Dynamic Elasticities and Flexibilities in a Quarterly Model of the U.S. Pork Sector

by

Karl D. Skold and Matthew T. Holt

Dynamic Elasticities and Flexibilities in a Quarterly Model of the U.S. Pork Sector

Karl D. Skold and Matthew T. Holt*

I. Introduction

Over the years there has been considerable discussion about the appropriate interpretation of price and quantity relationships in simultaneous systems of supply and demand equations. This discourse has focused on two related issues. The first being the relationship between flexibilities and elasticities in a multi-commodity context (Foote; Meinken, Rojko, and King; Harlow; Waugh; Houck) and the second being the appropriate derivation and interpretation of elasticities between endogenous variables in a simultaneous system (Meinken, Rojko, and King; Buse; Colman and Miah; Chavas, Hassan and Johnson). As a result of this discussion, it is now widely accepted that partial elasticities and flexibilities, as typically derived for single equations, are not valid measures of net effects in a simultaneous setting. Instead, it is necessary to evaluate what are referred to as "total" elasticities and flexibilities if appropriate inferences are to be made (Buse).[1]

Until recently, methods for deriving total elasticities in a dynamic simultaneous equations framework were not available. Consequently, while economists have continued to evaluate structural econometric models by deriving reduced forms and by examining the resulting mean-paths of endogenous variables (Freebairn and Rausser; Arzac and Wilkinson), there has been no known attempt to obtain the total response relationships implied by an estimated linear econometric model. Although procedures for obtaining total response measures were not available for some time, Chavas, Hassan, and Johnson have recently illustrated that partial reduced forms obtained for simultaneous dynamic systems can be used to derive analytical expressions for total price and quantity effects.

The purpose of this paper is to illustrate the potential for deriving total price and quantity effects for a dynamic simultaneous system of supply and demand equations. In this paper we build upon the methodological framework for deriving total price and quantity relationships considered by Chavas, Hassan, and Johnson. In particular, we show how their analytical results can be extended, using numerical simulations, beyond their restrictive two-variable lag model. The result is that total elasticities and flexibilities can be obtained even when the lag structure on endogenous variables is not arbitrarily constrained. The application is with a quarterly model of the U.S. pork sector which is similar in design to the models reported by Harlow, Arzac and Wilkinson, and others. The hog sector seems especially suited for examining the

---

The authors are Pre Doctoral Research Associate and Post Doctoral Research Associate, Center for Agricultural and Rural Development, Deptartment of Economics, Iowa State University, Ames, Iowa.

implications of total price and quantity effects since production occurs sequentially and since there are well defined biological time lags governing supply response.

In the next section, the concepts of partial and total price and quantity effects are reviewed using a standard market model. These results are then extended to a dynamic linear system. The third section reports the estimates of a structural model of the U.S. pork sector. In the fourth section, the results from previous sections are used to derive dynamic elasticities for selected exogenous variables, and total response elasticities and flexibilities for key endogenous variables. Important conclusions of the study and several suggestions for future research are discussed.

II. Partial and Total Effects in a Simultaneous System

General Results

A standard market model consists of equations explaining the demand for and supply of a particular good or product. If equilibrium is assumed, then quantity transacted and price are determined simultaneously. Using Chavas, Hassan, and Johnson's notation, a hypothetical market model can be expressed as:

$$Y_{1s} = f_s(Y_2, \underline{X}), \tag{1}$$

$$Y_{1d} = f_d(Y_2, \underline{X}), \tag{2}$$

$$Y_{1s} = Y_{1d}, \tag{3}$$

where Y_{1s} is quantity supplied, Y_{1d} is quantity demanded, Y_2 is price, and $\underline{X}$ is a k-dimensional vector of exogenous variables conditioning supply and demand. The relationships between quantities and price are frequently summarized using the elasticity concept. That is,

$$\epsilon_s = (\delta Y_{1s}/\delta Y_2)(Y_2/Y_{1s}), \tag{4}$$

$$\epsilon_d = (\delta Y_{1d}/\delta Y_2)(Y_2/Y_{1d}), \tag{5}$$

where ϵ_s and ϵ_d denote the elasticities of supply and demand, respectively.

A more typical situation encountered in applied work is to have a model where the values of more than two variables are determined endogenously. A generalized representation of the market model in (1) - (3) would then include an equaiton for each endogenous variable. In general each endogenous variable would be conditioned on the values of all remaining endogenous variables. In this case, the supply and demand equations in (1) and (2) become

$$Y_{1s} = f_s(Y_2, \ldots, Y_G, \underline{X}), \tag{6}$$

$$Y_{1d} = f_d(Y_2, \ldots, Y_G, \underline{X}), \tag{7}$$

where $Y_2, \ldots, Y_G$ represent the remaining endogenous variables. The partial elasticities in (4) and (5) are clearly not appropriate in the present case since changes in price will affect the values of the remaining endogenous variables. Consequently, there would be secondary feedbacks resulting from a price change not reflected in the partial derivatives $(\delta Y_{1s}/\delta Y_2)$ and $(\delta Y_{1d}/\delta Y_2)$. To capture the total effect of a price change, the total derivatives of equations (6) and (7) must be considered. The supply and demand elasticities are then

$$\epsilon_s = [\delta Y_{1s}/\delta Y_2 + \sum_{i=3}^{G} (\delta Y_{1s}/\delta Y_i)(\delta Y_i/\delta Y_2)]\ (Y_2/Y_{1s}), \tag{8}$$

$$\epsilon_d = [\delta Y_{1d}/\delta Y_2 + \sum_{i=3}^{G} (\delta Y_{1d}/\delta Y_i)(\delta Y_i/\delta Y_2)](Y_2/Y_{1d}). \tag{9}$$

Using partial elasticities to convey essential information pertaining to parameter values in a simultaneous system is not appropriate since the secondary feedback effects represented by the summation terms in (8) and (9) are excluded (Buse; Chavas, Hassan, and Johnson).

Total Response in a Static System

The purpose of the present exercise is to derive expressions for the multipliers and elasticities between endogenous variables.[2] That is, we wish to obtain $\epsilon_{ij} = (\delta Y_{it}/\delta Y_{jt})(Y_{jo}/Y_{io})$ implied by the simultaneous system. Assuming that the structural model is linear in both parameters and variables, the simultaneous supply-demand system can be written as

$$\underline{Y}_t\beta + \underline{X}_t\Gamma + \underline{E}_t = \underline{0}, \quad t = 1,\ldots,T. \tag{10}$$

In equation (10), $\underline{Y}_t$ is a G-dimensional vector of observations on endogenous variables at time t, $\underline{X}_t$ is a K-dimensional vector of exogenous variables at time t, β is a (GxG) parameter matrix associated with endogenous variables, Γ is a (KxG) parameter matrix associated with predetermined variables, and $\underline{E}_t$ is a G-dimensional vector of additive disturbance terms with mean zero and variance-covariance matrix Σ. The equations in (10) can be ordered so that the i'th endogenous variable Y_{it} is determined by the i'th equation. The implication is that the diagonal elements in β will be unity. It is also assumed that the vector $\underline{X}_t$ does not contain lagged endogenous variables.

To obtain expressions for the total effects, the system in (10) is partitioned into two subsystems. The first contains the equations for the endogenous variables of interest (Y_{it}, Y_{jt}) while the second contains the equations for the remaining G-2 endogenous variables. Without loss of generality, assume that i=1 and j=2 and that the equations in (10) are arranged so that Y_1 is first and Y_2 is second in the ordering. The system in (10) can then be partitioned as

$$[Y_1,\ Y_2,\ \underline{Y}.] \begin{bmatrix} \beta_{11} & \beta_{12} & \underline{\beta}_{1.} \\ \beta_{21} & \beta_{22} & \underline{\beta}_{2.} \\ \underline{\beta}_{.1} & \underline{\beta}_{.2} & \beta_{..} \end{bmatrix} + \underline{X}[\underline{\Gamma}_1,\ \underline{\Gamma}_2, \Gamma.] + [E_1, E_2, \underline{E}_.] = \underline{0} \tag{11}$$

where the t subscript has been dropped for notational convenience and the dimensions of the partitions for the β and Γ matrices are implied by the partition of $\underline{Y}$. Using (11), the structural model can be compressed into a lower-dimensional system where only Y_1 and Y_2 are determined endogenously.

The total effects between Y_1 and Y_2 can then be derived from this lower-order system.

Assuming that $\beta_{..}$ is non-singular, the partial reduced form for the remaining G-2 endogenous variables (11) can be obtained. The reduced form for the second subsystem is then

$$\underline{Y}_{.} = -Y_1\underline{\beta}_{1.}\beta_{..}^{-1} - Y_2\underline{\beta}_{2.}\beta_{..}^{-1} - \underline{X}\Gamma_{.}\beta_{..}^{-1} - \underline{E}_{.}\beta_{..}^{-1} \qquad (12)$$

The partial reduced form in (12) expresses the G-2 endogenous variables in the second subsystem as a function of the exogenous variables $\underline{X}$ and the endogenous variables, Y_1 and Y_2, from the first subsystem. Consequently the system in (12) shows how the endogenous variables in the second subsystem will adjust if there is a shock to one of the endogenous variables from the first subsystem, Y_1 or Y_1. The partial reduced form in (12) can be substituted for $\underline{Y}_{.}$ in the first subsystem, thus obtaining expressions for the Y_1 and Y_2 which are functions only of Y_1, Y_2, $\underline{X}$, error terms, and model parameters.

Making these substitutions in the first equation, and collecting and rearranging terms, yields the following reduced form:

$$\begin{aligned} Y_1 = &-Y_2[\beta_{21} - \underline{\beta}_{2.}\beta_{..}^{-1}\underline{\beta}_{.1}][\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} \\ &-\underline{X}[\underline{\Gamma}_1 - \Gamma_{.}\beta_{..}^{-1}\underline{\beta}_{.1}][\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} \\ &-w_1[\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} \end{aligned} \qquad (13)$$

where $w_1 = E_1 - \underline{E}_{.}\beta_{..}^{-1}\underline{\beta}_{.1}$. Equation (13) expresses Y_1 as a function of Y_2, exogenous variables $\underline{X}$, and the system error terms. Also incorporated into (13) are the adjustments that would occur in $\underline{Y}_{.}$ as a result of a change in Y_1 or Y_2. Thus, all essential structural information implied by the system in (10) has been compressed into a single equation relating Y_1 to Y_2. Similar substitutions will obtain an expression relating Y_2 to Y_1.

From (13), the multiplier for Y_1 resulting from a change in Y_2 can be readily inferred,

$$(\delta Y_1/\delta Y_2) = -\frac{\beta_{21} - \underline{\beta}_{2.}\beta_{..}^{-1}\underline{\beta}_{.1}}{\beta_{11} - \underline{\beta}_{1.}\underline{\beta}_{..}^{-1}\underline{\beta}_{.1}} \qquad (14)$$

The multiplier in (14) measures the total effect of a change in Y_2 on Y_1. The corresponding total elasticity is obtained by multiplying $(\delta Y_1/\delta Y_2)$ by the ratio (Y_{20}/Y_{10}) where Y_{20} and Y_{10} are appropriate reference values. If Y_1 corresponds to the demand equation, then the multiplier in (14) measures total demand quantity response as price is exogenously altered. Similar interpretations apply if Y_1 represents the supply equation.

Corresponding to (14), the total effect for a change in Y_2 as Y_1 is exogenously altered is

$$(\delta Y_2/\delta Y_1) = -\frac{\beta_{12} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.2}}{\beta_{22} - \underline{\beta}_{2.}\beta_{..}^{-1}\underline{\beta}_{.2}} \tag{15}$$

and the total flexibility is determined by evaluating $(\delta Y_2/\delta Y_1)(\delta Y_{10}/\delta Y_{20})$. Observe that the inverse of the total flexibility implied by (15) does not equal the inverse of the total elasticity implied by (14), a conclusion consistent with the results obtained by Meinken, Rojko, and King; Houck; Colman and Miah; and others. Furthermore, the total effects identified in (14) and (15) are not, in general, equal to the partial effects corresponding to (4) and (5). To see this, observe that the multiplier in (14) will equal $-\beta_{21}$ if $\underline{\beta}_{.1} = 0$ or if $\underline{\beta}_{1.} = \underline{\beta}_{2.} = \underline{0}$. The implication in the first case is that Y_1 does not depend on the remaining endogenous variables $\underline{Y}$. The second condition implies that Y_1 and Y_2 do not enter as conditioning variables in the second subsystem. In any event, the total effect in (14) will equal the partial effect if and only if one of the two conditions identified above holds.

Total Response in a Dynamic System

The above results do not hold for systems of equations where lagged endogenous variables are included. The typical structural system includes dynamic components which reflect partial adjustments in supply response, lags in expectation formation, or habit persistence in consumption. There also are many instances where lag distributions arise naturally in the model specification. For instance, many agricultural models account for biological growth or production lags directly in the supply equations. Linear models are also estimated frequently with autoregressive error terms. The autoregressive error structure represents an additional source of dynamic interaction. The previous results obtained for static models can then be extended to a dynamic setting.

Consider the case where the system in (10) contains lagged values of endogenous variables. For purposes of illustration only first-order lags are included, although the extension to higher-order lags is straight forward. The dynamic representation of the structural model is

$$\underline{Y}_t\beta + \underline{X}_t\Gamma + \underline{Y}_{t-1}\phi + \underline{E}_t = \underline{0} \tag{16}$$

where ϕ is a GxG parameter matrix corresponding to the first-order lags on endogenous variables. The dynamic system in (16) is more general than the one considered by Chavas, Hassan, and Johnson since they examined only the restrictive case where lags occur in Y_1 and Y_2. As before, the system in (16) can be partitioned into two subsystems: one for the endogenous variables Y_{1t} and Y_{2t} and one for the remaining G-2 endogenous in $\underline{Y}_{.t}$. The ordering is also assumed to be such that Y_{1t} is determined by the first equation and Y_{2t} is determined by the second equation.

Making the partition gives

$$[Y_{1t}, Y_{2t}, \underline{Y}_{.t}] \begin{bmatrix} \beta_{11} & \beta_{12} & \underline{\beta}_{1.} \\ \beta_{21} & \beta_{22} & \underline{\beta}_{2.} \\ \underline{\beta}_{.1} & \underline{\beta}_{.2} & \beta_{..} \end{bmatrix} + \underline{X}_t[\underline{\Gamma}_1, \underline{\Gamma}_2, \Gamma_.] + \underline{Y}_{t-1}[\underline{\phi}_1, \underline{\phi}_2, \phi_.] + [E_{1t}, E_{2t}, \underline{E}_{.t}] = \underline{0} \tag{17}$$

As before, the reduced form for the second subsystem can be obtained from (17) and is given by

$$\underline{Y}_{.t} = -Y_{1t}\underline{\beta}_{1.}\beta_{..}^{-1} - Y_{2t}\underline{\beta}_{2.}\beta_{..}^{-1} - \underline{X}_t\Gamma_{.}\beta_{..}^{-1} - \underline{Y}_{t-1}\phi_{.}\beta_{..}^{-1} - \underline{E}_{.t}\beta_{..}^{-1} \qquad (18)$$

Equation (18) is similar to equation (12) but with first-order lags included for the endogenous variables. Substituting for $\underline{Y}_{.t}$ in the first equation in the partition in (17), collecting terms on Y_1, Y_2, $\underline{X}_t$, and $\underline{Y}_{t-1}$ and making several algebraic simplifications gives the dynamic reduced form for Y_{1t}:

$$\begin{aligned} Y_{1t} = &-Y_{2t}[\beta_{21} - \underline{\beta}_{2.}\beta_{..}^{-1}\underline{\beta}_{.1}][\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} \qquad (19) \\ &-\underline{X}_t[\underline{\Gamma}_1 - \Gamma_{.}\beta_{..}^{-1}\underline{\beta}_{.1}][\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} \\ &-\underline{Y}_{t-1}[\underline{\phi}_1 - \phi_{.}\beta_{..}^{-1}\underline{\beta}_{.1}][\beta_{11} - \underline{\beta}_{1.}\beta_{..}^{-1}\underline{\beta}_{.1}]^{-1} + V_1 \end{aligned}$$

The reduced form in (19) differs from that in (13) in that lagged values of the endogenous variables enter the equation. Chavas, Hassan, and Johnson suggest applying the transformations described by Chow to reparameterize the model. Dynamic flexibilities and elasticities can then be obtained in the usual manner. While this approach would work in the special case where only lags in Y_1 and Y_2 appear in the model, it is inappropriate in the more general case considered here. This is because a change in Y_2 will have a delayed impact on the values of $\underline{Y}_{.}$ as can be observed from equation (18). Delayed changes in $\underline{Y}_{.}$ will, in turn, affect the intermediate run multipliers for Y_1. Thus, even though current values of $\underline{Y}_{.}$ do not enter the reduced form for Y_{1t}, the presence of a general lag structure in the endogenous variables means that simple analytical methods cannot be applied.

One alternative is to simulate equations in (18) and (19) numerically. The equations should be ordered so that Y_{1t}, as determined from (19), is evaluated first. Then, since Y_{2t} is treated as exogenous, values for the remaining endogenous variables in the second subsystem defined by (18) can be inferred. In the second iteration, lagged values corresponding to $\underline{Y}_{.t-1}$ enter equation (19) to determine the new value of Y_{1t}. The reference value for Y_{2t} can be altered and the implied total response multipliers and elasticities for Y_{1t} can be evaluated. The whole process is repeated iteratively for a suitable number of periods. Although this procedure does not result in analytical expressions for the total flexibilities and elasticities, it does provide a convenient way of measuring the total effects in a general dynamic model since all potential dynamic adjustments are accounted for.[3]

III. Model Structure and Specification

A structural model of the U.S. pork sector is used, in conjunction with the concepts discussed in the previous section, to derive total price and quantity relationships. The pork industry is a likely candidate for investigating dynamic adjustments using a total response framework since well defined biological lags exist which effectively limit short-term supply response. The pork sector has also been associated with a fairly predictable cyclical component (Shonkwiler and Spreen). A quarterly time frame is used since many of the sequential production activities occur naturally within this time interval. The specified model is block recursive since current production levels are not determined contemporaneously with price. In addition, price determination occurs at the retail level with the demand equation being estimated in price dependent form since short-term production is essentially fixed. Farm prices are, in turn, determined directly through a separate linkage equation.

The complete model for the U.S. pork sector consists of seven behavioral equations and two identities. The model was estimated with quarterly data for the 1968 to 1985 time period. Except for the estimation procedure used was two-stage least squares (2SLS), the barrow and gilt slaughter equation was estimated by ordinary least squares (OLS) since this equation does not contain any contemporaneous endogenous variables. When necessary, the estimated equaitons were corrected for first- and fourth-order autocorrelation. Structural parameter estimates, along with partial elasticities, and other important measures of fit, are reported in Table 1. Variable definitions and data sources are listed in the appendix.

The present model differs from previous ones in that supply is viewed as a sequential process. Consequently, the specification of the supply equations is based upon the biological sequence of production. As a direct result of the biological structure, economic variables are allowed to condition only the sows farrowing and sow slaughter equations. The remaining supply equations, including pig crop and barrow and gilt slaughter, are specified simply as technical relationships.

Although producers receive a variety of economic signals when making production decisions, the set of economic conditioning variables in the supply equations is limited to output prices, the price of feed, and interest rates. This information set, while parsimonious, does include the major price and cost signals that affect short-term profitability.

The supply component begins with the level of sows farrowing (Equation 20). Sows farrowing reflects producers' decisions about breeding herd expansion and contraction, and thus their adjustments in production capacities. The explanatory variables in the sows farrowing equation include the previous period's farm price and a distributed lag of feed costs. In addition, sows farrowing lagged four quarters is included to reflect adjustment costs associated with expanding the underlying breeding herd. Feed costs are included with an imposed distributed lag structure. The estimated coefficients have the expected signs and with the exception of farm price, are significant at conventional levels.

Pig crop (Equation 21) is determined directly by the level of farrowings. Time trend, T65, is included to represent increases in litter size and reduced death loss over the sample period. The estimated coefficient on farrowings indicates that the average litter size is approximately 5.6 pigs. Of course this does not reflect the technological improvement captured by the time trend. A zero-one dummy variable was also included to account for the redefinition of pig crop which occurred in 1974 (Blanton, 1983).

Barrow and gilt slaughter (Equation 22) depends on the size of the pig crop from the previous three quarters. A three period lag on pig crop seems reasonable since ther is a five-to-six month lag between birth and slaughter. The sum of the estimated coefficients on lagged pig crop implies that 86 percent of the three previous pig crops are slaughtered. A time trend is included to account for changes in rate-of-gain resulting from better feeding practices over the sample.

Sow slaughter (Equation 23) reflects the rate of culling from the breeding herd, or the disinvestment decisions of producers. Distributed lags for farm price, feed costs, and interest rate are included as explanatory variables, as well as the previous period's farrowings. Lagged farrowings represents the stock of available sows for slaughter. All estimated coefficients have the expected signs. In addition, the farm price variable is statistically significant while the interest rate feed cost variables are not.

Domestic pork production (Equation 24) multiplies barrow and gilt slaughter and sow slaughter by their respective live weights. Commercial pork production (Equation 25) transforms domestic pork production into a carcass weight equivalent. A time trend was included to reflect technological improvement in this transformation over the sample.

Pork retail price (Equation 26) was estimated in price dependent form and includes beef and chicken prices, per capita food expenditures, total domestic disappearance, ant the lagged retail price as explanatory variables. The lagged retail pork price is included to reflect price stickiness at the retail level. All estimated coefficients have the expected signs and, with the exception of chicken price and food expenditures, are statistically significant.

The pork farm price (Equation 27) depends directly on the retail price. Also included in the equation are an index of marketing costs for meat packers and a time trend. Increases in marketing costs expand the retail-farm margin, and thus reduce the farm level price. The time trend is included to capture changes in meat processing (Wholgenant and Mullen, 1987).

The model is closed with an identity that derives total domestic disappearance (Equation 28). Included in the identity is the variable OTHER which incorporates net imports, net cold storage stocks, military use, and shipments. These variables were treated as exogenous in the present exercise.

IV. Partial and Total Elasticities and Flexibilities

The dynamic behavior of the quarterly hog model can be examined through mean-path multipliers and elasticities with respect to exogenous variables. Although this method provides important information about model behavior, it does not give any indication about the dynamic relationships between endogenous variables. Hence, in addition to intermediate run elasticities and flexibilities for selected exogenous variables, total elasticities and flexibilities are presented for selected endogenous variables.

Dynamic Response with Respect to Exogenous Variables

The reduced form equations for the pork model are dynamic and represent a system of higher-order stochastic difference equations. The dynamic features can be attributed to the biological lags imposed on certain endogenous variables in the supply equations and the autoregressive error structure. Mean-path multipliers and elasticities are typically derived for a system of first-order stochastic difference equations. Consequently, the model must be transformed from a system of higher-order difference equations into the first-order difference equation system. A complete discussion of the methods involved for reparameterizing the model into a first-order system can be found in Chow (pp. 152-54). Additional complications arise in the present case because of the presence of the first- and fourth-order autoregressive error terms in some structural equations. However, Chow (pp. 61-62) also describes an appropriate transformation to use when the model contains autocorrelated residuals.

The methods described by Chow were applied to the structural model of the U.S. pork sector and intermediate multipliers and elasticities were obtained. Intermediate run multipliers measure the cumulative effects of a change in an exogenous variable on an endogenous variable when the change has persisted for several periods. Intermediate run elasticities can then be obtained from the appropriate multipliers.

Intermediate run elasticities for selected endogenous variables with respect to feed cost, the retail price of beef, and the retail price of chicken are reported in Table 2. The results indicate that changes in feed cost have small impacts on production initially, but the response increases in magnitude over time. In addition, as illustrated in Figure 1, the initial impact on total pork production is positive since the only adjustments which can occur initially is in sow slaughter. It takes several periods before production actually declines and prices rise. These results are intuitively appealing given the biological lags involved in adjusting production.

The impacts of demand shifters (e.g., beef price and chicken price) on production levels and prices gave similar results (Table 2). For instance, increasing the beef price has no impact on total pork production initially (Figure 1). But after one period, production declines as prices rise and, thus, sow slaughter decreases. This pattern continues for several periods until increased farrowing levels filter through the market, resulting in higher production. The biggest impact on farm pork price after approximately a four-to-five period delay. The impacts on retail price are similar. Again, these results conform with known biological relationships which constrain short-term production adjustments. The increased farm price results in higher production levels after approximately two periods. After a four or five period delay, production has increased sufficiently to dampen the effects of increased beef or chicken prices. The intermediate price flexibilities then decline monotonically and approach a new steady state level.

Dynamic Response with Respect to Endogenous Variables

Insights into the relationships between endogenous variables can be obtained by examining total price and quantity effects. Since the estimated pork model includes a general lag structure on the endogenous variables, the methods described in section II were used to obtain numerical estimates of total elasticities and flexibilities.[5]

Total elasticities with respect to the farm price of hogs and total flexibilities for the farm price with respect to the remaining endogenous variables are reported in Table 3. In general, the total production response is small as indicated by the elasticities for farrowings, pig crop, sow slaughter, barrow and gilt slaughter, and pork supply. The production elasticities do, however, increase in magnitude over time (Figure 4). Also, price responsiveness declines at successive stages of the production process. This result also conforms with prior notions about the relative inability to adjust output at later stages of the production process.

The approximate long-run elasticity for pork production is 0.232. By comparison, Meilke, Zwart, and Martin report long run elasticities for pork production in the U.S. between 0.43 and 0.48 and MacAulay reports 0.50 for the same coefficient. Although the long-run production response obtained here is smaller than those reported previously, it should be emphasized that the earlier estimates were obtained using standard partial elasticity concepts.

The flexibilities reported in Table 3 also confirm that total elasticities are not the inverse of total flexibilities. As expected, the price impacts increase at each stage of production with commercial pork production having the largest impact on farm price in the long run (-1.00). The farm price flexibility with respect to sow slaughter is small and, at any point in time, is approximately a tenth the size of the corresponding barrow and gilt slaughter flexibility. This result is entirely plausible since sow slaughter has historically accounted for about ten percent of total pork production.

The relationships between farm price and retail price are also of interest. The retail price flexibility with respect to the farm price indicates that initially, an increase in the farm price results in a higher retail price. The intuition is that sow slaughter levels are reduced at the same time farrowing levels are increasing. After several periods, the higher price levels result in increased barrow and gilt slaughter. The result is that the long-run retail price flexibility is negative (-0.17), but small. Conversely, farm price flexibilities with respect to retail price are positive and are all greater than one (Table 3). An exogenous increase in the retail price results in oscillatory behavior in farm prices (Figure 4). The time between peaks varies between eight and twelve quarters which corresponds roughly with the emerging three-year hog cycle reported by Shonkwiler and Spreen. The largest impact comes after eight quarters when the farm price flexibility reaches 1.72. Although the oscillations continue, they dampen out after approximately thirty quarters and approach an approximate long-run level of 1.67.

V. Conclusions

Partial elasticities and flexibilities provide incomplete information in a simultaneous system of supply and demand equations. This is because partial elasticities are not evaluated in a general equilibrium context where all other endogenous variables are allowed to adjust freely. Total elasticities and flexibilities offer a more appropriate means of characterizing static and dynamic relationships among endogenous variables in a systems framework. The conceptual framework for measuring total price and quantity effects has been available for some time (Buse). However, analytical procedures for deriving total response relationships in dynamic settings were not available until recently (Chavas, Hassan, and Johnson). Even so, these methods have not been adopted in evaluating model results.

In this paper we show how measures of total response can be incorporated in a simultaneous model with a general lag structure and autoregressive errors. This represents an important extension to previous research since total response elasticities and flexibilities have not been derived previously in this context. The empirical application was with a quarterly model of the U.S. pork sector. The results suggest that total supply elasticities are generally smaller than those reported elsewhere which were obtained in a partial response context. Of course, the elasticities reflect the underlying model structure. The application of this procedure may be limited to other models, since linearity is required. Thus, an area of future research is to extend these methods to nonlinear systems.

Notes

1. A total elasticity measures the change in an endogenous variable caused by a change in another endogenous variable when all remaining variables in the system are allowed to adjust accordingly.

2. The methods for obtaining multipliers and elasticities between endogenous and exogenous variables are not reviewed here since there results are well knows and have been extensively covered elsewhere (e.g., Chow; Fomby, Hill, and Johnson).

3. This method is quite similar to the approach frequently used to obtain multipliers and elasticities for exogenous variables in nonlinear structral models (Fair).

4. The dynamic interactions implied by the autoregressive error structure must also be accounted for when obtaining total price and quantity effects. In the present case, the structural model was converted to a system of quasi difference equations by using methods similar to those described by Fomby, Hill, and Johnson (pp. 525-26). The resulting transformed system, which has a stationary error process, was used to obtain all total response results.

Table 1. Structural parameter estimates for the U.S. quarterly pork model

(20) Hogs Farrowing (2SLS)

$$FARROW_t = \underset{(2.20)}{1083.15^{a}} + \underset{\substack{(4.04)\\ {[0.66]}}}{0.66}\, FARROW_{t-4} + \underset{\substack{(1.42)\\ {[0.13]}}}{10.31}\, FPPK_{t-1} - \underset{\substack{(-.05)\\ {[-0.18]}}}{123.93}\, FEEDPS_t$$

$$+ \underset{(1.58)}{251.42}\, JS2 + \underset{(1.84)}{137.62}\, JS3 + \underset{(0.99)}{64.91}\, JS4$$

$$R^2 = 0.88 \qquad u_t = \underset{(7.48)}{0.81}\, u_{t-1} - \underset{(-2.63)}{0.34}\, u_{t-4} + \epsilon_t$$

(21) Pig Crop (2SLS)

$$PCUS_t = \underset{(1.10)}{3627.19} + \underset{\substack{(4.85)\\ {[0.77]}}}{5.59}\, FARROW_t + \underset{(1.32)}{110.91}\, T65 + \underset{(1.85)}{2084.45}\, JS2$$

$$+ \underset{(1.36)}{937.99}\, JS3 + \underset{(1.28)}{768.72}\, JS4 - \underset{(-1.19)}{1136.76}\, DMPC$$

$$R^2 = 0.79 \qquad u_t = \underset{(-2.02)}{-0.24}\, u_{t-1} + \epsilon_t$$

(22) Barrow and Gilt Slaughter (OLS)

$$BGSUS_t = \underset{(-1.48)}{-1560.09} + \underset{\substack{(5.59)\\ {[0.31]}}}{0.27}\, PCUS_{t-1} + \underset{\substack{(5.79)\\ {[0.36]}}}{0.31}\, PCUS_{t-2} + \underset{\substack{(6.00)\\ {[0.32]}}}{0.28}\, PCUS_{t-3}$$

$$+ \underset{(4.52)}{68.60}\, T65 + \underset{(5.63)}{1679.66}\, JS2 - \underset{(-1.21)}{531.18}\, JS3 + \underset{(2.81)}{1503.22}\, JS4$$

$$R^2 = 0.90 \qquad D.W. = 1.53$$

(23) Sow Slaughter (2SLS)

$$SSUS_t = \underset{(-0.14)}{-59.55} - \underset{\substack{(-2.84)\\ {[-0.51]}}}{16.10}\, FPPKS_t + \underset{\substack{(1.74)\\ {[0.25]}}}{66.69}\, FEEDPS_t^{c} + \underset{\substack{(0.11)\\ {[0.01]}}}{1.31}\, IFCLS_t^{d}$$

$$+ \underset{\substack{(3.70)\\ {[1.20]}}}{0.46}\, FARROW_{t-1} + \underset{(0.09)}{199.04}\, JS2 + \underset{(7.03)}{7.57}\, JS3 + \underset{(3.90)}{274.98}\, JS4$$

$$R^2 = 0.74 \qquad u_t = \underset{(2.00)}{0.44}\, u_{t-1} + \epsilon_t$$

(24) Domestic Pork Production[e]

$$PPF_t = -5158269.6 + 19505.2\, LWBG_t + 237.1\, BGSUS_t$$

$$+ 1187.5\, LWS_t + 449.5\, SSUS_t$$

Table 1. Continued

(25) Commercial Pork Production

$$TOTSPK_t = \underset{\substack{(51.3)\\ [0.94]}}{0.63}\ (PPF_t/1000) + \underset{(3.12)}{20.04}\ T65$$

$R^2 = 0.98$ $\qquad u_t = \underset{(8.45)}{0.92}\ u_{t-1} + \epsilon_t$

(26) Pork Retail Price (2SLS)

$$RPPK_t = \underset{(18.10)}{58.19} + \underset{\substack{(8.19)\\ [0.59]}}{0.59}\ RPPK_{t-1} - \underset{\substack{(-3.80)\\ [-0.58]}}{0.02}\ TOTDPK_t + \underset{\substack{(3.55)\\ [0.29]}}{0.21}\ RPBF4_t$$

$$+ \underset{\substack{(1.24)\\ [0.11]}}{0.22}\ RPCK_t + \underset{\substack{(0.90)\\ [0.05]}}{0.02}\ FEXP_t - \underset{(-2.97)}{4.78}\ JS2 + \underset{(0.10)}{0.19}\ JS3 + \underset{(1.45)}{2.39}\ JS4$$

$R^2 = 0.99$

(27) Pork Farm Price (2SLS)

$$FPPK_t = \underset{(-4.22)}{-10.02} + \underset{\substack{(10.99)\\ [1.83]}}{0.57}\ RPPK_t - \underset{\substack{(-3.78)\\ [-0.28]}}{0.04}\ MKTCOST_t - \underset{(-1.94)}{0.91}\ T65$$

$$+ \underset{(1.14)}{0.55}\ JS2 + \underset{(1.85)}{0.99}\ JS3 - \underset{(-2.68)}{1.25}\ JS4$$

$R^2 = 0.97$ $\qquad u_t = \underset{(3.80)}{0.66}\ u_{t-1} - \underset{(-2.08)}{0.36}\ u_{t-4} + \epsilon_t$

(28) Total Domestic Disappearance

$$TOTDPK_t = TOTSPK_t + OTHER_t$$

[a] Structural parameters estimates are accompanied by their asymptotic t-ratios in parentheses and corresponding elasticities, evaluated at sample means, in brackets.

[b] $FPPKS_t = 0.5\ FPPK_{t-1} + 0.3\ FPPK_{t-2} + 0.2\ FPPK_{t-3}$

[c] $FEEDPS_t = 0.5\ FEEDP_t + 0.3\ FEEDP_{t-1} + 0.2\ FEEDP_{t-2}$, where

$FEEDP = (6/7)\ (PCO4/0.56) + (1/7)\ (PSOYM/20)$

[d] $IFCLS_t = 0.5\ IFCL_{t-1} + 0.3\ IFCL_{t-2} + 0.2\ IFCL_{t-1}$

[e] The identity used to derive domestic pork production (PPF) was PPF = BGSUS * LWBG + SSUS * LWS. Equation (5) was linearized using a first-order Taylor series approximation (see Chow, 1975, pp. 131-133).

Table 2. Mean elasticities for the selected endogenous variables

Period	Farrow	PCUS	SSUS	BGSUS	TOTSPK	FPPK	RPPK
			Percent change in feed costs				
0	-0.089	-0.069	0.126	0.000	0.012	-0.011	-0.006
1	-0.144	-0.111	0.096	-0.021	-0.011	0.004	0.002
2	-0.178	-0.137	0.078	-0.059	-0.049	0.046	0.025
3	-0.173	-0.133	0.026	-0.105	-0:098	0.114	0.062
4	-0.223	-0.171	0.006	-0.127	-0.121	0.175	0.095
5	-0.252	-0.193	-0.086	-0.146	-0.148	0.235	0.128
10	-0.300	-0.230	-0.315	-0.213	-0.234	0.465	0.252
15	-0.300	-0.230	-0.400	-0.235	-0.264	0.557	0.302
20	-0.308	-0.237	-0.415	-0.237	-0.267	0.580	0.315
25	-0.309	-0.238	-0.424	-0.236	-0.267	0.584	0.317
30	-0.309	-0.237	-0.425	-0.236	-0.267	0.584	0.317
35	-0.307	-0.236	-0.425	-0.237	-0.268	0.584	0.317
∞	-0.308	-0.236	-0.424	-0.236	-0.267	0.584	0.317
			Percent change in beef retail price				
0	0.000	0.000	0.000	0.000	0.000	0.527	0.286
1	0.068	0.052	-0.138	0.000	-0.013	0.852	0.462
2	0.109	0.084	-0.224	0.016	-0.006	1.039	0.564
3	0.133	0.102	-0.329	0.045	0.011	1.135	0.616
4	0.145	0.112	-0.388	0.079	0.038	1.168	0.634
5	0.194	0.149	-0.417	0.099	0.055	1.173	0.636
10	0.281	0.216	-0.237	0.190	0.159	1.005	0.545
15	0.304	0.233	-0.110	0.226	0.206	0.876	0.475
20	0.306	0.235	-0.067	0.237	0.221	0.828	0.449
25	0.310	0.238	-0.058	0.238	0.222	0.816	0.443
30	0.311	0.238	-0.053	0.237	0.222	0.814	0.442
35	0.310	0.238	-0.053	0.237	0.222	0.814	0.442
∞	0.309	0.238	-0.053	0.237	0.222	0.814	0.442
			Percent change in chicken retail price				
0	0.000	0.000	0.000	0.000	0.000	0.194	0.105
1	0.025	0.019	-0.051	0.000	-0.005	0.313	0.170
2	0.040	0.031	-0.082	0.006	-0.002	0.382	0.207
3	0.049	0.038	-0.121	0.016	0.004	0.417	0.226
4	0.053	0.041	-0.143	0.029	0.014	0.429	0.233
5	0.071	0.055	-0.153	0.036	0.020	0.431	0.234
10	0.103	0.079	-0.087	0.070	0.058	0.369	0.200
15	0.112	0.086	-0.041	0.083	0.076	0.322	0.175
20	0.112	0.086	-0.025	0.087	0.081	0.304	0.165
25	0.114	0.087	-0.021	0.087	0.082	0.300	0.163
30	0.114	0.087	-0.020	0.087	0.082	0.299	0.162
35	0.114	0.087	-0.019	0.087	0.082	0.299	0.162
∞	0.114	0.087	-0.020	0.087	0.082	0.299	0.162

Figure 1. Intermediate run elasticities, U.S. pork production

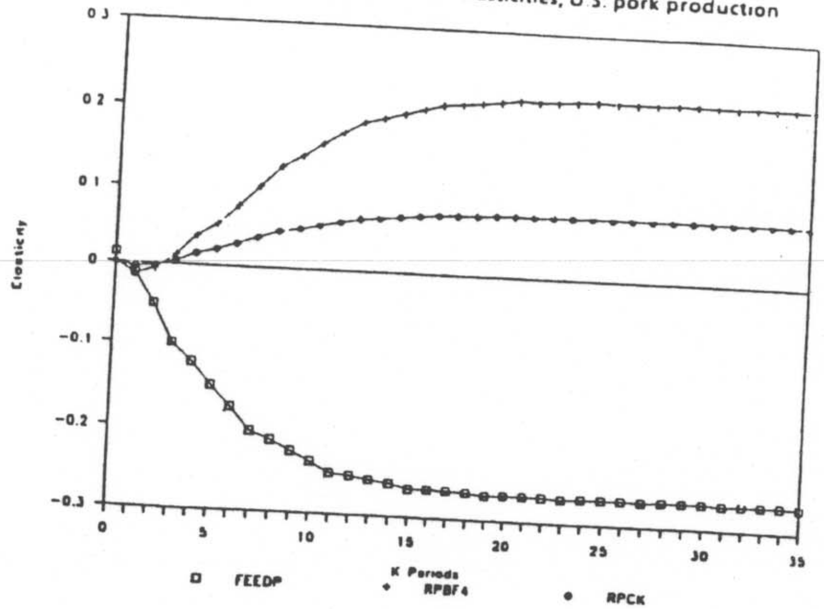

Figure 2. Intermediate run flexibilities, pork farm price

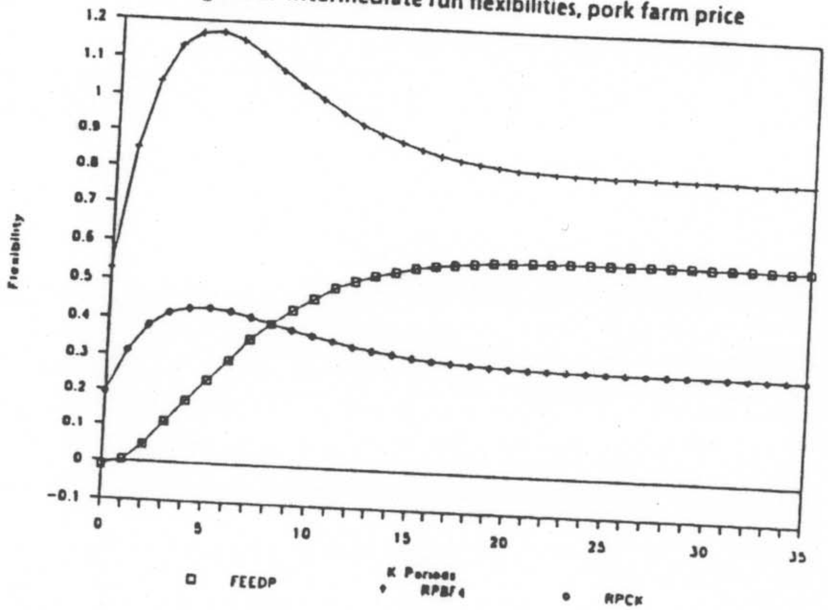

Figure 3. Total elasticities, U.S. pork production

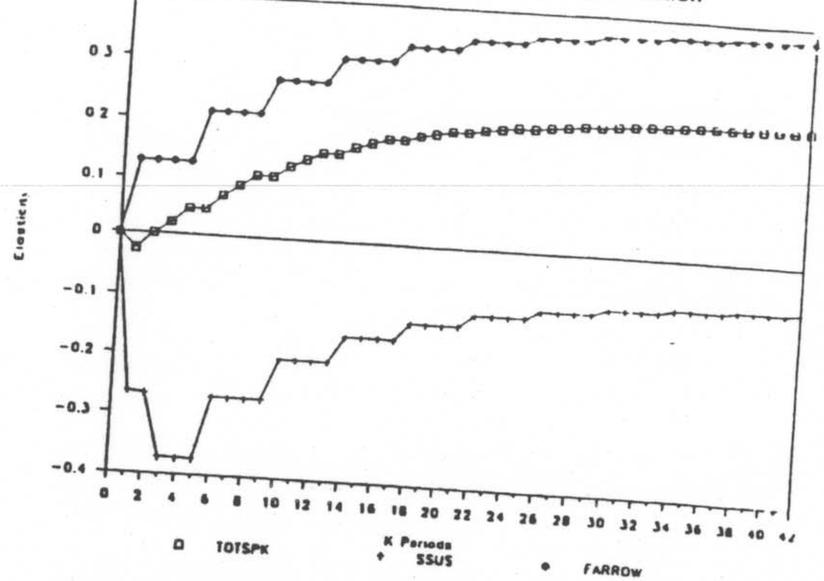

Figure 4. Total flexibilities, pork farm price with respect to the pork retail price

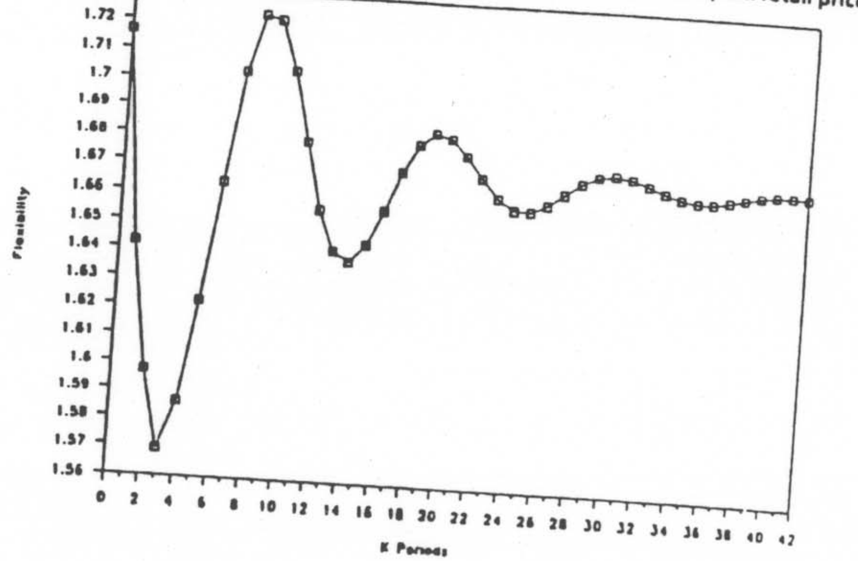

Table 3. Total elasticities with respect to farm price for selected endogenous variables

	Elasticities[a]						Flexibilities[b]					
Period	FARROW	PCUS	SSUS	BGUS	TOTSPK	RPPK	FARROW	PCUS	SSUS	BGUS	TOTSPK	RPPK
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.056	-0.485	-0.587	1.716
1	0.128	0.099	-0.263	0.000	-0.025	0.009	-0.151	-0.124	-0.074	-0.648	-0.778	1.642
2	0.129	0.099	-0.266	0.031	0.000	0.005	-0.319	-0.311	-0.082	-0.721	-0.863	1.597
3	0.129	0.099	-0.372	0.066	0.020	-0.003	-0.491	-0.519	-0.085	-0.761	-0.905	1.569
4	0.129	0.099	-0.373	0.099	0.046	-0.016	-0.595	-0.643	-0.087	-0.792	-0.939	1.586
5	0.214	0.164	-0.373	0.099	0.046	-0.024	-0.664	-0.722	-0.088	-0.818	-0.967	1.622
10	0.271	0.207	-0.201	0.178	0.128	-0.080	-0.782	-0.852	-0.085	-0.857	-1.013	1.704
15	0.308	0.236	-0.156	0.227	0.174	-0.118	-0.780	-0.840	-0.079	-0.836	-0.997	1.644
20	0.333	0.255	-0.126	0.256	0.200	-0.141	-0.790	-0.843	-0.078	-0.841	-1.009	1.683
25	0.361	0.276	-0.106	0.268	0.213	-0.154	-0.784	-0.832	-0.077	-0.832	-1.001	1.659
30	0.368	0.282	-0.084	0.278	0.223	-0.162	-0.787	-0.833	-0.077	-0.834	-1.006	1.673
35	0.373	0.286	-0.078	0.285	0.229	-0.167	-0.785	-0.830	-0.077	-0.831	-1.003	1.665
40	0.376	0.288	-0.075	0.288	0.232	-0.170	-0.786	-0.830	-0.077	-0.831	-1.004	1.669

Note: All elasticities and flexibilities are derived by simulating the model at the means of the sample data.

[a]Total elasticities are with respect to the farm price.

[b]Total flexibilities are for the farm price with respect to selected endogenous variables.

References

Arzac, E.R. and M. Wilkinson. "A Quarterly Econometric Model of the United States Livestock and Feed Grain Markets and Some of its Policy Implications." American Journal of Agricultural Economics 61(1979):297-308.

Blanton, B.J. "A Quarterly Econometric Model of the United States Pork Sector." M.S. Thesis, University of Missouri-Columbia, May 1983.

Board of Governors of the Federal Reserve System. Agricultural Finance Databook. Division of Research and Statistics, Washington, D.C., December 1984.

Buse, R.C. "Total Elasticities - A Predictive Device." Journal of Farm Economics 40(1958):881-891.

Chavas, J.P., Z.A. Hassan, and S.R. Johnson. "Static and Dynamic Elasticities and Flexibilities in Systems of Simultaneous Equations." Journal of Agricultural Economics 32(1981):177-187.

Chow, G.C. Analysis and Control of Dynamic Economic Systems. New York: John Wiley and Sons, 1975.

Colman, D. and H. Miah. "Are Direct Elasticities the Inverse of Direct Flexibilities." Journal of Agricultural Economics 24(1973):364-368.

Fair, R.C. "Estimating the Uncertainty of Policy Effects in Nonlinear Models." Econometrica 48(1980):1381-1391.

Fomby, T.B., R.C. Hill, and S.R. Johnson. Advanced Econometric Methods. New York: Springer-Verlag, 1984.

Foote, R.J. Analytical Tools for Studying Demand and Price Analysis. USDA, Agricultural Handbook 146, 1958.

Freebairn, J.W., and G.C. Rausser. "Effects of Changes in the Level of U.S. Beef Imports." American Journal of Agricultural Economics 57(1975):676-688.

Harlow, A.A. "A Recursive Model of the Hog Industry." Agricultural Economics Research 14(1962):1-12.

Houck, J.P. "Price Flexibilities and Price Elasticities." Journal of Farm Economics 47(1965):789-792.

MacAulay, T.G. "A Forecasting Model for the Canadian and U.S. Pork Sectors." in Commodity Forecasting Models for Canadian Agriculture vol. 11, Agriculture Canada, 1978.

Martin, J.W. "U.S. Agricultural Policy and the Demand for Imported Beef." Ph.D dissertation. Iowa State University, 1982.

Meinken, K.W., A.S. Rojko, and G.A. King. "Measurement of Substitution in Demand." Journal of Farm Economics 33(1956):733-735.

Shonkwiler, J.S., and T.H. Spreen. "Statistical Significance and Stability of the Hog Cycle." Southern Journal of Agricultural Economics 18(1986):227-233.

U.S. Department of Agriculture. Agricultural Prices. National Agricultural Statistics Service, Agricultural Statistics Board, Washington, D.C. various issues 1967-1986.

U.S. Department of Agriculture. Hogs and Pigs. ERS, Washington, D.C., various issues 1967-1988.

U.S. Department of Agriculture. Livestock and Poultry Situation and Outlook. ERS, Washington, D.C., various issues 1967-1986.

U.S. Department of Agriculture. Livestock and Meat Statistics. Statistical Bulletin 552 and annual supplements, ERS, Washington, D.C., various issues 1973-1983.

U.S. Department of Commerce. Survey of Current Business. Bureau of Economic Analysis, Washington, D.C., various issues 1967-1986.

U.S. Department of Labor. Employment and Earnings of the United States. Washington, D.C., various issues 1967-1986.

Waugh, F.V. Demand and Price Analysis: Some Examples from Agriculture. USDA, Technical Bulletin 1316, 1964.

Wholgenant, M.K., and J.D. Mullen. "Modeling the Farm-Retail Price Spread for Beef." Western Journal of Agricultural Economics 12(1987):119-125.

Appendix. Variable definition and source

Label	Definition	Units	Source[a]
FARROW	Hogs Farrowing	1000 head	Hogs and Pigs
PCUS	Pig Crop	1000 head	Hogs and Pigs
BGSUS	Barrow and Gilt Slaughter	1000 head	LMS
SSUS	Sow Slaughter	1000 head	LMS
PPF	Domestic Pork Production	pounds	BGSUS * LWBG + SSUS * LWS
TOTSPK	Commercial Pork Production	million pounds	LPSO
TOTDPK	Domestic Disappearance	million pounds	LPSO
FPPK	Pork Farm Price (Barrow and Gilts, Omaha 182, 210-240 pounds)		
RPPK	Pork Retail Price	dollars/cut	LPSO
LWBG	Live Weight Barrow and Gilts	cents/pound	LPSO
LWS	Live Weight Sows	pounds	LMS
PCO4	Average Corn Price	pounds	LMS
PSOYB	Average Soymeal Price	dollars/bushel	AP
IFCL	Interest Rate on Feeder Cattle Loans	dollars/ton	AP
RPBF4	Beef Retail Price	percent	AFDB
RPCK	Chicken Retail	cents/pound	LPSO
FEXP	Per Capita Food Expenditures	cents/pound	LPSO
FOODEXP	Food Expenditures	dollars/person	FOODEXP/POPN4
POPN4	U.S. Population	millions of dollars	
MKTCOST	Index of Marketing Costs	millions	SCB
			SCB
PPIFP	Producer Price Index of Fuels and Related Products and Power	1967 = 100	0.5 x (PPIFP + IMPHRE)
IMPHRE	Index of Earnings of Employees in Packing Plants	1967 = 100	SCB
OTHER	Net Stock, Net Imports, Military Use, Shipments	1967 = 100	EEUS
T65	Time Trend	million pounds	LPSO
JS2, JS3, JS4	Seasonal Dummy Quarters 2,3,4	Beginning in 1965 equals 1.00, 1.25, . . .	
DMPC	Dummy Variable	If year ≥ 1974 = 1, equals 0 otherwise	

[a]Hogs and Pigs represents the Hogs and Pigs report, LMS is the Livestock and Meat Statistics, AP is the Agricultural Prices, LPSO is the Livestock and Poultry Situation and Outlook, AFDB is the Agricultural Finance Databook, SCB is the Survey of Current Business, and EEUS is Employment and Earnings of the United States.