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Demand System Estimation with Upward Sloping Supply

Thomas I. Wahl and Dermot J. Hayes*

The theory underlying demand systems estimation was developed for individual consumers facing set prices. When economists began to apply these systems at the aggregate level, they realized that many of the assumptions underlying the theory were violated. To date, most of the work done on this "aggregation problem" has focused on the conditions under which the individual's cost or the indirect utility function allowed for aggregation across populations with different income levels.

These early authors understood that changes in aggregated demand would cause price increases, but apart from pointing out this obvious fact, little else was done. This neglect was justified. Many of the products included in early demand systems were broad aggregates of manufactured goods from which data were collected at the retail level on an annual basis. Under these conditions, it seems reasonable to assume that the supply curve is perfectly elastic. Within a year, the producers of manufactured products can respond to demand increases by adding shifts or by increasing the speed of current production. Any demand-induced price changes that occur at the wholesale level will be absorbed by retailers. Also, the aggregate price indices used to represent the prices of a bundle of commodities tend to muffle the impact of individual price changes.

Work has now progressed to the point at which demand systems are as convenient to estimate and as familiar as the ad hoc models used by agricultural economists in the past. Consequently, we now see demand system estimates of subcomponents of the food group. An example might be an annual model of the Japanese meat demand system using retail disappearance and prices. It is obvious that the supply curve in the preceding example should not be treated as perfectly elastic, yet this assumption is invariably used in the literature where meat demand systems have been estimated. Strangely, it is much more difficult to find models with single-equation ad hoc demand equations that ignore supply. The vast literature on the estimation of simultaneous equations systems is evidence of the importance of this topic in the past. It seems possible that the applications of the new demand systems have overlooked this obvious problem. Yet while it exists, the parameter estimates from demand systems may exhibit simultaneous equations bias, thereby reducing their effectiveness in policy analysis.

The purpose of this paper is to compare estimates from a demand system in which supply is assumed to be perfectly elastic with estimates from a system that allows for an upward sloping supply curve. A linear approximate Almost Ideal Demand System (LA/AIDS) of Japanese meat expenditures is used as an example.**

First, previous work and the properties of the LA/AIDS model are reviewed. Then estimation procedures for both the perfectly elastic and the upward sloping supply curve models are discussed. The methodology to test for endogenous prices is then presented, and the results of both models are compared. And finally, the important results are summarized.

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**The Japanese information is used because it is well behaved in that it satisfies the theoretical restrictions and because it is readily available (Hayes, Wahl, and Williams).

Previous Work

Deaton and Muellbauer (1980) derive the LA/AIDS from the PIGLOG class of preferences, which permits perfect aggregation over consumers and results in representative demand functions that are correctly estimated by using market-level data. However, the perfectly elastic supply assumption is maintained (Deaton and Muellbauer 1980, 313). Attfield (1985) examines the endogeneity of total expenditures using Deaton and Muellbauer's AIDS specification and data. Attfield finds that the rejection of homogeneity may be reinterpreted as the rejection of exogeneity of expenditures when homogeneity is assumed to be part of the maintained hypothesis. Chalfant (1987), after estimating a globally flexible version of the LA/AIDS for meat demand in the United States, adds the caveat that the endogeneity of prices may be remedied by using Three-Stage Least Squares (3SLS) if a simultaneous equations bias is suspected. Phlips (1983) states that the observed prices are the result of the equalization of supply and demand and that the data for estimating demand functions have been simultaneously affected by supply. Phlips further states that a simultaneous model should be constructed but then justifies not pursuing the problem further because, in practice, simultaneity is ignored.

Thurman (1986) uses the Wu-Hausman model specification test to examine the endogeneity of all prices, as well as only chicken price, in a single-equation ad hoc demand equation for poultry meat by using ordinary least squares (OLS) and an instrumental variables (IV) estimator. Chalfant and Wallace (1988) estimate the Wu-Hausman test statistic by using Iterated Seemingly Unrelated Regressions (ITSUR) and Iterated Three-Stage Least Squares (IT3SLS) to examine the endogeneity of output in a cost-minimization problem for the trucking industry.

The LA/AIDS Model

Deaton and Muellbauer's AIDS is a first-order approximation to any demand system, aggregates perfectly over consumers, and allows symmetry and homogeneity to be easily imposed and tested. The estimated share equations of the LA/AIDS are of the form

$$w_i = a_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (X/P), \tag{1}$$

where w is the share, p is price, X is total expenditures on the group, and P is Stone's price index. Homogeneity, Slutsky symmetry, and adding up can be imposed on this system by restricting the parameters of the system.

The system in (1) can be estimated by using a systems estimator. However, because the shares sum to one, the equations form a singular system, and therefore, one of the equations must be deleted from the system for estimation. However, for the system to be invariant to the equation deleted, initial restrictions must be imposed upon the covariance matrix. Berndt and Savin (1875) demonstrate that use of the same autocorrelation correction for each of the estimated equations makes the system invariant to the equation deleted. ITSUR or IT3SLS, which is an IV estimator, can be used to estimate the system.

The "best" choice of instruments for 3SLS estimation is not known. The literature on the choice of instruments emphasizes the exogeneity of the instruments. Fomby, Hill, and Johnson (1984) suggest that the instruments chosen should be uncorrelated with the error term but at the same time should have the highest correlation with the elements of the right-hand side (RHS) variables matrix. For the instruments to be uncorrelated with the error terms, the instruments must be exogenous with respect to the endogenous RHS variables. For agricultural demand systems, this implies that the instruments must be exogenous to the supply process. However, there are an infinite number of combinations of exogenous supply-side variables, and the choice introduces an element of variability in the results even with the same model and data (Gallant 1977). Chalfant uses low-order interaction terms of the exogenous spatial characteristics of the firm's operating authority to create the instruments to estimate the translog cost function of trucking firms. Thurman used feed price, the price of labor, energy input prices, and feed efficiency as instruments to estimate an ad hoc poultry meat demand equation.

Although the potential choice of instruments is limited only by the number of variables available, sets of variables that are not affected by the supply determination process, such as population, highly aggregated price indices, or per capita gross income measures, seem to be reasonable choices for instruments. The first-stage R^2 (FSR^2) statistics are suggested as an indication of the quality of the instruments. The FSR^2 statistic measures the fraction of the variation of the endogenous RHS variable that remains after projection through the instruments (SAS Institute, Inc.). If the FSR^2 is almost one, the instruments are almost perfectly correlated with the endogenous RHS variables, and little is gained from the use of the instruments. However, if the FSR^2 only approaches one, the use of the instruments can account for simultaneous equations bias.

Following (1), an LA/AIDS for Japanese meat expenditures is specified. The five meats in the system are Wagyu beef, import-quality beef (domestic dairy beef and imported beef), pork, chicken meat, and fish. For singularity reasons already mentioned, the fish equation is deleted from the estimation. The system is corrected for autocorrelation by using the procedure suggested by Berndt and Savin. The system is estimated by using both ITSUR and a two-step procedure that is equivalent to IT3SLS. In the first stage, the endogenous RHS variables are regressed upon the chosen instruments. The predicted values from these regressions are then substituted for the actual variables in an ITSUR estimation of the transformed system. Homogeneity is imposed upon the own-price term in both models. Symmetry is tested and accepted at the 95 percent level by both estimation techniques. Table 1 presents the estimated coefficients and standard errors for the AIDS system estimations using ITSUR and IT3SLS for Japanese meat expenditures. The five meat prices are treated as endogenous RHS variables for the IT3SLS estimation. The FSR^2's are presented at the bottom of the table. Total meat expenditures and Stone's index of meat prices are assumed to be exogenous. The instruments for the 3SLS estimation are created from variables that are exogenous to the supply of each meat. These include the producer price index, the price of corn, the consumer price index, population, average annual family income, and livestock birthrates by type.

The estimated elasticities and associated standard errors are presented in Table 2. As expected, all the own-price elasticities are negative for either estimation technique. The Hicksian cross-price elasticities indicate complementarity between chicken and import-quality beef and between pork and chicken. These results are consistent with previous estimations of Japanese meat demand systems.* The Marshallian own-price elasticities for each meat increase when prices are assumed to be endogenous. This result may indicate that models that ignore simultaneity of prices consistently underestimate the magnitude of own-price elasticities. However, the robustness of this result needs to be examined by using other data sets. The chicken meat expenditure elasticities seem to be the most sensitive to the estimation technique, increasing from 0.49 to 1.13. Using ITSUR, chicken meat would be interpreted to be a normal good. Using 3SLS estimation techniques, chicken meat would be interpreted to be a luxury good. The choice of variables to treat as endogenous seems to influence the resulting elasticities.

Hausman (1978) proposes a test for model specification on the basis of the existence of a consistent, asymptotically efficient estimator under the null hypothesis and an estimator that is not asymptotically efficient under the null hypothesis but that is consistent under both the null and the alternative hypotheses. Wu (1973) proposes a similar test. The Wu-Hausman test for model specification can be used to examine the null hypothesis that the prices on the RHS of (1) are exogenous. For systems estimation, ITSUR is consistent and asymptotically efficient under the null hypothesis. IT3SLS is consistent under both the null and the alternative but is inefficient under the null hypothesis. Using ITSUR and IT3SLS, the Wu-Hausman test measures the discrepancy between the sets of parameters, β and β^*, respectively, given a consistent estimator of the variance under the null. Defining q as $\beta-\beta^*$ and $V(q) = var(\beta) - var(\beta^*)$ as a consistent estimator of the variance of q, the test is of the form

$$M = q'\{V(q)\}^{-1}q, \qquad (2)$$

*See Hayes, Wahl, and Williams.

Table 1. Estimated Parameters and Standard Errors for ITSUR and IT3SLS Estimations of an AIDS of Japanese Meat Expenditure Shares[a]

Share	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	β_i	α_i	R^2	DW
ITSUR									
Wagyu Beef	-0.027	0.007 (0.014)	0.040 (0.016)	0.008 (0.012)	-0.029 (0.01)	0.017 (0.110)	0.126 (0.034)	0.849	1.619
IQ Beef[b]	0.007	0.026	0.002 (0.014)	-0.026 (0.010)	-0.010 (0.016)	0.057 (0.098)	0.216 (0.031)	0.912	2.124
Pork	0.040	0.002	0.003	-0.045 (0.014)	-0.001 (0.025)	-0.127 (0.160)	-0.262 (0.051)	0.691	2.061
Chicken Meat	0.008	-0.026	-0.045	-0.001	0.063 (0.016)	-0.058 (0.102)	-0.049 (0.033)	0.653	1.496
Fish	-0.029	-0.010	-0.001	0.063	-0.023	0.111	0.969		
IT3SLS[c]									
Wagyu Beef	-0.056	0.021 (0.017)	0.042 (0.020)	0.006 (0.016)	-0.015 (0.019)	0.015 (0.114)	0.146 (0.035)	0.844	2.191
IQ Beef	0.021	-0.013	0.020 (0.016)	-0.020 (0.013)	-0.007 (0.018)	-0.095 (0.099)	0.342 (0.031)	0.941	1.831
Pork	0.042	0.020	-0.080	-0.005 (0.019)	0.026 (0.032)	-0.190 (0.191)	-0.444 (0.061)	0.529	2.049
Chicken Meat	0.006	-0.020	-0.005	-0.001	0.107 (0.020)	0.153 (0.119)	0.161 (0.038)	0.442	2.090
Fish	-0.015	-0.007	0.026	0.178	-0.021	0.063	0.793		

[a] Standard errors are in parentheses. Symmetry and homogeneity are imposed. The homogeneity restriction is imposed upon the own-price coefficient.
[b] IQ beef is import-quality beef, which includes both imported beef and domestic Japanese dairy beef.
[c] The FSR^2 for Wagyu beef, IQ beef, pork, chicken meat, and fish, respectively, are 0.957, 0.941, 0.884, 0.881, and 0.981.

Table 2. Hicksian Elasticities and Standard Errors for ITSUR and IT3SLS Estimations of Japanese Meat Expenditures[a]

Type Expenditure	Elasticities	Hicksian Elasticities ITSUR Model		Hicksian Elasticities IT3SLS Model		Marshallian Elasticities ITSUR Model		Marshallian Elasticities IT3SLS Model	
Wagyu Beef									
	Wagyu Beef	-1.49	(0.52)	-2.07	(0.48)	-1.57	(0.42)	-2.14	(0.47)
	IQ Beef[b]	0.23	(0.30)	0.51	(0.37)	0.12	(0.30)	0.42	(0.36)
	Pork	0.97	(0.38)	1.02	(0.44)	0.75	(0.36)	0.80	(0.48)
	Chicken	0.28	(0.28)	0.25	(0.35)	0.13	(0.27)	0.10	(0.37)
	Fish	0.01		0.28		-0.78		-0.48	
	Expenditure	1.35	(0.69)	1.31	(0.71)	1.35	(0.69)	1.31	(0.71)
IQ Beef									
	Wagyu Beef	0.14	(0.18)	0.29	(0.21)	0.05	(0.17)	0.19	(0.21)
	IQ Beef	-0.61	(0.19)	-1.07	(0.22)	-0.75	(0.17)	-1.25	(0.20)
	Pork	0.19	(0.23)	0.40	(0.26)	-0.08	(0.18)	0.06	(0.21)
	Chicken	-0.19	(0.17)	-0.12	(0.19)	-0.39	(0.12)	-0.37	(0.16)
	Fish	0.46		-0.49		-0.51		-0.74	
	Expenditure	1.66	(0.36)	2.11	(0.37)	1.66	(0.39)	2.11	(0.37)
Pork									
	Wagyu Beef	0.29	(0.11)	0.30	(0.13)	0.28	(0.10)	0.31	(0.13)
	IQ Beef	0.10	(0.12)	0.21	(0.13)	0.08	(0.09)	0.22	(0.11)
	Pork	-0.82	(0.23)	-1.34	(0.26)	-0.85	(0.17)	-1.32	(0.22)
	Chicken	-0.16	(0.15)	0.08	(0.17)	-0.18	(0.09)	-0.10	(0.13)
	Fish	0.58		0.74		0.49		0.83	
	Expenditure	0.22	(0.31)	0.15	(0.38)	0.22	(0.31)	0.15	(0.38)
Chicken									
	Wagyu Beef	0.12	(0.12)	0.10	(0.15)	0.10	(0.11)	0.05	(0.14)
	IQ Beef	-0.14	(0.12)	-0.09	(0.14)	-0.18	(0.09)	-0.18	(0.12)
	Pork	-0.22	(0.21)	0.11	(0.24)	-0.30	(0.13)	-0.06	(0.18)
	Chicken	-0.89	(0.16)	-0.87	(0.19)	-0.94	(0.12)	-1.00	(0.15)
	Fish	1.13		0.73		0.83		0.07	
	Expenditure	0.49	(0.28)	1.13	(0.33)	0.50	(0.28)	1.13	(0.33)
Fish									
	Wagyu Beef	0.01		0.01		0.02		-0.07	
	IQ Beef	0.07		0.11		0.07		-0.02	
	Pork	0.16		0.22		0.21		-0.02	
	Chicken	0.22		0.29		0.14		0.12	
	Fish	-0.45		-0.62		-0.45		-1.48	
	Expenditure	1.19		1.46		1.10		1.45	

[a]All models have Slutsky symmetry and homogeneity imposed. Elasticities are calculated for 1986. The standard errors, in parentheses, are calculated as suggested by Chalfant.
[b]IQ beef is import-quality beef, which includes both imported beef and domestic Japanese dairy beef.

where M has a chi-square distribution with n degrees of freedom and n is the number of parameters in β that are directly affected by the correction for endogeneity. Using the results presented in Table 1, in which all five prices are assumed to be endogenous, the calculated value of M is 118. The critical value of the χ_{10}^2 statistic at the 1 percent level of significance is 23.21. Therefore, the null hypothesis that the prices on the RHS of (1) are exogenous is rejected.

All the prices on the RHS of (1) may not be endogenous. The biological lags in the livestock production process have the effect of fixing the available supply of meat. Wagyu beef would be considered to have a relatively fixed supply because of the production lags; however, Wagyu beef price is successfully fixed by the Japanese government in an effort to support the incomes of Wagyu producers (BAE 1985). That is, Wagyu beef prices are determined exogenously. Imported beef, which is subject to import quota restrictions, is a substitute for domestic dairy beef. Domestic dairy beef, because of the length of the livestock production process, has a relatively fixed supply. Therefore, import-quality beef, which is the combination of domestic dairy beef and imported beef, has a relatively fixed supply, and thus, prices may be endogenous. Pork supply is relatively fixed, and prices may therefore be endogenous. The production cycle for chicken is much shorter than a year, allowing producers to respond to price changes within the observation period; therefore poultry prices may or may not be exogenous. Because the endogeneity of all meat prices is uncertain, each of the prices should be tested individually for endogeneity. This would provide additional information about the system as well as guidance in the choice of prices to correct for endogeneity.

The first five columns of Table 3 present the Hicksian elasticities from sequentially estimating the system with one price specified as endogenous. The Wu-Hausman test statistics are listed across the top of the table. The test for endogeneity of Wagyu beef price indicates that it may be exogenous. This may be because of the government price stabilization efforts. Import-quality beef, pork, chicken meat, and fish reject exogeneity. For comparative purposes, the last column presents the ITSUR elasticities from Table 1, where all five meats are assumed to be exogenous. Overall, the results emphasize the sensitivity of the elasticities to model specifications. The largest changes seem to occur for the elasticities of the price that is treated as endogenous. The Wu-Hausman statistics for chicken meat and fish are lower than those for import-quality beef and pork. This may reflect the shorter production lag for chicken and fish sectors.

Conclusions

Previous applications of demand systems to the food sector have overlooked the endogeneity of prices in agricultural markets. We demonstrate how to test and compensate for endogeneity of prices in an AIDS. The specific example used is the Japanese meat expenditure system. The results indicate that four of the prices are endogenous. The only price found to be exogenous is set by government price-stabilization efforts.

The corrected model had consistently larger Marshallian own-price elasticities. This may be due to some statistical aberration in this particular data set or perhaps due to more fundamental properties of the adjustment mechanism.

The results presented suggest a strategy for estimation of demand systems of the food sector. First, exogeneity of prices should be tested. Prices that reject exogeneity should be treated as endogenous RHS variables by using an IV estimator such as IT3SLS.

Table 3. Hicksian Elasticities after Correction for Endogenous Prices

	System Corrected for Endogeneity of					ITSUR Results
	Wagyu beef	IQ Beef	Pork	Chicken	Fish	
Wu-Hausman Statistic	6.87	27.70[a]	30.00[a]	20.16[a]	20.48[a]	
Wagyu Beef						
Wagyu Beef	-0.85	-1.58	-1.49	-1.46	-1.60	-1.49
IQ Beef[b]	-0.21	0.24	0.29	0.12	0.17	0.23
Pork	0.82	1.08	0.85	0.85	0.75	0.97
Chicken	0.05	0.03	0.21	0.37	0.18	0.28
Fish	0.18	0.23	0.22	0.17	0.49	0.01
Expenditure	0.63	0.87	0.92	1.28	0.63	1.35
IQ Beef						
Wagyu Beef	-0.12	0.14	0.12	0.07	0.10	0.14
IQ Beef	-0.40	-0.88	-0.58	-0.63	-0.72	-0.61
Pork	0.14	0.29	0.18	0.35	0.29	0.19
Chicken	-0.13	0.01	-0.25	-0.14	-0.18	-0.19
Fish	0.51	0.43	0.52	0.25	0.51	0.46
Expenditure	1.71	1.33	2.21	1.49	2.07	1.66
Pork						
Wagyu Beef	0.24	0.33	0.25	0.25	0.22	0.29
IQ Beef	0.75	0.15	0.09	0.18	0.15	0.10
Pork	-0.81	-0.78	-0.79	-1.01	-0.77	-0.82
Chicken	-0.17	-0.16	0.04	-0.13	-0.07	-0.16
Fish	0.63	0.48	0.61	0.46	0.66	0.58
Expenditure	0.17	0.26	0.35	0.39	0.14	0.22
Chicken						
Wagyu Beef	0.02	0.13	0.09	0.16	0.07	0.12
IQ Beef	-0.09	0.11	-0.18	-0.10	-0.13	-0.14
Pork	-0.24	-0.23	0.06	-0.19	-0.10	-0.22
Chicken	-0.83	-0.75	-0.98	-0.88	-0.68	-0.89
Fish	1.14	0.95	1.01	1.02	0.84	1.13
Expenditure	0.58	0.83	0.73	0.57	1.11	0.49
Fish						
Wagyu Beef	0.01	0.02	0.01	0.01	0.04	0.01
IQ Beef	0.07	0.06	0.07	0.05	0.07	0.07
Pork	0.17	0.13	0.17	0.13	0.18	0.16
Chicken	0.22	0.19	0.20	0.20	0.17	0.22
Fish	-0.49	-0.40	-0.47	-0.39	-0.47	-0.45
Expenditure	1.24	1.07	1.17	1.07	1.12	1.19

[a]Significant at the 1 percent level. The 1 percent significance level for a χ_5^2 is 15.09.
[b]IQ beef is import-quality beef, which includes both imported beef and domestic Japanese dairy beef.

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