=1$  and  $w_0$  is the forward contract price of  $x$  at  $t=0$ . From (2) and (3) we know the period 1 demand for inputs  $x_1$  and  $z$ , and by substituting into (4) we obtain

$$\pi = p y - w_1 \left[ \frac{y}{a} - x_0 \right] - r_1 \frac{y}{b} - w_0 x_0 \quad (5)$$

During the preplanting period ( $t=0$ ) both  $p$  and  $w_1$  are random elements in the decision process, while both  $r$  and  $y$  are given. Thus the problem in the

preplanting period is to choose  $x_0$  so as to maximize a linear function of the mean and variance of profit, conditional on information available at  $t=0$ :

$$\max E(\pi|\Omega_0) - \frac{\lambda}{2} (\pi) - \frac{\lambda}{2} \text{Var}(\pi|\Omega_0) \quad (6)$$

where  $\Omega_0$  is information available at  $t=0$  and  $\lambda$  is a measure of the agent's risk aversion. The mean and variance of profit are:

$$E(\pi|\Omega_0) = E(p|\Omega_0)y - E(w_1|\Omega_0)\left[\frac{y}{a} - x_0\right] - r\left[\frac{y}{b}\right] - w_0x_0; \text{ and} \quad (7a)$$

$$\begin{aligned} \text{Var}(\pi|\Omega_0) &= y^2 \text{Var}(p|\Omega_0) + \left[\frac{y}{a} - x_0\right]^2 \text{Var}(w_1|\Omega_0) \\ &\quad - 2y\left[\frac{y}{a} - x_0\right]\text{Cov}_0(w_1, p|\Omega_0) \end{aligned} \quad (7b)$$

The first order condition for this problem is

$$E(w_1|\Omega_0) - w_0 - \lambda[y\sigma_{pw}^2 - \left(\frac{y}{a} - x_0\right)\sigma_w^2] = 0 \quad (8)$$

where  $\sigma_{pw} = \text{Cov}(w_1, p|\Omega_0)$  and  $\sigma_w^2 = \text{Var}(w_1|\Omega_0)$

Solving for  $x_0$  we obtain the following input demand function for  $x_0$ .

$$x_0 = \frac{y}{a} + \frac{E(w_1|\Omega_0) - w_0}{\lambda\sigma_w^2} - \frac{y\sigma_{pw}}{\sigma_w^2} \quad (9)$$

From an empirical perspective, it would be of value to arrive at a decision rule for forward purchasing  $x$ . This can be accomplished by implementing a "forward contract ratio", where  $x_0$  is some proportion of total  $x$ . Dividing (9) by  $x$  and recalling that  $x = \frac{y}{a}$  we obtain our optimal contract ratio,

$$\frac{x_0}{x} = 1 + \frac{[E(w_1|\Omega_0) - w_0]a}{\lambda\sigma_w^2 y} - \frac{a\sigma_{pw}}{\sigma_w^2} \quad (10)$$

Notice from equation (10) that if the forward contract price is equal to the expected future spot price then the middle term drops out and we are left with the simple rule,

$$\frac{x_0}{x} = 1 - \frac{a\sigma_{pw}}{\sigma_w^2} \quad (11)$$

It is easily verified that (11) is also the forward contract ratio that minimizes the variance of profits (ignoring effects on expected profits). Given this simplified rule (11), increases in the covariance between output

and input prices lead to reductions in risk and, hence, reductions in the quantity forward contracted. Similarly, increases in the variance of input prices leads to an increase in the amount forward contracted.<sup>1</sup>

Returning to (10), now assume that the forward contract price is at a discount to the expected future spot price;  $E(w_1|\Omega_0) > w_0$ . Then a risk neutral farmer would want to forward contract large amounts in  $t=0$  and resell them on the spot market at the (expected) higher price at  $t=1$ . However, this is a risky strategy, implying that the more risk averse a farmer is, the less forward contracting the farmer would be willing to undertake (i.e.  $\partial x_0/\partial \lambda < 0$ ).

#### IV. Empirical Results

In order to implement the optimal forward contract ratio, we need estimates of the variables in equation 10. Suppose initially that,

$$E(w_1|\Omega_0) = w_0, \quad (12)$$

Then we get the simple decision rule (11), as before. Equation (11) is also appropriate if the individual is infinitely risk averse or if one pursues a minimum variance objective.

To estimate the simple decision rule represented by equation (11) we need the input-output coefficient, "a," and the conditional covariance between input and output prices along with the conditional variance of the input price. We estimated these variables for the case of fertilizer used in corn production on a representative corn belt farm. A plot of the estimated correlogram of the spot fertilizer price data indicated the possibility of a nonstationary data series. Dickey and Fuller suggest a procedure to test the null hypothesis that the price level has a unit root. The result of this test failed to reject the null hypothesis and the conclusion of nonstationarity was reached. Since the null hypothesis of unit roots could not be rejected, a first difference specification was estimated.

Assuming that the formation of both input and output price expectations is based on the information set at preplanting time (period 0), then the following estimation models are specified,

$$\Delta w_t^S = \beta_{10} + \beta_{11} \Delta w_0^S + \beta_{12} \Delta p_0 + \epsilon_{1t}, \text{ and} \quad (13a)$$

$$\Delta p_t^S = \beta_{20} + \beta_{21} \Delta w_0^S + \beta_{22} \Delta p_0 + \epsilon_{2t}, \quad (13b)$$

where,  $\Delta w_t^S = w_t^S - w_{t-1}^S$  and  $\Delta p_t = p_t - p_{t-1}$ .

The null hypothesis of all the parameters in both (13a) and (13b) equal to zero was rejected at the 5 percent error level (Table 1).

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<sup>1</sup>These results assume  $\sigma_{pw} > 0$ .

To implement the optimal forward contract ratio (equation 11) two estimates are needed. First, is an estimate of the ratio  $\sigma_{pw} / \sigma_w^2$ . The residuals from (13a) and (13b) were used to obtain estimates of these conditional variances and covariances. Using the forecast errors from equations (13a) and (13b),  $\sigma_{pw} = 0.0597$  and  $\sigma_w^2 = 63.987$ ; hence,  $\sigma_{pw} / \sigma_w^2 = 0.00093$ . Myers and Thompson have shown that this ratio can be equivalently obtained as the estimate of  $\delta_1$  in the following regression,

$$\Delta p_t = \delta_0 + \delta_1 \Delta w_t^S + \delta_2 \Delta w_{t-1}^S + \delta_3 \Delta p_{t-1} + u_t. \quad (14)$$

Second, the remaining unknown component in equation (12) is an estimate of the production function parameter "a". Given our problem, the parameter "a" in equation (1) is defined as an input-output coefficient: bushels of corn per ton of nitrogen fertilizer. Based on a previous study of the effect of nitrogen on corn yield by (Vitosh, et.al) the application of 115 pounds of nitrogen per acre can be expected to yield 100 bushels of corn. This relationship implies a value of the parameter "a" of 1740. If "a" were as large as 1740 and a covariance-variance ratio of 0.00093, the optimal forward contract ratio would be negative. Since we have not bounded our solution to lie between zero and one, negative values are possible. However, this should be interpreted as  $x_0=0$  since we want to rule out  $x_0 < 0$ .

Initially, we believed that the decision maker's desire to reduce risk would be an important element in the decision to forward contract inputs. However, these results show that for fertilizer used by corn producers, even highly risk averse (variance minimizing) farmers would have little incentive to forward contract. Given values of  $\sigma_w^2$ ,  $\sigma_{pw}$  and "a" estimated here, there appears to be very little the farmer can do to reduce the variance of profit. It is apparent that the risk reduction incentive to forward contract is small, as evidenced by the magnitude of the  $\sigma_{pw} / \sigma_w^2$  ratio. Given that forward contracting of fertilizer occurs, one might conclude that the impetus is largely due to the presence of discount premiums associated with these forward purchases.

In this simple technology model, the influence of the input-output coefficient for fertilizer was substantial. It is important to recognize that input-output coefficients will vary with the input employed, and hence so too will their impact on the contract ratio. For example, one could anticipate some agricultural chemicals as having a very specific role in risk management, possibly overriding price discounts as the incentive to forward contract.

Returning to the optimal contract ratio, if we assume that the expected spot price exceeds the forward contract price (i.e.  $E(w_1 | \Omega_0) > w_0$ ), then equation (10) holds. By using the limited forward contract data, discount premiums were determined for purposes of obtaining the range of discounts. Using this information, hypothetical discounts ( $\alpha$ ) and levels of risk aversion ( $\lambda$ ) were chosen to illustrate forward contract ratios' (Table 2). Results support earlier conclusions that the more risk averse a farmer is, the

less forward contracting he will undertake. This may be in part due to the possibility of an unfavorable price change in the next period. Also, the contract ratios increase with larger alpha values, indicating that a major incentive to forward purchase inputs is due to the presence of discount premiums.

## V. Summary

Forward contracting of inputs by farmers is a growing activity. This practice allows the decision maker to better manage price risk and ensure reliable supplies and quality. Little attention, however, has been directed toward this form of risk management.

In this paper we discuss the decision choices facing farmers who forward purchase inputs. A mean-variance model is used to derive an optimal rule for forward contracting. By interpreting this rule, we found that, in the absence of discount premiums and, if the objective is to minimize the variance of income, a farmer has little incentive to forward contract. Finally, using the limited forward contract data available, we illustrate how forward contract ratios' change due to changes in discount premiums and risk aversion levels. This empirical application supports earlier conclusions regarding the importance of discounting to farmers when they are deciding whether or not to forward purchase their inputs.



TABLE 1  
OLS Estimates of Fertilizer and Corn Price Expectations Models

Dependent Variable	Independent Variables			
	Constant	$\Delta w_{t-1}^S$	$\Delta p_{t-1}$	F-Statistic (3,72)
$\Delta w_t^S$	-0.340 (0.965)	0.469 (0.107)	-8.345 (5.887)	9.85
$\Delta p_t$	-0.003 (0.019)	0.005 (0.002)	0.017 (0.114)	4.52

Where:  $\Delta w_t^S = w_t^S - w_{t-1}^S$ ,  $\Delta p_t = p_t - p_{t-1}$  and  
the numbers in parentheses are standard errors.

TABLE 2  
Optimal Fertilizer Forward Contract Ratios Under Alternative Degrees  
of Risk Aversion ( $\lambda$ ) and Discounts ( $\alpha$ )

Risk Parameter ( $\lambda$ )	Discount ( $\alpha$ )			
	0.96	0.94	0.92	0.90
0.025	0.00 <sup>a</sup>	0.00 <sup>a</sup>	0.02	0.18
0.01	0.19	0.58	0.98	1.00 <sup>b</sup>
0.0075	0.45	0.98	1.00 <sup>b</sup>	1.00 <sup>b</sup>

Where: a = negative values constrained to be zero, and

b = values greater than 1.0

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