Forecasting Quarterly Demand for Meats with a Mixed Structure-Time Series Model

by

Kuo S. Huang

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I. INTRODUCTION

Numerous quarterly models have been developed for U.S. meat commodities. Most recently, Westcott and Hull, and Stillman have formulated some quarterly forecasting models of livestock and poultry for use in their analyses of situation and outlook programs and related activities. However, most of the quarterly models use a partial demand approach to specify the demand component of the models. Thus the estimated demand parameters for meats may not be consistent with the classical demand theory. To improve the specification of the demand component, this paper is aimed at developing an efficient and theoretically consistent econometric model for forecasting U.S. quarterly demand for meats.

Since the demands for meats and other food commodities are inter-related, an ideal quarterly demand model for meats should be obtained within the framework of a complete demand system that considers all food commodities and the postulates of classical demand theory. Thus far it is possible to directly estimate an annual U.S. food demand system. However, it is difficult to estimate a quarterly demand system because the required quarterly time series data across all concerned food commodities are not available. To resolve this problem, one may assume that both the demand structures for annual and quarterly observations are similar. Accordingly, one may obtain the price and expenditure elasticities for meats from an annual theoretically consistent demand system as a priori information for the specification of the quarterly demand model. The error of using these demand elasticities may be attributed in part to the residual series of the model, and is subject to further time series analysis.

To improve the forecasting capability of the quarterly demand model for meats, this study combines the economic structural component of the model with a time series analysis for the unexplained residuals. By applying such a mixed structure-time series model, one can provide an economic explanation of the dependent variable that can be explained by the structural component of the model, and a time-series explanation of the dependent variable that cannot be explained structurally. Thus the model is likely to provide better forecasts than the regression model alone or a time-series model alone.

In this paper, a brief discussion on the specification and estimation of a mixed structure-time series model is given at the beginning. The empirical results of a quarterly demand model for meats with a focus on its forecasting performance follows.

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II. MODEL SPECIFICATION AND ESTIMATION

The statistical model for the quarterly per capita consumption of beef, pork, and chicken is specified as a log-linear function of meat prices, per capita expenditure, seasonal dummy variables, and an autoregressive process of residuals with a lag up to k quarters as follows:

$$\log Q_{it} = \sum_{j=1}^{m} \alpha_{ij} \log P_{jt} + \beta_i \log M_t + \sum_{j=1}^{3} \theta_{ij} D_{jt} + \theta_{i0} + u_{it} \qquad (1)$$

and $$u_{it} = \varepsilon_{it} - \sum_{j=1}^{k} \delta_{ij} u_{i,t-j}, \qquad i=1,2,..,m \qquad (2)$$

where variables at time t are Q_{it} (per capita quantity demand for the ith commodity), P_{jt} (price of the jth commodity deflated by consumer price index), M_t (per capita expenditure deflated by consumer price index), and D_{jt}s (dummy variables assigned for 1 sequentially from the second to fourth quarters, and others to be zeros); u_{it} and ε_{it} are respectively random disturbances in which ε_{it} is assumed to be normal and independently distributed, $N(0,\sigma^2 I)$.

The structural parameters α_{ij}s and β_is in equation (1) are price and expenditure elasticities respectively. In this study, these demand parameters are obtained from a theoretically consistent annual demand system. Alternatively, this study also estimates the demand parameters directly for comparison. A set of quarterly dummy variables is also included to reflect the possibility of seasonal variation in the demand for meat commodities.

Moreover, as indicated by Muth, there is little empirical interest in assuming that the disturbance term in a structural model is completely unpredictable. It is desirable for this study to assume that the disturbance in the demand equation can be predicted based on past observations. Since the expected values of the disturbance could be related to economic conditions prevailing in the past quarters, one may assume that the disturbance is not independent over time but follows an autoregressive process. Consequently, equation (2) provides the predictive information for the movements of random disturbance.

The proposed statistical model can be viewed as a mixed structure-time series model. The model not only provides a structural explanation of quantity demand for meats in equation (1), but also replicates the past behavior of residuals by specifying an autoregressive process in equation (2).

To estimate the model, one needs a three-step estimation procedure because the disturbance terms in the autoregressive process are not observable. First, some preliminary estimates of the structural parameters in equation (1) are estimated by ordinary least squares. Because of potential serial correlation in the estimated residual series, the preliminary estimates are perhaps unbiased but inefficient, thus requiring a further estimation. Second, the estimated residuals from the first step are used to fit the autoregressive process in equation (2). Given an appropriate order for the lags, one can obtain the autoregressive coefficients in the equation by solving the so-called Yule-Walker equation as stated in Box and Jenkins. Third, based on the estimated autoregressive structure, the structural parameters are reestimated by the application of an Aitken estimation procedure suggested by Gallant and Goebel.

III. EMPIRICAL ESTIMATION RESULTS

The data used for this study are quarterly data covering the first quarter of 1960 to the third quarter of 1987 with a total of 111 observations for each variable. Per capita consumption data for beef, pork, and chicken are compiled by the U.S. Department of Agriculture. The price data are consumer price indexes (1967=100) for all urban consumers from the Department of Labor. The personal consumption expenditure data are compiled by the U.S. Department of Commerce. All the prices and expenditure data are deflated by the consumer price index for all items.

At the beginning, this study obtains the price and expenditure elasticities for meats from a theoretically consistent demand system estimated by Huang. The demand system, which consists of 1,722 price and expenditure elasticities for 40 food items and 1 nonfood category, was estimated from a set of annual data series covering 1953-1983. The structure of the complete demand system can be described as below.

The demand system for n commodities is represented as a set of n linear equations with n(n+1) parameters:

$$\dot{q}_1 = e_{11} \dot{p}_1 + e_{12} \dot{p}_2 + \dots + e_{1n} \dot{p}_n + g_1 \dot{m}$$

$$\vdots$$

$$\dot{q}_n = e_{n1} \dot{p}_1 + e_{n2} \dot{p}_2 + \dots + e_{nn} \dot{p}_n + g_n \dot{m} \qquad (3)$$

where

$\dot{q}_i$ = relative change in the quantity of the ith commodity

$\dot{p}_i$ = relative change in the price of the ith commodity

$\dot{m}$ = relative change in consumption expenditure

e_{ij} = demand elasticity of the ith commodity with respect to the price of the jth commodity, and

g_i = income (expenditure) elasticity of the ith commodity.

Classical demand theory specifies that the price and income elasticities in the demand system are interdependent. They are related through the so-called Engel aggregation, homogeneity, and symmetry as follows:

$$\text{Engel aggregation:} \quad \sum_{i=1}^{n} w_i \, g_i = 1 \tag{4}$$

$$\text{Homogeneity:} \quad \sum_{j=1}^{n} e_{ij} + g_i = 0 \qquad \text{for } i = 1,2,..,n \tag{5}$$

$$\text{Symmetry:} \quad e_{ji}/w_i + g_j = e_{ij}/w_j + g_i \qquad \text{for } i,j = 1,2,..,n \tag{6}$$

where w_i is the expenditure weight of the ith commodity, and e_{ij} and g_i are as defined above. These parametric restrictions are incorporated into the estimation of the complete demand system (3) by a constrained maximum likelihood procedure.

The figures in the upper part of table 1 are a priori information of price and expenditure elasticities obtained from Huang's demand system. Since the demand elasticities of the model developed in this case are constrained by the prior information, we may define this model as the "constrained case." These elasticities are theoretically consistent, and thus can be regarded as reliable estimates representing the demand structure of meat commodities. The direct-price elasticities are -0.6166 (beef), -0.7297 (pork), and -0.5308 (chicken). The expenditure elasticities for beef and pork are almost the same at about 0.45, while that of chicken is relatively small at 0.37. The estimated cross-price elasticities show significant substitute relationships among beef, pork, and chicken. For example, the quantity of beef demanded will increase by 0.1087 percent and 0.0572 percent for a 1-percent increase in pork and chicken price respectively.

Given the meat price and expenditure elasticities, according to equation (1), one may subtract the price and expenditure effects from the actual values of quantity demanded and redefine an adjusted quantity demanded variable in the equation as a function of seasonal dummy variables only. After an initial ordinary least square fitting, one may obtain an estimated residual series. The residuals in part reflects the omission of other price effects other than meat prices, and the use of a priori demand elasticities which are estimated from a functional form and data different from the quarterly model as specified in equation (1).

On the basis of estimated Durbin-Watson statistics -- beef (0.0814), pork (0.2788), and chicken (0.1117) -- they imply severe serial correlations in the estimated residual series. One may apply an autoregressive process for the residual series of each equation, and then substitute the model for the implicit error term in the original quarterly demand equation.

To determine the number of lags for use in fitting equation (2), some autocorrelations with various lags are calculated. The results indicate

Table 1 -- Estimation Results for Constrained Cases

Independent variable	Beef	Pork	Chicken
Price:			
Beef	-0.6166 (.0483)	0.1910 (.0390)	0.2927 (.0698)
Pork	.1087 (.0220)	-.7297 (.0327)	.2635 (.0492)
Chicken	.0572 (.0136)	.0908 (.0170)	-.5308 (.0608)
Expenditure:	.4549 (.0585)	.4427 (.0624)	.3645 (.0863)
Seasonal dummy:			
Spring	-.0042 (.0042)	-.0648 (.0121)	.0838 (.0092)
Summer	.0244 (.0044)	-.0615 (.0165)	.0797 (.0113)
Fall	.0067 (.0043)	.0593 (.0122)	-.0086 (.0093)
Intercept	1.4755	1.2548	-.7313
Residual:			
Lag 1	-.8062	-.7720	-.7813
Lag 2	-.1388	.1576	-.0182
Lag 3	-.0324	-.3552	-.2083
Lag 4	.0225	-.0854	-.0666
Lag 5	.1370	.2158	.3054
Lag 6	-.1637	.0864	-.1724
Lag 7	-.1075	-.0353	-.0159
Lag 8	.1715	-.1857	-.0895
Lag 9	-.0251	.0549	.0251
Lag 10	-.0258		.1715
Lag 11	-.0321		-.0072
Lag 12	.0607		-.0830
Adjusted R-square:			
Structural model	-0.0154	0.2961	0.1216
Mixed model	.9173	.8722	.9126
RMS/MEAN (percent)	.8331	1.0970	1.1927

Note: Meat price and expenditure elasticities are obtained from Huang. The structural model in the regression fitting refers to the quantity demanded by excluding price and expenditure effects as a function of seasonal dummy variables. The figures in parentheses are the estimated standard errors.

that the degrees of autocorrelations are descending with the increase of lags. This study chooses those lags with the autocorrelation higher than 0.5; the maximum lags are 12 for beef and chicken, and 9 for pork. These lags are then used for fitting the autoregressive processes; their estimated coefficients are compiled in the lower section of the table.

The final estimated coefficients for the dummy variables are listed in the middle section of the table. These estimates show the potential seasonal shifts as deviated from the winter season. Among those of significant estimates, the consumption of beef has a positive shift in summer; the consumption of pork has a positive shift in fall, and negative shifts over spring and summer; while the consumption of chicken has positive shifts in spring and summer.

To compare the goodness of fit between the structural model and the mixed structure-time series model, this study computes the adjusted coefficient of determination, adj.R^2, from the ordinary coefficient of determination, R^2, as adj.$R^2 = 1 - (1 - R^2)(n-1)/(n-k)$, where n and k are the numbers of sample observations and independent variables, respectively. The adj.R^2 is a better measurement than the R^2 for comparing the goodness of fit, because the former is not sensitive to the number of independent variables in a equation, while the latter is likely to increase its value with the increase of independent variables. The structural model, which relates the adjusted quantity series to seasonal dummy variables, has the adj.R^2 values less than 0.3 for each equation. It is striking to find that, by applying the mixed structure-time series model, their adj.R^2 values have increased substantially to about 0.9.

An ex post simulation is conducted over the sample period excluding the observations assigned for 12 lags covering the first quarter of 1963 to the third quarter of 1987. To provide an average measure of forecasting performance, one may calculate an error measurement defined as the ratio between the the root-mean-square error (RMS) and the sample mean (MEAN) of quantity demanded over the simulated period. These errors expressed in percentage are: 0.833 (beef), 1.097 (pork), and 1.193 (chicken).

As another measure, one may evaluate how well the model simulates turning points over the sample period. A simple way is to compare the sign of the actual change in quantity demanded to the corresponding simulated change. The turning point errors reflect the number of signs in the simulated quantity changes not consistent with the actual quantity changes. The numbers of sign errors out of a total 99 sample observations are 32 for beef, 12 for pork, and 21 for chicken. A graphic comparison of actual per capita quantity demanded (measured in logarithm) in contrast with those of simulation results are presented in figures 1-3.

This study also directly estimates the demand parameters in equation (1) without imposing any parametric constraints. We may call this the "unconstrained case" of the mixed structure-time series model. Their estimation results are compiled in the appendix. The direct-price elasticities are -0.3421 (beef), -0.7204 (pork), and -0.3998 (chicken). Their expenditure elasticities are 0.1389 (beef), 0.0640 (pork), and 1.0430

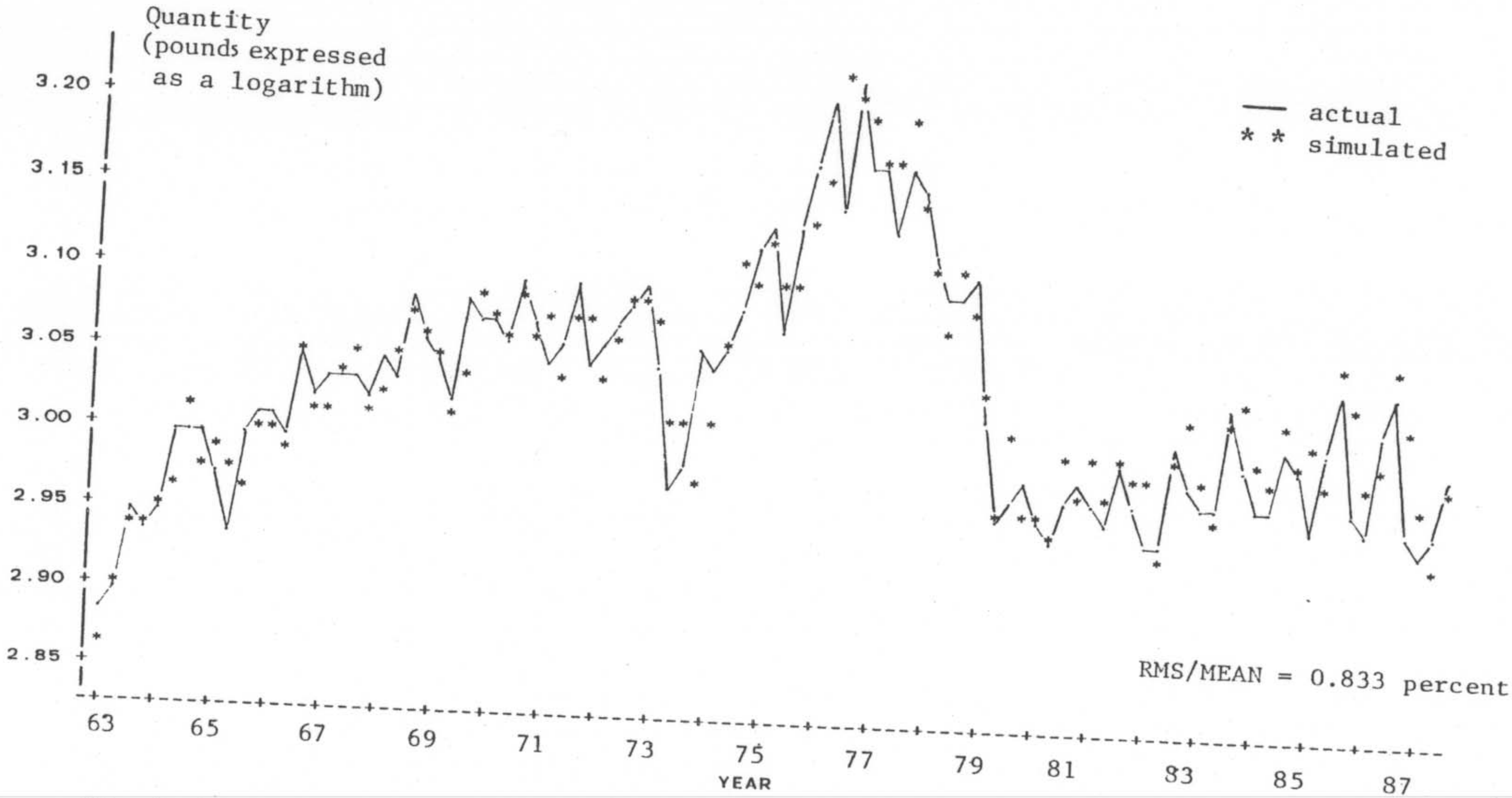

Figure 1. Per capita beef consumption by quarter, 1963 - 1987

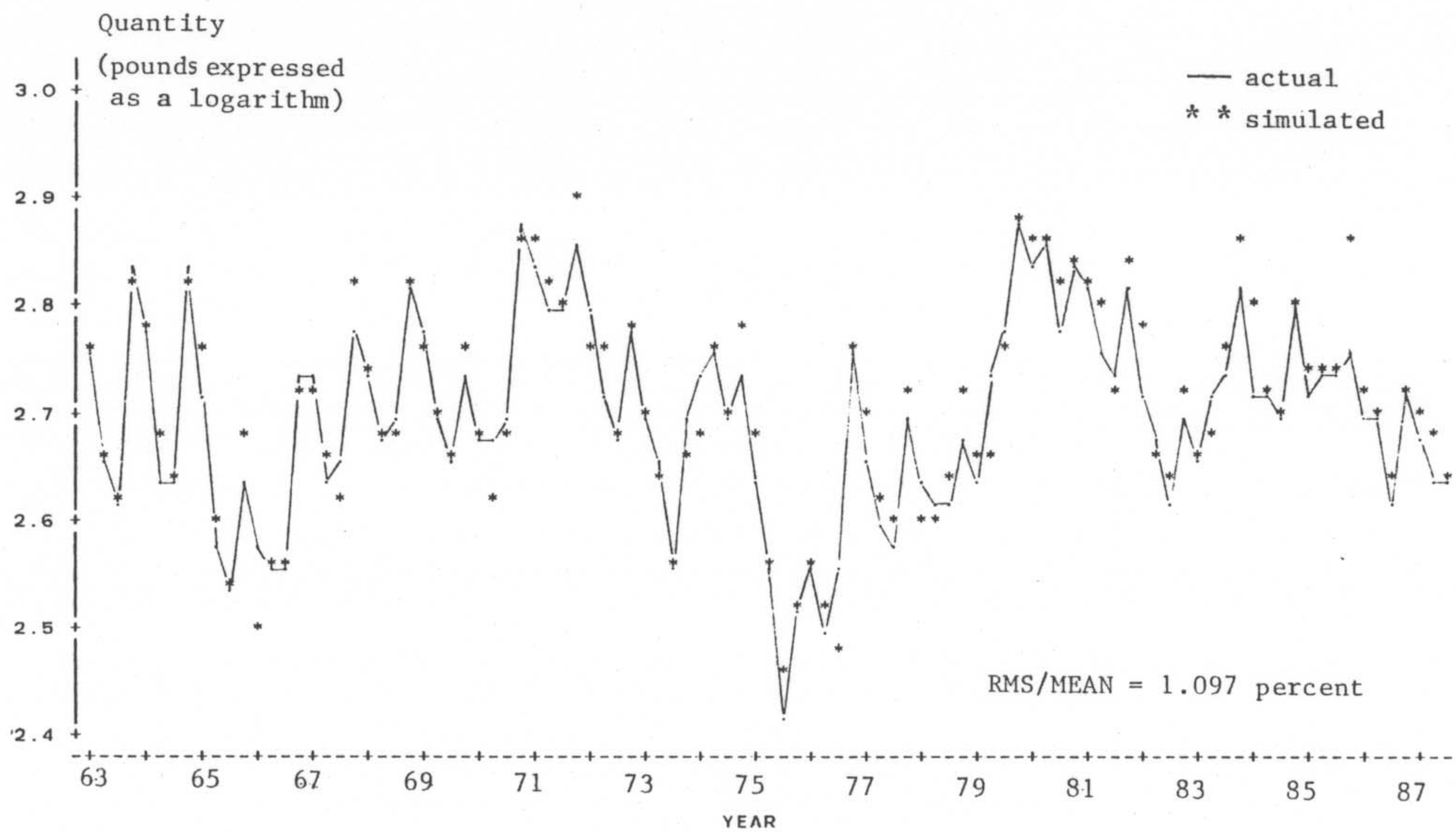

Figure 2. Per capita pork consumption by quarter, 1963 - 1987

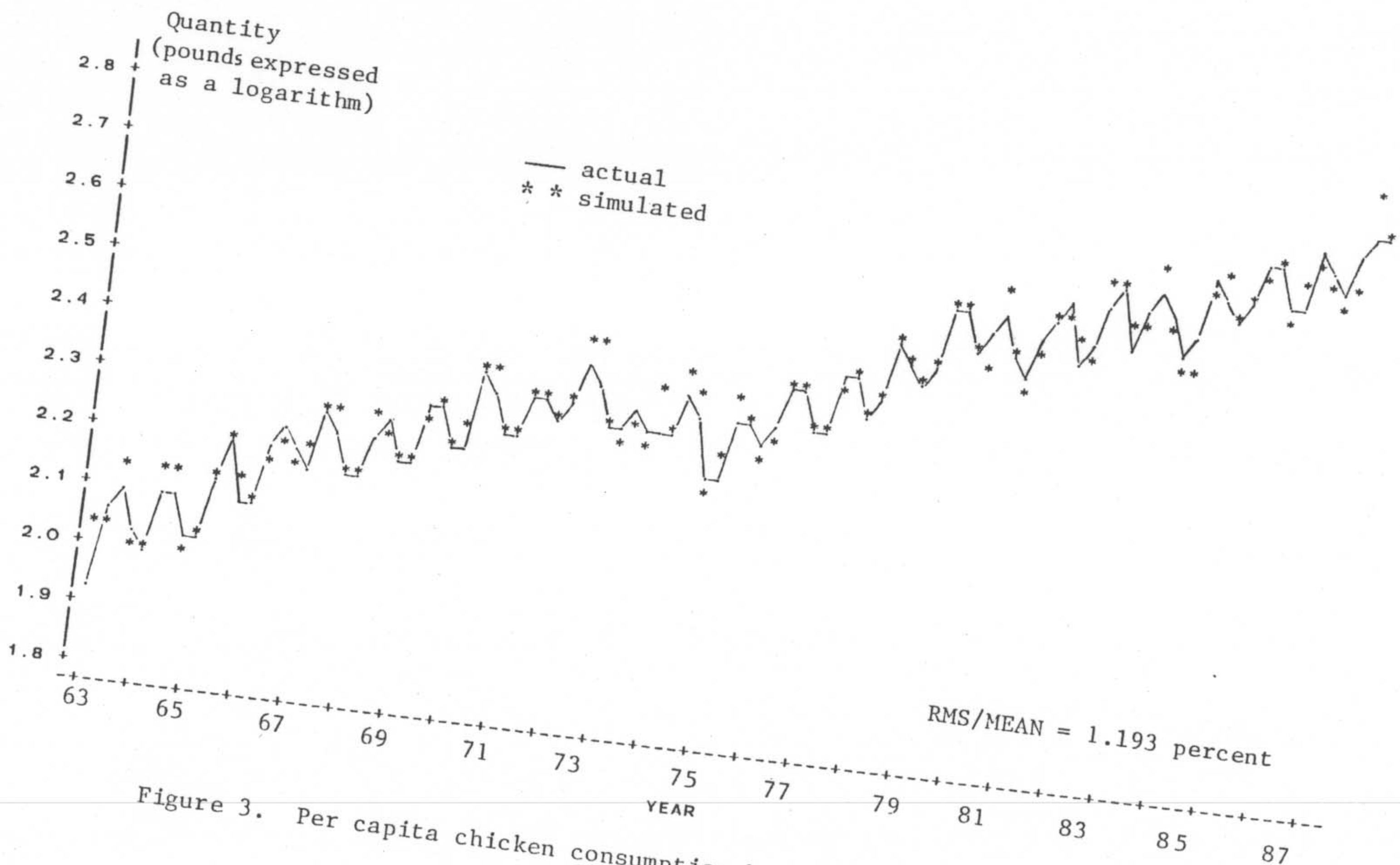

Figure 3. Per capita chicken consumption by quarter, 1963 - 1987

(chicken). These estimated demand elasticities, however, are not theoretically consistent, and are obviously deviated from the elasticities used in the constrained case.

IV. CONCLUSIONS

A mixed structure-time series model for quarterly demand for meats has been developed in this study by incorporating the demand elasticities obtained from a theoretically consistent demand system. This model is particularly useful for meat commodity analysts who like to incorporate the interdependent nature of demand for foods into the forecasting framework, but have difficulty in estimating a complete demand system directly.

In addition, the model developed in this study has provided an efficient forecasting instrument for quantity demanded for meats. The forecasting performance of the model is significantly better than that of using the structural model alone. On the basis of errors being about 1 percent in the simulated per capita quantity demanded over the sample period, the model can serve as a useful analytical tool for meat commodity analysts monitoring the demand for meats in the short run. In fact, the methodology of mixed structure-time series modeling can be extended to a wide variety of commodity forecasting problems.

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APPENDIX

Estimation Results for the Unconstrained Case of the Model

This study also estimates the unconstrained case of the mixed structure-time series model for comparison. The demand parameters in this model are directly estimated from equation (1) without imposing any parametric constraints. An initial ordinary least square fitting of the demand equations shows that high serial correlation is likely in the error terms. Their estimated Durbin-Watson values are 0.35 (beef), 1.02 (pork), and 1.02 (chicken). These estimates imply that an autoregressive procedure is necessary to improve the statistical efficiency of structural estimates.

Following a similar procedure used in the constrained case, the number of lags in equation (2) is determined by the magnitude of autocorrelations from the estimated residual series in the initial model fitting. Again, this study choses the lags on the basis of estimated autocorrelations about 0.5 or higher. Accordingly, two lags are assigned for beef equation, and one lag for pork and chicken equations. The final estimation results for the mixed structure-time series models are compiled in the appendix table.

In the table, in addition to the estimated cross-price elasticities, the estimated direct-price elasticities are -0.3421 (beef), -0.7204 (pork), and -0.3998 (chicken). Their estimated expenditure elasticities are 0.1389 (beef), 0.0640 (pork), and 1.0430 (chicken). The estimated seasonal pattern listed in the middle section of the table is similar to that in the constrained case.

The adjusted coefficients of determination, adj.R^2, are presented at the bottom of the table. One is related to the initial structural model, and the other is related to the mixed structure-time series model. The forecasting performance of beef has significantly improved by using the mixed structure-time series model; its adj.R^2 value increases from 0.5403 for the initial structure model to 0.8837. Slight improvements in the forecasting performance are found for pork and chicken.

An ex post simulation is conducted over the sample period covering the first quarter of 1963 to the third quarter of 1987. The error measurements, which are defined as the ratio between the the root-mean-square error and the sample mean of quantity demanded over the simulated period expressed in percentage, are: 0.907 (beef), 1.154 (pork) and 1.252 (chicken).

Appendix table -- Estimation Results for Unconstrained Cases

Independent variable	Beef	Pork	Chicken
Price:			
Beef	-0.3421	0.5773	0.0578
	(.1038)	(.0768)	(.0793)
Pork	.2670	-.7204	-.0016
	(.0786)	(.0687)	(.0712)
Chicken	-.0810	.0141	-.3998
	(.0636)	(.0664)	(.0691)
Expenditure:	.1389	.0640	1.0430
	(.0986)	(.0727)	(.0754)
Seasonal dummy:			
Spring	-.0026	-.0651	.0821
	(.0055)	(.0073)	(.0077)
Summer	.0221	-.0585	.0842
	(.0064)	(.0083)	(.0087)
Fall	.0047	.0616	-.0077
	(.0056)	(.0076)	(.0079)
Intercept	2.6289	2.7781	-4.4022
Residual:			
Lag 1	.8664	.4902	.4827
Lag 2	-.0709		
Adjusted R-square:			
Structural model	0.5403	0.8214	0.9675
Mixed model	.8837	.8652	.9758
RMS/MEAN (percent)	.9077	1.1540	1.2518

Note: The figures in parentheses are the estimated standard errors.