Forecasting Meat Prices Through an Inverse Demand System

by

Kuo S. Huang

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I. INTRODUCTION

The objective of this study is to develop a framework for use in forecasting meat prices. Red meats and poultry have generated the most important source of farmers' incomes. The cash receipts from farm marketing for cattles and calf, hog, sheep and lamb, and poultry and eggs were $53.8 billion in 1986, about 40 percent of total farmers' cash receipts from livestock and crops (USDA). On the other hand, consumers spent $81.8 billion (about 26 percent of total expenditures for food consumed at home) on red meats and poultry consumed at home in 1987 (USDC). Thus any change in meat prices may have significant impacts on both producers and consumers. To assess these impacts, an adequate forecasting model for meat prices is required.

To formulate a model for forecasting meat prices, a partial demand approach, in which a particular meat price is a function mainly of the meat quantity demanded and income, has commonly been used. The major drawback of this approach is its neglect of the possible interdependent nature of demand between meat and other commodities that are included in consumers' budgeting. To improve the partial demand approach, this study uses a complete inverse demand system developed by Huang as a framework for forecasting meat prices.

In this paper, to familiarize you with the Huang's inverse demand system, a brief description about the rationale of its model specification is given at the beginning. It is followed by an application of the demand system to forecasting meat prices.

II. AN INVERSE DEMAND SYSTEM

To understand the forecasting model used in this study, it would be helpful to summarize here some basic results of an inverse demand system developed in Huang. Let q denote an n-coordinate column vector of quantities demanded for a "representive" consumer, p an n-coordinate vector of the corresponding prices, $m = p'q$ the consumer's expenditure, and U(q) the utility function, assumed nondecreasing and quasi-concave in q. By applying the Hotelling-Wold identity, one can obtain an inverse demand system, in which prices are functions of quantities demanded and income, from a differentiable direct utility function as:

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$$r_i = U_i(q) / \sum_{j=1}^{n} q_j \, U_j(q) \qquad i=1,2,..,n \qquad (1)$$

in which $U_i(q)$ is the marginal utility of the ith commodity, and $r_i = p_i/m$ is the normalized price of the ith commodity, and the budget constraint is $r'q = 1$ for an n-coordinate vector of the normalized prices r. All income flexibilities are implicitly constrained to unitary values on the basis of an inverse demand system.

This inverse demand system is as important as the quantity dependent system for food demand analysis. Agricultural economists (ie, Houck, Waugh and Fox) have long recognized that lags between farmers' decisions on production and commodities marketed may predetermine quantities with price adjustments providing the market clearing mechanism. Consequently, quantities rather than prices are appropriate instrumental or control variables in the analysis of many types of agricultural policies and problems. For example, the production of corn and soybeans in the United States has been affected by the drought recently, especially in the eastern Corn Belt. Much attention has been focused on the drought that has threatened feed crops and livestock production, and may have caused substantial impacts on meat prices.

To explore the nature of the inverse demand system, one may illustrate the price effects of a marginal change in the quantity demanded as follows. Following Anderson's suggestion, one may regard the "scale slope" of quantities demanded as playing the same role as the income slope in ordinary demand. Consider figure 1 for two commodities, X and Y. Income and the quantity of X are fixed, but the quantity of Y is variable. The primary interest is to evaluate the response of commodity prices to the quantity change of Y.

For an equilibrium quantity vector, q_a, which consists of OC of X and OA of Y, the consumer attains a utility indifference curve U_a, and pays a normalized price vector, say r_a, which is obtained from a tangent line L_a of U_a at q_a for satisfying $r_a'q_a = 1$. For a marginal increase in Y from OA to OB, the equilibrium shifts from q_a to q_b, and a higher utility indifference curve, U_b, is attained. Drawing a tangent line L_b of U_b at q_b indicates that the consumer will pay a new normalized price vector, say r_b, for satisfying $r_b'q_b = 1$.

Furthermore, by connecting a ray to q_b from the origin, one finds that the ray intersects with the initial utility indifference curve U_a at q_c, which is a quantity vector scaled down proportionally to q_b. At the quantity vector q_c, the consumer will pay a normalized price vector, say r_c, which is obtained from the tangent line L_c of U_a at q_c for satisfying $r_c'q_c = 1$.

The movement of quantity vectors from q_a to q_b is equivalent to moving from q_a to q_c along the initial utility indifference curve U_a, and then moving from q_c to q_b along the change of scale in quantity demanded. Thus, an increase in the quantity of a commodity affects the quantity demanded of that commodity in two ways: (a) it induces the consumer to substitute the commodity for other commodities, and (b) it gives the consumer a higher

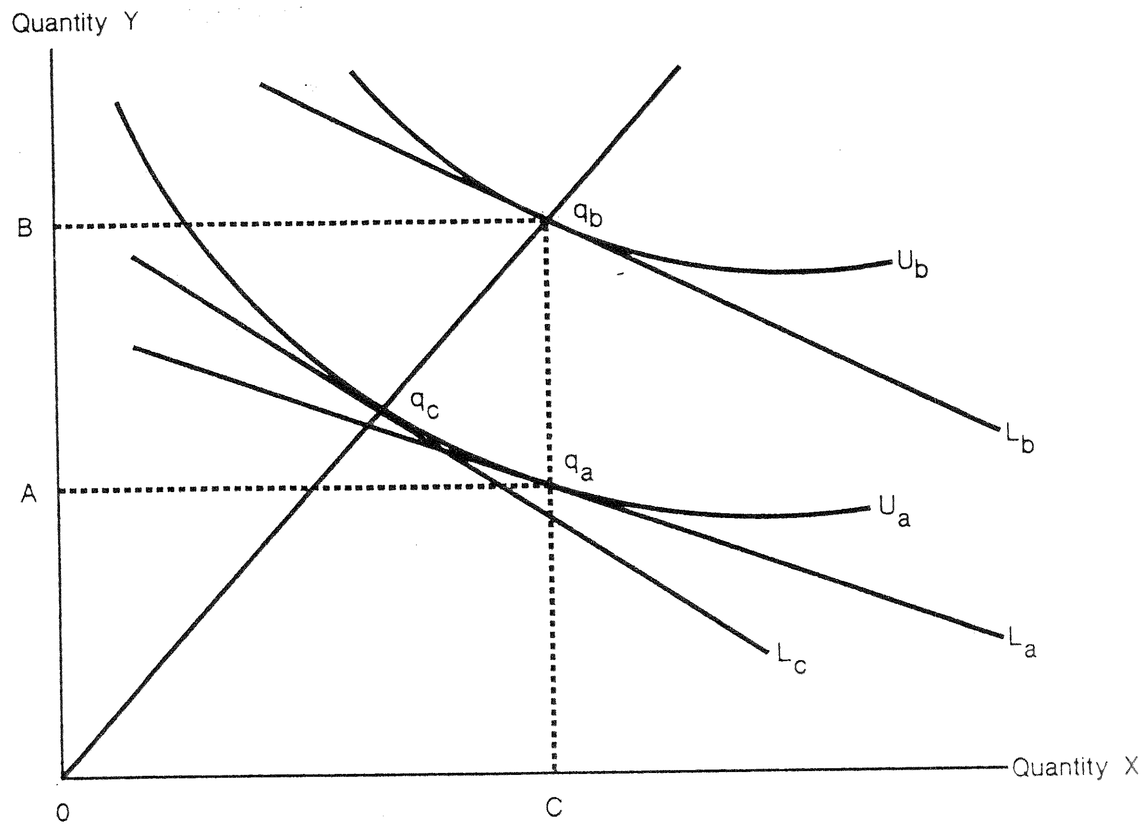

Figure 1. Decomposition of price effects for quantity changes

level of satisfaction, as indicated by an increase in the scale of quantity demanded.

The consumer can be induced to make such movements at different equilibrium levels of quantities demanded by paying different prices. The total change in the price vector (r_b - r_a) in response to a marginal increase of quantity Y can be divided into two parts. One is a scale effect, which shows the price difference (r_b - r_c) in response to a proportional change of quantity vectors from q_c to q_b. The other is the Antonelli substitution effect, which is reflected by the price difference (r_c - r_a) in response to a substitution of more quantity of Y for X, given that the consumer stays on the same indifference curve. The Antonelli effect is the counterpart of the Slutsky substitution effect in the ordinary demand case.

To incorporate these comparative static properties into an inverse demand system, one may redefine the quantity variable in the demand system (1) as $q = s\, q^*$. The variable s is the factor of proportionality between q and q*, where q* is defined by the intersection of a line from the origin through q and the base period utility curve. Accordingly, the inverse demand system is a function of the scale variable s and the reference vector in commodity space q* as

$$r_i = f_i(s\, q^*) \qquad i = 1,2,\ldots,n \qquad (2)$$

in which the price effect in response to a change in the reference quantity should be interpreted as the compensated price effect at the base period utility level.

In specifying a functional form of the inverse demand system (2), one may direct approximation of the conceptual demand relationships without imposing any rigid assumptions on the form of utility structure. Given a demand structure consisting of n commodities, according to (2), a complete inverse demand system can be defined as a set of linear equations with n(n+1) demand parameters:

$$\begin{aligned} r_1' &= f_{11}^*\, q_1' + f_{12}^*\, q_2' + \ldots. + f_{1n}^*\, q_n' + g_1\, s' \\ &\cdot \\ &\cdot \\ &\cdot \\ r_n' &= f_{n1}^*\, q_1' + f_{n2}^*\, q_2' + \ldots. + f_{nn}^*\, q_n' + g_n\, s' \end{aligned} \qquad (3)$$

where r_i' is relative change in the normalized price of the ith commodity; q_i' relative change in the reference quantity of the ith commodity; s' is relative change in the scale of quantity demanded; f_{ij}^* is compensated price flexibility of the ith commodity with respect to a quantity change in the jth commodity; and g_i is scale flexibility of the ith commodity.

Based on the properties of an inverse demand system as discussed in figure 1, one may develop the interdependent relationships among the demand parameters in equation (3). Some linear theoretical constraints among the compensated flexibilities of an inverse demand system obtained by Anderson are as follows:

Scale aggregation: $$\sum_{i=1}^{n} w_i\, g_i = -1 \qquad (4)$$

Homogeneity: $$\sum_{j=1}^{n} f_{ij}^* = 0 \qquad i=1,2,\ldots,n \qquad (5)$$

Symmetry: $$f_{ji}^*/w_i = f_{ij}^*/w_j \qquad i,j = 1,2,\ldots,n \qquad (6)$$

where $w_i = p_i q_i/m$ is the expenditure weight of the ith commodity. These parametric restrictions across demand equations are useful working hypotheses for studying the consumer's behavior and are desirable as part of the empirical estimation process. Besides, the uncompensated price flexibilities (say f_{ij}'s) can be derived from the compensated price flexibilities by using the following identity:

$$f_{ij} = f_{ij}^* + g_i\, w_j \qquad i,j = 1,2,\ldots,n. \qquad (7)$$

An empirical inverse demand system for 13 aggregate food categories and a nonfood sector for 1947 through 1983 was estimated and published in Huang. The demand system is specified on the basis of equation (3), while the parametric contraints defined in equations (4) to (6) were incorporated into estimation simultaneously. A constrained maximum likelihood method was used.

In the demand system, the food categories are: (1) beef and veal, (2) pork and other red meats, (3) poultry, (4) fish, (5) eggs, (6) dairy products, (7) fats and oils, (8) fresh fruits, (9) fresh vegetables, (10) processed fruits and vegetables, (11) cereal and bakery products, (12) sugar and sweeteners, and (13) nonalcoholic beverages. Our attention here, however, is given to the part of the demand system related to the demand for beef, pork, and poultry.

For fitting the demand system, the price index is obtained from The Consumer Price Index (USDL). Per capita consumption expenditure is calculated from total consumption expenditure, obtained from Personal Consumption Expenditures, divided by the civilian population of 50 States at July 1 of each year (USDC). The quantity index for the nonfood sector is calculated from the current value of the nonfood per capita expenditure divided by the consumer price index of all items less food.

The quantity data and expenditure weights for foods are compiled from Food Consumption, Prices, and Expenditures (USDA-ERS). The expenditure weights are calculated as the average of those weights available in the following two periods: 1957-59 and 1967-69. The indexes for composite food categories are calculated by the Laspeyres indexes procedure for individual foods in a specific category with base year 1967. The composite food and nonfood quantity indexes are further aggregated into a general Laspeyres quantity index to represent a scale variable in the inverse demand system. This quantity index is then used to deflate the quantity index of each category and thus obtain the corresponding reference quantity in the base period.

III. APPLICATION TO MEAT PRICE FORECASTING

In this section, major focus is on the evaluation of the potential analytic and forecasting capability of an inverse demand system contained in Huang for use in meat price analysis. The estimated inverse demand system serves at least two major functions: (a) to provide a quantitative representation of the economic structure of food demands, and (b) to provide a quantitative model for forecasting and analyzing food consumption behavior. Accordingly, the materials that follow in this section first present some analytical results of estimated demand price flexibilities for meats and poultry, and then evaluate the forecasting performance for these prices.

Table 1 contains some estimated demand information for meats and poultry. The diagonal entries of the table are compensated direct-price flexibilities. They show how much commodity price must change to induce the consumer to absorb marginally more of that commodity while maintaining the same utility level in the base period. The compensated direct-price flexibility for beef of -1.082 indicates that a 10 percent increase in the quantity of beef and veal would require a price decrease of about 11 percent, holding constant the quantities of all other commodities. Other direct-price flexibilities for pork and poultry are -1.222 and -1.059 respectively.

Table 1 -- Estimated compensated flexibilities

Dependent variable	Independent variables -- quantities				
	Beef	Pork	Poultry	Others	Scale
Prices:					
Beef	-1.082	-0.052	-0.135	. . .	-1.496
	(.053)	(.041)	(.019)		(.222)
Pork	-.066	-1.222	-.102	. . .	-1.269
	(.051)	(.070)	(.027)		(.296)
Poultry	-.468	-.279	-1.059	. . .	-1.963
	(.067)	(.074)	(.067)		(.401)
Expenditure share (percent)	2.98	2.34	0.86		

Source: Compiled from Huang. The figures in parentheses are the estimated standard errors.

The estimated compensated cross-price flexibilities are shown in the off-diagonal entries of table 1. These flexibilities should be interpreted with caution. A marginal increase of the quantity of one good may have a substitution effect on the other goods, and the price of other goods should be lower to induce consumers to purchase the same quantity of the other goods. Accordingly, the cross-price flexibilities in the table reflect substitution if the sign is negative and complementarity if the sign is positive. For example, the compensated cross-price flexibility between the price of beef and the quantity of poultry is -0.135 which implies that the two commodity categories are substitutes. A marginal 10 percent increase in the quantity of poultry is associated with a 1.35 percent decrease in the price of beef to induce consumers to purchase the same quantity of beef. As expected, the compensated cross-price flexibilities show that there is substitution between red meats and poultry.

The estimated scale flexibilities in the last column of table 1 show the potential response of commodity price to a proportionate increase in all commodities. For example, the scale flexibility for beef is -1.496, which indicates that a proportionate increase in all commodities by 10 percent would decrease the price of this meat category by about 15 percent. All the estimated scale flexibilities for beef, pork, and poultry are negative as expected, with magnitudes larger than 1 in absolute value.

In practical application, the essential role of estimated scale flexibilities is to relate compensated to uncompensated price flexibilities on the basis of equation (7). The derived uncompensated direct-price flexibilities for red meats and poultry are summarized in table 2. As expected, the uncompensated direct-price flexibilities are larger in absolute value than those of compensated flexibilities. They are beef (-1.126), pork (-1.252), and poultry (-1.076).

Table 2 -- Derived uncompensated flexibilities

Dependent variable	Independent variables -- quantities		
	Beef	Pork	Poultry
Prices:			
Beef	-1.126	-0.087	-0.148
Pork	-.103	-1.252	-.113
Poultry	-.526	-.325	-1.076

Source: Computed on the basis of compensated flexibilities in table 1.

Now we focus on the evaluation of the potential forecasting capability of the demand system. Recall that the demand system for n commodities can be represented by

$$\underset{(n\times 1)}{r_t'} = \underset{(n\times n)}{F_q}\,\underset{(n\times 1)}{q_t'} + \underset{(n\times 1)}{g}\,\underset{(1\times 1)}{s_t'}$$

where r_t', q_t' and s_t' are relative changes in normalized prices, reference quantities, and the scale of quantity demanded at year t, respectively; F_q is an n x n compensated price flexibility matrix, and g is a vector of scale flexibilities. The implementation of this demand system is rather straightforward. For conducting forecasting, one may update the information on relative changes in quantities and forecast the normalized prices.

The immediate forecasting results from the model are in terms of relative changes in normalized prices. The projected relative changes, however, can easily be transformed into price levels, say, a vector of r_t, on the basis of normalized price level available in the previous year, r_{t-1} as follows:

$$\underset{(n\times 1)}{r_t} = (\underset{(n\times n)}{I} + \underset{(n\times 1)}{D_r})\,\underset{(n\times 1)}{r_{t-1}}$$

where D_r is a diagonal matrix with the elements of the projected vector of the relative changes in normalized price in the diagonal, and I is a unit matrix. In case of an ex ante forecast when the lagged price level is unknown, the projected price in the previous year should be substituted. Finally, one may transform forecasted normalized price into the actual price level by simply multiplying the index of per capita consumption expenditure.

To evaluate the forecasting performance of the model, an ex post simulation is conducted here for comparing the difference between actual and simulated values over the sample period. Two types of RMS (root-mean-square) simulation errors are used to measure the average forecasting performance over the period. In case of relative changes in normalized price, the RMS simulation error defined below can be used. That is

$$\text{RMS error} = [\sum_{t=1}^{T} (r_t'' - r_t')^2 / T]^{1/2} \times 100$$

where r_t'' and r_t' are simulated and actual values of normalized price changes, respectively.

On the other hand, to measure the average performance of normalized price and actual price levels, the RMS percent error can be used. The error is defined as follow:

$$\text{RMS percent error} = [\sum_{t=1}^{T} (y_t'' - y_t')^2 / T]^{1/2} / y^* \times 100$$

where y_t'', y_t' and y^* are simulated, actual values and sample mean of a concerned price index, respectively.

Table 2 summarizes the forecasting performance for three types of simulated meat prices: normalized price changes, normalized price index, and price index. The simulation covers 1954-1987, with available updated observations for 4 years beyond the sample period in the estimation of the demand system. In the table, the average simulated errors for normalized price changes are 4.09 percent for beef, 4.67 percent for pork, and 4.55 percent for poultry. The simulated errors for normalized price index are in a range between 3.81 and 4.19 percent. The simulated errors for price index with a range between 5.02 and 5.70 percent are slightly larger than those for normalized price changes.

Table 3 -- Forecasting performance over the period 1948-87

Simulations	Root-mean-square errors		
	Beef	Pork	Poultry
		- - Percent - -	
Normalized price changes	4.09	4.67	4.55
Normalized price index	4.19	4.94	3.81
Price index	5.70	5.30	5.02

Source: Computed.

In general, the forecasting performance measured in terms of the root-mean-square errors is in a range between 3.81 and 5.70 percent for all cases. Despite the various transformations required for obtaining the price index for meats, their errors are quite close to those of normalized price changes which are simulated directly from the demand system. Graphic presentation of the actual and simulated results is presented in the appendix figures. These figures provide a better intuitive feel of forecasting performance and help to ascertain the consistency with the error measurements obtained in table 3.

IV. CONCLUSIONS

An inverse demand system developed by Huang has been used for forecasting meat prices. The demand system explores the interdependent nature of demand for foods and provides some analytical information regarding the price variations in response to quantity changes for 13 food categories and one nonfood sector.

The inverse demand system can be applied straightforwardly for forecasting the prices of meats and other food commodities. An ex post simulation for the price changes of beef, pork, and poultry indicates that the simulated errors are less than 6 percent in all cases. Accordingly, one may conclude that the inverse demand system is a useful model for forecasting meat prices. Furthermore, one may define some potential changes for quantity supplies of foods under various scenarios, and simulate the program effects of controlling market supplies on meat prices.

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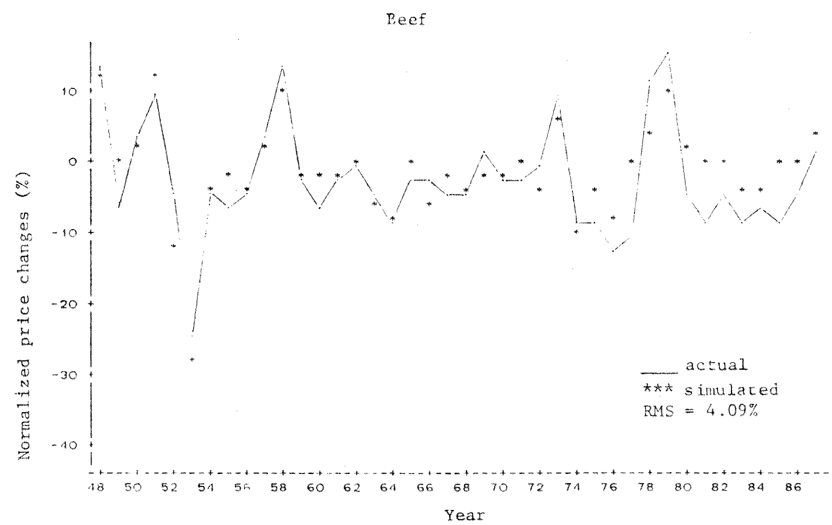
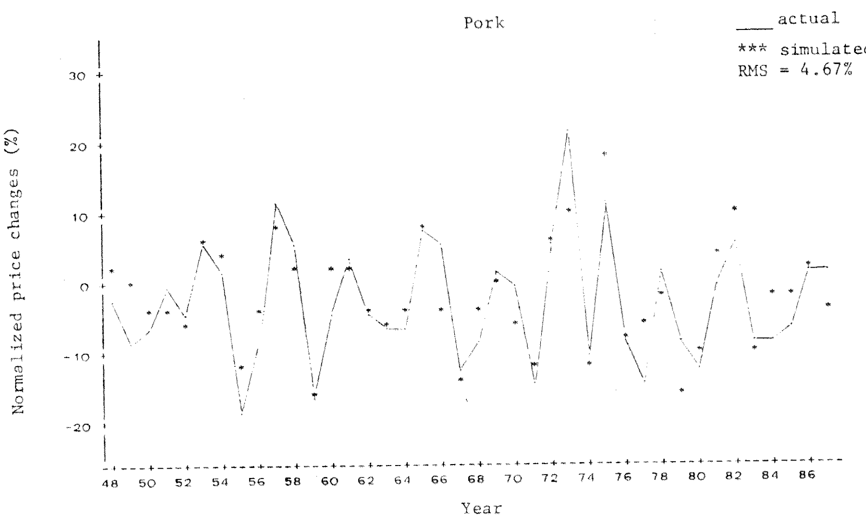
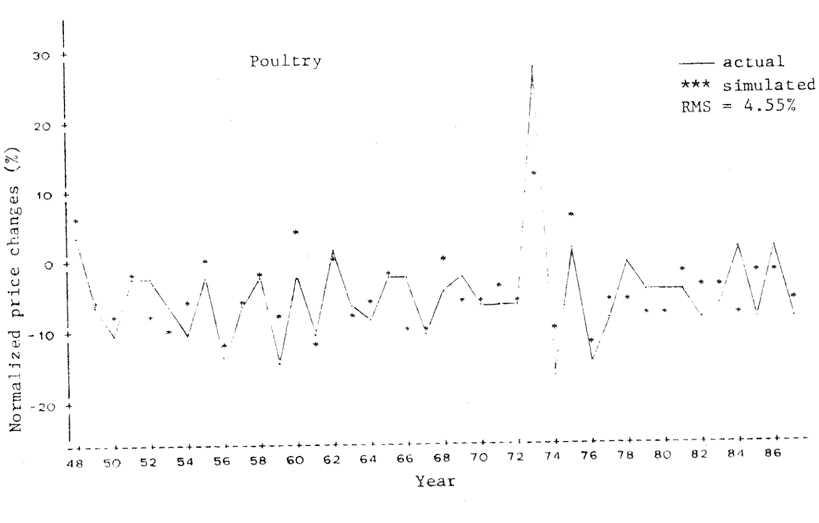
Appendix figures:

Graphic comparison of actual and simulated price changes

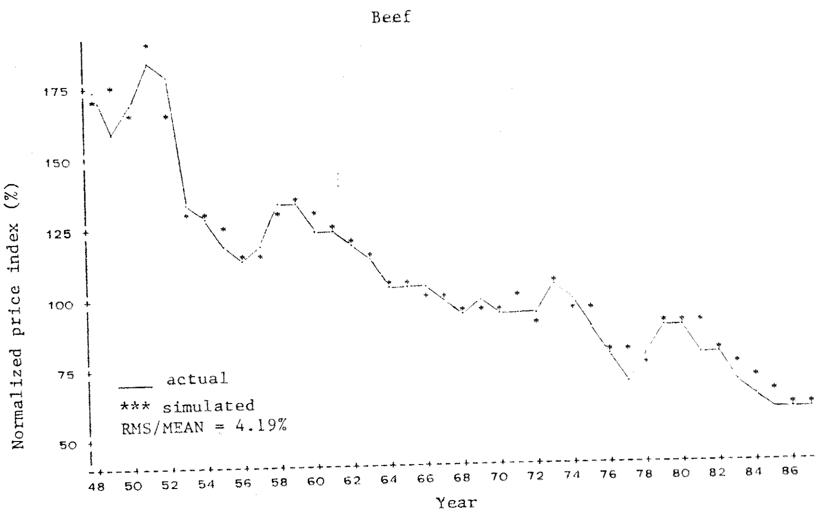


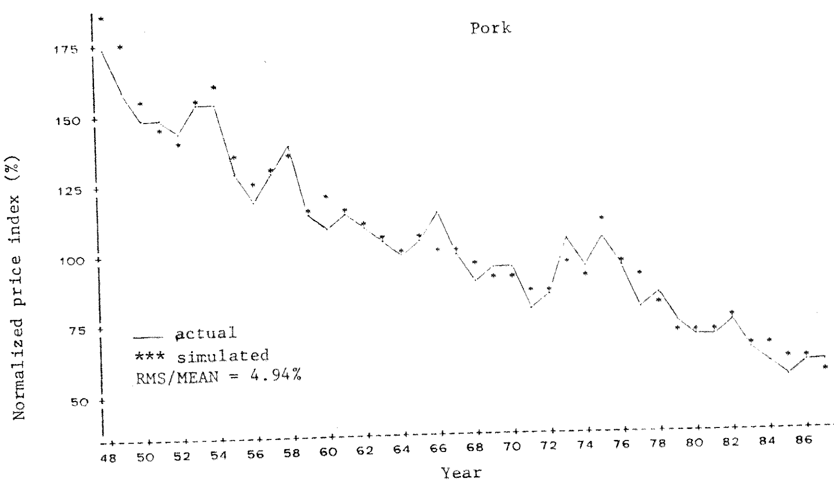


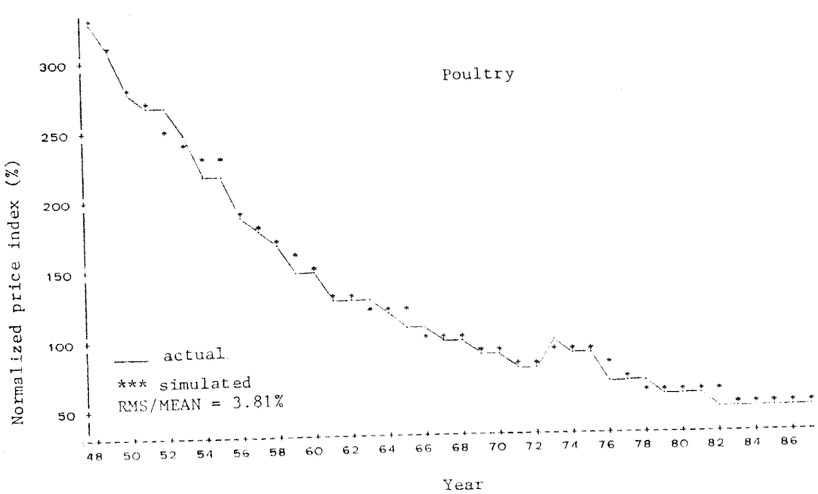


Appendix figures:

Graphic comparison of actual and simulated prices (continued)







Appendix figures:

Graphic comparison of actual and simulated prices (continued)