The Effects of Price Supports on the Valuation of Options on Agricultural Futures Contracts

by

Mario J. Miranda and Joseph W. Glauber

The Effects of Price Supports on the Valuation of Options on Agricultural Futures Contracts

Mario J. Miranda and Joseph W. Glauber *

In their seminal paper, Black and Scholes derived formulas for determining the fair values of European put and call options on traded securities. Black subsequently derived similar formulas for the special case of options on commodity futures contracts. The basic assumptions underlying the Black-Scholes option pricing formula are that the markets for options, bonds, and futures are frictionless, the risk-free rate of return is constant over the life of the option, and the conditional probability distribution of the commodity's price at the option's expiration date is lognormal (see Jarrow and Rudd). Under these assumptions, the Black-Scholes formula expresses the premium of a put or call option on a futures contract in terms of the futures price, the volatility of the futures price, the strike or exercise price, the risk-free rate of return, and the time remaining until the expiration of the option.

The robustness of the Black-Scholes option pricing formula to the assumptions underlying its derivation has been the focus of considerable research (see Hauser and Neff for a survey). For options on some agricultural futures, the validity of the lognormal price distribution assumption has been questioned due to the incidence of price distorting government price support programs. Government purchases of an agricultural commodity under a support program (e.g., nonrecourse loan defaults) effectively place a floor on market price. Moreover, government releases of stocks previously acquired under such a program restrain upward price movements and, if the government stockpile is sufficiently large, can effectively place a ceiling on price. Thus, probability distributions admitting high concentrations of probability mass near the support and release prices are perhaps more appropriate for modeling the distribution of supported agricultural prices than the smooth and regular lognormal distribution assumed by Black-Scholes theory.

Efforts to derive analytic option pricing formulas assuming distributions other than the lognormal have been largely ineffective. This has led to the development of numerical option pricing models. Best known among the numerical models is the binomial option pricing model of Cox, Ross and Rubenstein. In this model, the futures price is assumed to follow a discrete two-state jump process, which implies a binomial conditional distribution for the percentage change in the commodity price at the expiration date. The pricing model is easily implemented on a microcomputer and converges to the Black-Scholes model as the intervals segmenting the time to expiration are reduced in size and increased in number.

One approach to the valuation of options in the presence of price supports has been to extend the binomial pricing model by introducing a reflecting barrier at the support price

*The authors are, respectively, Assistant Professor, Department of Agricultural Economics and Rural Sociology, The Ohio State University, and Economist, Economic Research Service, U.S. Department of Agriculture. This research was supported by U.S.D.A. Cooperative Research Agreement 58-3AEK-7-00035.

(Gordon). This approach has the drawback that the stochastic process describing price movements is memoriless. That is, price always behaves the same way, in a probabilistic sense, after encountering the reflecting barrier (i.e., the support price). As such, the model ignores the fundamental forces that drive price to the support level. In addition, the reflecting barrier model cannot accommodate releases of government stocks since market price can conceivably rise above the release price if the government stockpile is depleted.

A numerical modeling strategy that provides an alternative to the binomial model for valuing options was developed by Cox and Ross, Boyle, and Parkinson. In this approach, an explicit stochastic model for the underlying commodity's price is constructed and the conditional distribution of the commodity's price at the expiration date is derived. Under the assumption of risk neutrality, the present value of an option equals the expected value of the option at the expiration date discounted at the risk-free rate or return. Option premia can thus be computed using numerical integration techniques such as Monte Carlo simulation or numerical quadrature. The main advantage of this method is that it is applicable regardless of the nature of the distribution function, making it well suited for valuing options when the price distribution is distorted by a price support program.

In the current paper, we employ the above strategy to construct a numerical pricing model for options on futures for agricultural commodities subject to government price supports. As a model of the underlying commodity market, we use the computable rational expectations (CRE) model of the U.S. soybean market developed by Miranda and Helmberger and Glauber et al. In this model, storers and farmers base their storage and planting decisions on future price expectations that are rational, in the sense of Muth, with respect to the effects of government policy. The conditional probability distribution of the commodity price is endogenous and can exhibit positive probability mass at the support and release prices. In the context of this model, we evaluate the performance of the Black-Scholes option pricing formula in the presence of price supports and examine, in general, the effects of price supports on the value of options on agricultural futures contracts.

In Section 1, we discuss the CRE model of the U.S. soybean market under a price support program. In Section 2, we explain how options may be valued using the direct numerical approach and traditional Black-Scholes' approach. In Section 3, we examine the effects of price supports on option premia and the biases that price supports may induce in the Black-Scholes option pricing formula. In Section 4, we consider the implications of our findings and provide directions for future research.

I. The Market Model

Miranda and Helmberger's Computable Rational Expectations (CRE) model of the U.S. soybean market is a dynamic, stochastic model in which market participants react rationally to the effects of a government price support program. In the model, the government attempts to stabilize market price through open market purchases and sales. At the support price $\bar{P}_S$, the government offers to buy and store unlimited quantities of the commodity; at the release price $\bar{P}_R$, it offers to sell any quantities in its possession.

A typical year t begins with a given supply S_t, which is composed of the preceding year's ending private stocks I_{t-1}, the preceding year's ending government stocks G_{t-1}, and new production, which equals the acreage planted the preceding year A_{t-1} times a random per-acre yield $\tilde{Y}_t$:

$$S_t = I_{t-1} + G_{t-1} + A_{t-1} \cdot \tilde{Y}_t. \tag{1}$$

Supply is either consumed or stored for future consumption. Denoting current consumption by C_t, ending private stocks by I_t, and ending government stocks by G_t, material balance requires that:

$$S_t = C_t + I_t + G_t. \tag{2}$$

Consumption is a function of current price P_t:

$$C_t = D_t(P_t). \tag{3}$$

The government purchases no stocks if the market price exceeds the support level and sells no stocks if the market price lies below the release level:

$$P_t > \bar{P}_S \Longrightarrow G_t \leq G_{t-1}, \tag{4}$$

$$P_t < \bar{P}_R \Longrightarrow G_t \geq G_{t-1}. \tag{5}$$

Since the government is willing to acquire unlimited stocks at the support price, the market price never falls below this level:

$$P_t \geq \bar{P}_S. \tag{6}$$

However, the market price can rise above the release level, though only if the government stockpile is first depleted:

$$G_t \geq 0. \tag{7}$$

$$P_t > \bar{P}_R \Longrightarrow G_t = 0. \tag{8}$$

Competition among the risk-neutral arbitrageurs eliminates expected speculative profit opportunities. This yields the familiar complementarity conditions:

$$I_t \geq 0, \tag{9}$$

$$P_t \geq \delta P^e_{t+1} - \kappa, \tag{10}$$

$$P_t > \delta P^e_{t+1} - \kappa \Longrightarrow I_t = 0. \tag{11}$$

Here, $\delta P^e_{t+1} - \kappa$ equals the price expected next period, appropriately discounted, minus the unit cost of storage (i.e., the expected marginal revenue from storing the commodity).

The acreage planted by farmers depends on the price they expect to receive the following year:

$$A_t = A_t(P^e_{t+1}). \tag{12}$$

Arbitrageurs and farmers form their price expectations rationally. That is, they forecast future prices using all the information available at the time the forecast is made:

$$P^e_{t+1} = \mathcal{E}_t P_{t+1}, \tag{13}$$

The above market model possesses no closed-form analytic solution, but can be solved, for any specific parameterization, using the stochastic-dynamic programming algorithm developed by Miranda. The numerical solution procedure centers on the derivation, for each period t, of the equilibrium price function $\tilde{P}_t$, which expresses the market price the followi period P_{t+1}, in terms of current private stocks I_t, current government stocks G_t, curre acreage planted A_t, and the following period's random yield $\tilde{Y}_{t+1}$:

$$P_{t+1} = \tilde{P}_t(I_t, G_t, A_t, \tilde{Y}_{t+1}). \tag{14}$$

The equilibrium price functions completely characterize the stochastic behavior of pric across time. If the time horizon is infinite, the discount rate is less than one, and the mode is structurally time-stationary, then the equilibrium price functions are identical in all period and an unique steady-state equilibrium price function $\tilde{P}$ exists.

For the purposes of this paper, we employ a steady-state model in which all market parameters, including the demand and supply functions and the distribution of the random yield, are assumed to be time-stationary. Parameter estimates were drawn from the econometric study of the U.S. soybean market by Glauber. Exogenous supply and demand shifters are assigned their 1977 values to obtain simplified exponential forms expressing quantity demanded, in billions of bushels, and acreage supplied, in millions of acres, as functions of market price and expected market price, respectively, in real (1977) dollars per bushel:

$$C_t = 5.18 P_t^{-0.613} \tag{15}$$

$$A_t = 13.0 {P^e_{t+1}}^{0.886}. \tag{16}$$

The random yields $\tilde{Y}_t$, measured in bushels per acre, were assumed to be independently and identically lognormally distributed as given by

$$\tilde{Y}_t = 29.4 \exp^{0.173\tilde{Z}} \tag{17}$$

where $\tilde{Z}$ is standard normally distributed. We assumed a annual storage cost of \$0.36 per bushel and an annual interest rate of 8 percent.

II. Options Valuation

In this section, we discuss two different methods for valuing options on agricultural futures contracts. The first method values options directly using the conditional distribution of price derived from the computable rational expectations (CRE) model presented in the preceding section. This method explicitly captures the distortions of the probability distribution induced by government price support programs. The second method values options using the Black-Scholes pricing formula, which assumes a smooth lognormal price distribution.

Given the equilibrium price function $\tilde{P}$, the cumulative probability distribution of next year's price conditional on the current levels of private stocks I_t, government stocks G_t, and acreage planted A_t is given by:

$$\begin{aligned} H(p; I_t, G_t, A_t) &= \Pr\{P_{t+1} \le p \mid I_t, G_t, A_t\} \\ &= \Pr\{\tilde{P}(I_t, G_t, A_t, \tilde{Y}_{t+1}) \le p \mid I_t, G_t, A_t\}. \end{aligned} \tag{18}$$

The conditional cumulative probability distribution function of price, H, describes the stochastic behavior of price one year in the future and provides the key to valuing put and call options on futures contracts.

Invoking the result of Cox and Ross, we assume risk neutrality and compute the present value of a option as the expected value of the option at the expiration date discounted at the risk-free rate of return. Specifically, having H, the conditional cumulative probability distribution function of next year's price P, the premium on an European put option expiring in one year's time is given by the Lebesgue-Stiltjes integral

$$\pi_p = e^{-r}\mathcal{E}\{\max(0, K - P)\} = e^{-r}\int_{-\infty}^{+\infty} \max(0, K - p)dH(p); \tag{19}$$

the premium on an European call option expiring in one year's time is given by:

$$\pi_c = e^{-r}\mathcal{E}\{\max(0, P - K)\} = e^{-r}\int_{-\infty}^{+\infty} \max(0, p - K)dH(p). \tag{20}$$

The expectation can be computed numerically, to any desired degree of accuracy, using standard numerical quadrature techniques.

We refer to the options premium formulas (19) and (20) as the Computable Rational Expectations or CRE option formulas, in order to distinguish them from the Black-Scholes option formulas below. It is interesting to note that the CRE option formulas observe Black's put-call parity theorem, which states that, at any strike price, the difference between the put and call premiums on a futures contract equals the discounted difference between the strike price and the futures price. On the assumption of rational expectations, the futures price is simply the expected price,

$$F = \mathcal{E}P = \int_{-\infty}^{+\infty} p dH(p); \tag{21}$$

thus,

(22)

$$\begin{aligned}
\pi_p - \pi_c &= e^{-r}[\mathcal{E}\{\max(0, K - P)\} - \mathcal{E}\{\max(0, P - K)\}] \\
&= e^{-r}[\int_{-\infty}^{+\infty} \max(0, K - p)dH(p) - \int_{-\infty}^{+\infty} \max(0, p - K)dH(p)] \\
&= e^{-r}[\int_{-\infty}^{K} (K - p)dH(p) - \int_{K}^{+\infty} (p - K)dH(p)] \\
&= e^{-r}[\int_{-\infty}^{+\infty} K dH(p) - \int_{-\infty}^{+\infty} p dH(p)] \\
&= e^{-r}[K - F]
\end{aligned}$$

The basic assumptions underlying the Black-Scholes options pricing formula on commodity futures are that the markets are frictionless, the risk-free rate of return is constant, and that the conditional distribution of the underlying commodity's price at expiration is log-normal. Under these assumptions, the Black-Scholes formula expresses the premium π of a put option on a futures contract in terms of the futures price F, the volatility of the price of the underlying commodity σ, the risk free rate of return r, the strike price K, and the time until the expiration of the option, which in this case is one year:

$$\pi_p = e^{-r}(F \cdot N(-d_1) - K \cdot N(-d_2)), \tag{23}$$

where

$$d_1 = (\log(F/K) + \sigma^2/2)/\sigma \tag{24}$$

$$d_2 = (\log(F/K) - \sigma^2/2)/\sigma \tag{25}$$

and N is the standard normal cumulative distribution function. The Black-Scholes formula for the premium of a call option on a futures contract expiring in one year is:

$$\pi_c = e^{-r}(F \cdot N(d_1) - K \cdot N(d_2)). \tag{26}$$

The strike price and the risk-free interest rate are exogenous and the futures price is given by (21). Thus, all that remains to be done in order to compute options premia using the Black-Scholes formula is to identify the price volatility σ in the context of the current model. This is straightforward, however, as σ is just the conditional standard deviation of the natural logarithm of price one year in the future:

$$\begin{aligned} \sigma^2 &= \mathcal{E}(\log(p))^2 - (\mathcal{E}\log(p))^2 \\ &= \int_{-\infty}^{+\infty} (\log(p))^2 dH(p) - (\int_{-\infty}^{+\infty} \log(p) dH(p))^2. \end{aligned} \tag{27}$$

III. Simulation Results

In the preceding sections, we discussed two alternative methods for computing options premia for European put and call options: the CRE options valuation method and the Black Scholes options valuation method. The former method explicitly captures the distortions of market prices caused by price support programs; the latter does not. In this section, we first compare the premia computed using both methods in order to assess the biases that the Black-Scholes model might exhibit in the presence of price supports. We then examine the effects of changes in government price support policies on options valuation by comparing CRE options premia under alternative market scenarios and policy regimes.

Throughout the analysis, our simulation results are evaluated with reference to a base model parameterization. In the base case, the support price is assumed to be \$5 and the release price of \$6. Government stocks are assumed to be 0.3 billion bushels and private speculative stocks are assumed to be nonexistent. Planned production is assumed to be 1.8 billion bushels, although actual production can vary substantially due to random yield variations. In order to ascertain the effects of private and government actions on the values of put and call options on futures contracts, simulations are conducted assuming different levels of support and release prices and different levels of government stocks and planned production. Unless stated otherwise, market parameters should be assumed to be at their base case levels.

Figure 1a illustrates typical cumulative probability distributions for price under the CRE and Black-Scholes models. The CRE distribution exhibits singularities, that is, positive probability mass, at both the support price of \$5 and the release price of \$6. The cumulative lognormal distribution of the Black-Scholes model exhibits the usual smoothness. On first observation, the lognormal appears to provide a poor fit to the truncated CRE distribution. The lognormal assigns positive probability to prices below the support level, whereas the

CRE distribution asserts that prices below the support level are not possible due to the government's stock acquisition policy. Similarly, the lognormal overestimates the probability that prices above the release level will occur; the CRE distribution captures the low probability of such prices occuring as a result of the downward price pressure created by government stock releases.

Figures 1b and 1c, however, suggest that the options premia derived from the CRE and Black-Scholes model do not differ a great deal. The CRE put option premium is zero at strike prices at or below the support price, whereas the Black-Scholes premium is positive due to the failure of the lognormal distribution to capture the impossibility of prices dropping below the support level. At very high strike prices, both the CRE and Black-Scholes put options premia converge to the intrinsic value of the option, which is the same in both cases. Similarly, at low strike prices, both the CRE and Black-Scholes call option premiums converge to the intrinsic value of the option.

Figure 1d more clearly illustrates the divergence between the CRE and Black-Scholes option valuations. Recall that both the CRE and Black-Scholes option valuations observe put-call parity, implying that the difference in the two valuations is the same for put options as it is for call options. In no instance is the difference between the CRE and Black Scholes options valuations much more than 3 cents. This suggests that the lognormality assumption of the Black-Scholes options pricing model need not lead to excessive valuation errors, even when the distribution of prices is distorted by price supports.

In Figure 1d, we observe that the Black-Scholes premium tends to overvalue options at both the support price of $5 and the release price of $6. This is due to the lognormal assigning excessive probability to the incidence of prices below the support and release prices. The Black-Scholes options pricing formula tends to undervalue options most for strike prices in the vicinity of the futures price of $5.59, that is, for at-the-money options. This is due to the higher probability density assigned to prices between the support and release prices by the lognormal distribution relative to the CRE distribution.

Figure 2a illustrates the effects of increasing the support and release prices by $1 from $5-$6 to $6-$7. The new cumulative probability distribution is now truncated at the new support price; moreover, the probability that of market price will drop to the support level rises. A higher release price, on the other hand, is less likely to be achieved and prices above the release price are rare if not impossible. As one would expect, the mean of the distribution shifts upwards. The futures price under the $5-$6 policy is $5.592 and the futures price under the $6-$7 is $6.318, a rise of 72.6 cents.

As seen in Figure 2b, raising the support and release levels shifts the price distribution to the right, causing put option premia to fall or remain the same at all strike prices. At strike prices below the old support price, put option premia remain zero; at strike prices between the old and new support price, the premium drops from a positive level to zero. At strike prices above the new support price, the drop in options premia approaches 67 cents, which equals the decrease in the intrinsic value of the option, that is, the discounted decrease in the futures price.

As seen in Figure 2c, raising the support and release levels causes call option values to rise or remain the same at all strike prices. At low strike prices, option premia under the new policy exceed the premia under the old policy by the difference in the intrinsic value, about 67 cents. At strike prices above $7, a level rarely achieved under either price support policy, call option premia approach zero under each policy.

Figure 3a illustrates the effects of different levels of government stocks. The upper graph corresponds to a scenario in which the government stockpile is empty. Although the usual truncation in the distribution appears at the support price of $5, the singularity at the release price of $6 disappears because the government is unable to control price hikes through stock releases. The lower graph corresponds to a scenario in which the government stockpile has 0.6 billion bushels, or twice the amount in the base case. Given a large stockpile, the government is able to defend the release price as a ceiling, that is, price will never rise above $6. The larger the level of government stocks, of course, the lower the futures price. When the stockpile is empty, the futures price is $6.122, but for when the stockpile is full, the futures price is $5.529, a difference of 59.3 cents.

As seen in Figure 3b, put option premia are insensitive to the level of government stocks at strike prices below the release price. Generally, the value of a put option is not affected by the distribution of prices above the strike price, provided the overall probability of exceeding the strike price remains the same; in other words, the value of a put option depends entirely on the nature of the distribution of prices below the strike price. As seen in Figure 3a, the probability distributions of price at the low and high stock levels are very similar for prices below the release price; accordingly, at strike prices below the release price, option premia are essentially the same for both stock levels. It is only above the release price that the distribution of price is significantly affected by the level of government stocks. For strike prices over the release price, an option in the high stocks scenario has no time value while an option in the low stocks scenario has a positive, albeit rapidly falling, time value. In the limit, as the strike price rises, the difference in the options values approaches the difference in intrinsic values, that is, the discounted difference in the futures prices or 54.7 cents.

As seen in Figure 3c, the effects of government stocks levels on call options premia for strike prices below the release price tend to reflect the difference in intrinsic value (about 54.7 cents). For strike prices above the release price, the value of call option is zero when stocks are large because the probability of price rising above the release price is zero. If the government stockpile is empty, on the other hand, there is a possibility that price will rise above the release level and the put option retains some value. Even so, as the strike price rises the put option premium approaches zero when the stockpile is empty.

In Figure 4a, we see the effects of private planting decisions on the distribution of prices. If planting decisions call for an expected production of 1.5 billion bushels, the probability that price will fall to the support level is small; accordingly, the singularity of the cumulative probability distribution at the support price is smaller than usual. High prices, however, are more likely when planned production is small and the singularity at the release price is more pronounced. If planting decisions call for a high expected production of 2.1 billion bushels, the probability that price will fall to the support level is high and the probability that price will rise to or above the release level is small. Accordingly, the singularity at the support price is more pronounced and the singularity at the release price is less pronounced. The futures price under a low production plan, $6.293, exceeds the futures price under a high production plan, $5.218, by $1.075.

The effects of private production plans on the valuation of put options is illustrated in Figure 4b. Again, the value of a put option is clearly zero at strike prices below the support price of $5. At strike prices above the support price, lower prices under the high production plan imply increased expected profit from an option at any fixed strike price. As the strike price rises, the difference in the values of options under the two levels of production reflect the difference in intrinsic value, that is, the discounted difference in the futures price, or about $0.992.

The effects of private production plans on the valuation of call options is illustrated in Figure 4c. At low strike prices, the value of a call option tends to be entirely intrinsic, regardless of the production plan, and the difference in the value of an option under the two planned production scenarios equals $0.992. If expected production is high, the high probability that price will fall to the support level implies that the value of a call option whose strike price exceeds the support price will be small; moreover, the certainty that price will not exceed the release level implies that a call option with a strike price above the release price is valueless. If expected production is low, prices will be higher and the value of a call option, at any strike price, is higher.

IV. Conclusion

In this paper we have developed a model for pricing options on agricultural futures that explicitly incorporates the price distorting effects of government price support programs. The lognormality assumption usually maintained under Black-Scholes theory is discarded and Miranda and Helmberger's Computable Rational Expectations model is used instead to derive conditional probability distributions for price that exhibit singularities at the support and release prices. Cox and Ross' numerical option pricing strategy is then used to compute put and option premia. The model allows us to explore the valuation of options under different counterfactual government price support regimes and has an advantage over competing agricultural options valuation models in that the futures price and price volatility are both endogenous and react to changes in government policy.

In the context of our model, we have explored the performance of the Black-Scholes pricing formula. Our results suggest that the Black-Scholes formula performs adequately, even in the presence of price supports, and is generally robust to the usual lognormal price distribution assumption. Small biases are observed, however, at strike prices near the support and release prices, where the Black-Scholes formula tends to overvalue both put and call options. At near at-the-money strike prices, the Black-Scholes formula appears to slightly undervalue put and call options. The degree of under- and overvaluation of options is due to the lognormal distribution's inability to capture the singularities present in the price distribution when price supports are operative.

Our conclusions regarding the performance of the Black-Scholes formula are based on the assumption that rational ex-ante volatilities are employed in the computation of options premia. An alternative hypothesis, that options in practice are efficiently priced, would imply that implied price volatilities differ from rational ex-ante volatilities in predictable ways. Specifically, implied volatilities exceed rational ex-ante volatilities at strike prices near the support price. At near at-the-money strike prices, on the other hand, rational ex-ante volatilities exceed implied volatilities. In principle, these are empirically testable hypotheses. Whether the asserted biases are sufficiently strong or the data is sufficiently rich to provide evidence of these phenomena is an open question.

References

Black, F. "Pricing of Commodity Contracts." *Journal of Financial Economics* 81(1973):637-59.

Black, F. and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economics* 81(1973):637-59.

Boyle, P. "Options: A Monte Carlo Approach." *Journal of Financial Economics* (1977):323-38.

Cox, J.C. and S.A. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics* 3(1976):145-66.

Cox, J.C., S.A. Ross, and M. Rubenstein. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7(1979):229-63.

Glauber, J.W. "The Role of Price Expectations in the Inter-temporal Allocation of Soybeans." Ph.D. thesis. University of Wisconsin–Madison, 1984.

Glauber, J.W., P.G. Helmberger, and M.J. Miranda. "Four Approaches to Commodity Price Stabilization: A Comparative Analysis." *American Journal of Agricultural Economics* , [forthcoming].

Gordon, J.D. "Calculating the Effects of Price Supports on Put Options Premiums." AAEA Selected Paper, August 1985.

Hauser, R.J. and D. Neff "Pricing Options on Agricultural Futures: Departures From Traditional Theories." *Journal of Futures Markets* , 5(1985):539-77.

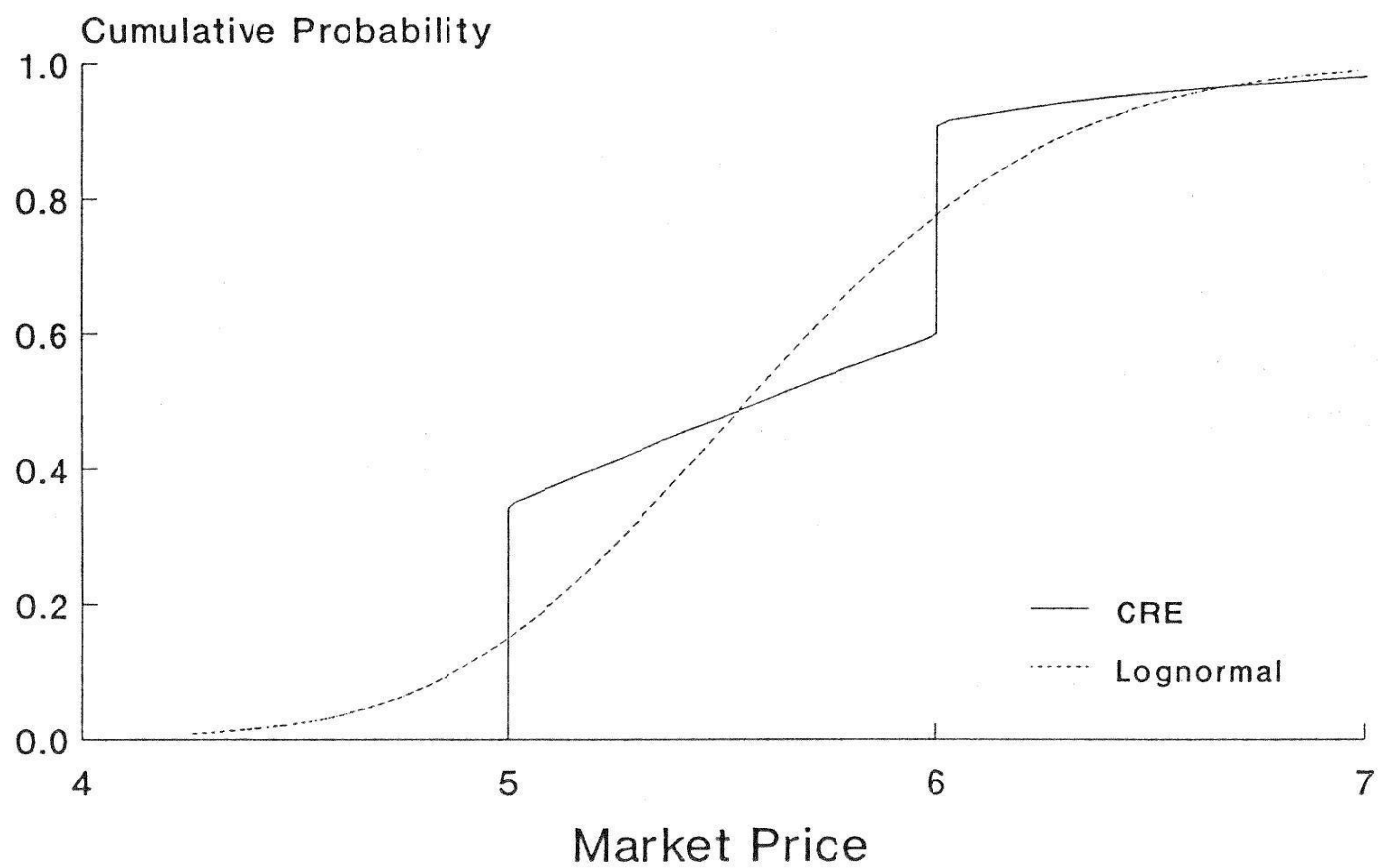
Jarrow, R.A. and A. Rudd. *Option Pricing.* Homewood, IL.: Irwin, 1985.

Miranda, M.J. "Analysis of Rational Expectation Models for Storable Commodities under Government Regulation." Ph.D. thesis. University of Wisconsin–Madison, 1985.

Miranda, M.J. and P.G. Helmberger. "The Effects of Price Band Buffer Stock Programs." *American Economic Review* 78(1988):46-58.

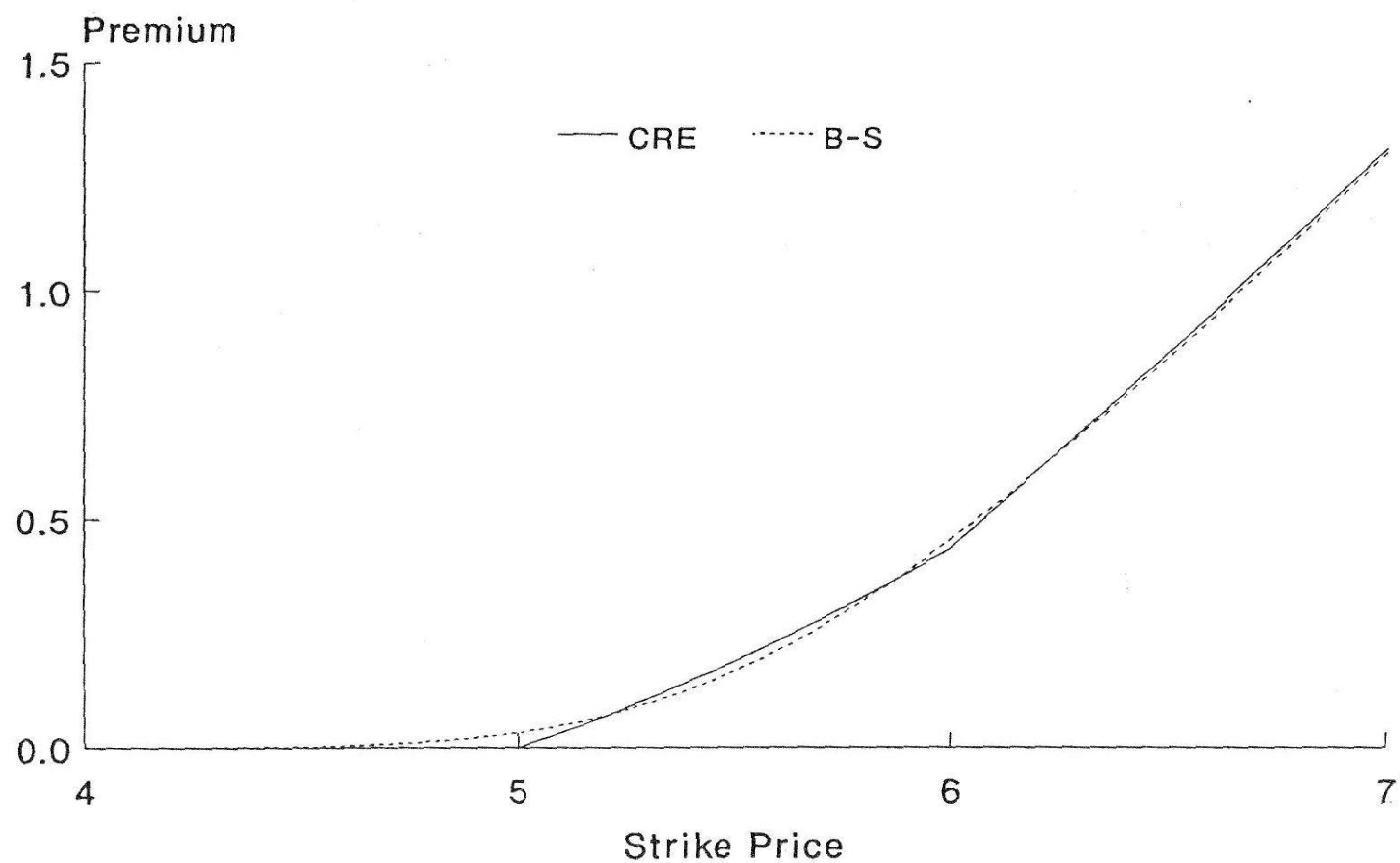
Parkinson, M. "Option Pricing: The American Put." *Journal of Business* (1977):21-36.

Figure 1a. Distribution of Price:
CRE Model vs. Lognormal



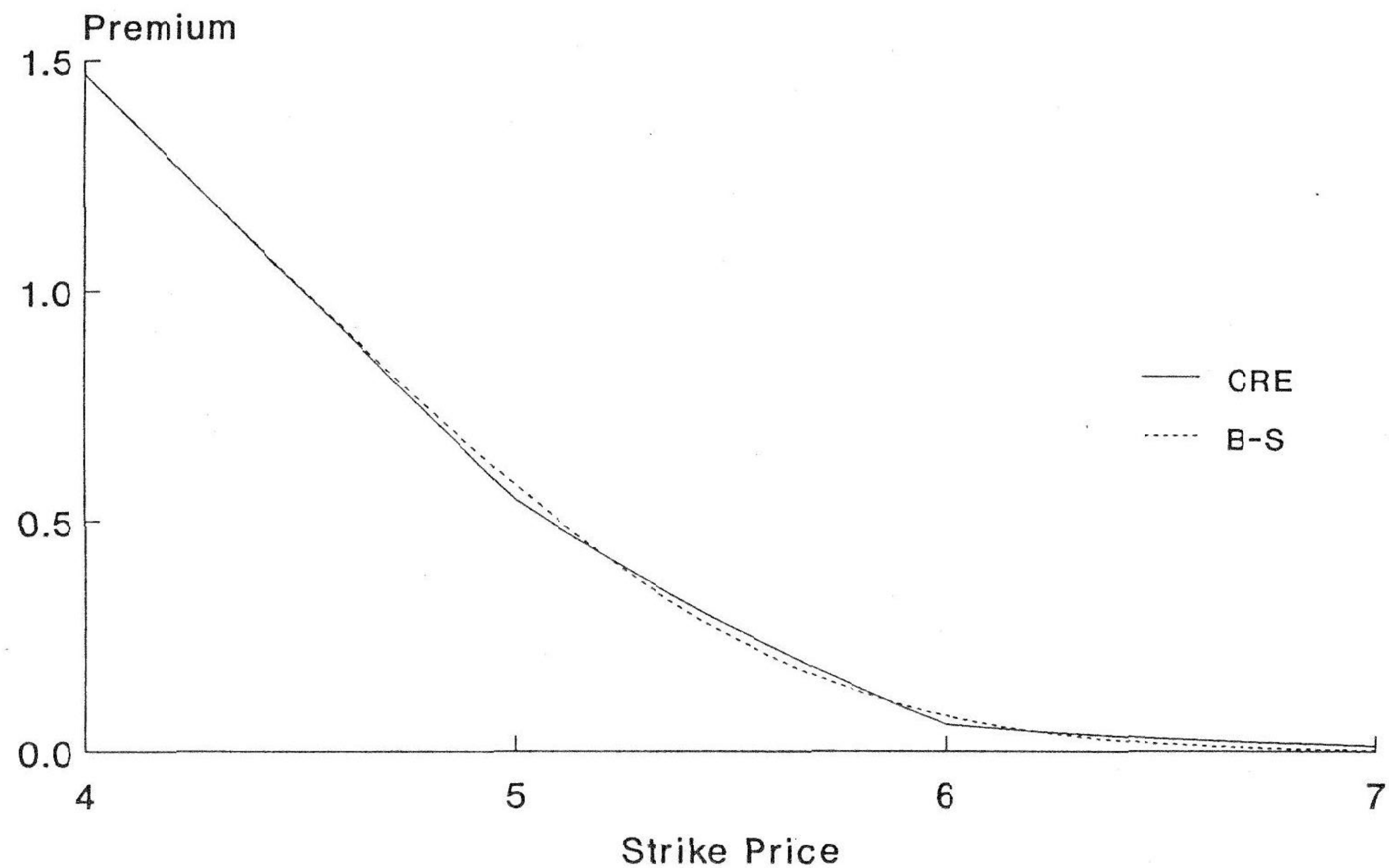
Support = $5.00
Release = $6.00
Futures = $5.59

Figure 1b. Put Option Premia:
CRE Model vs. Black-Scholes



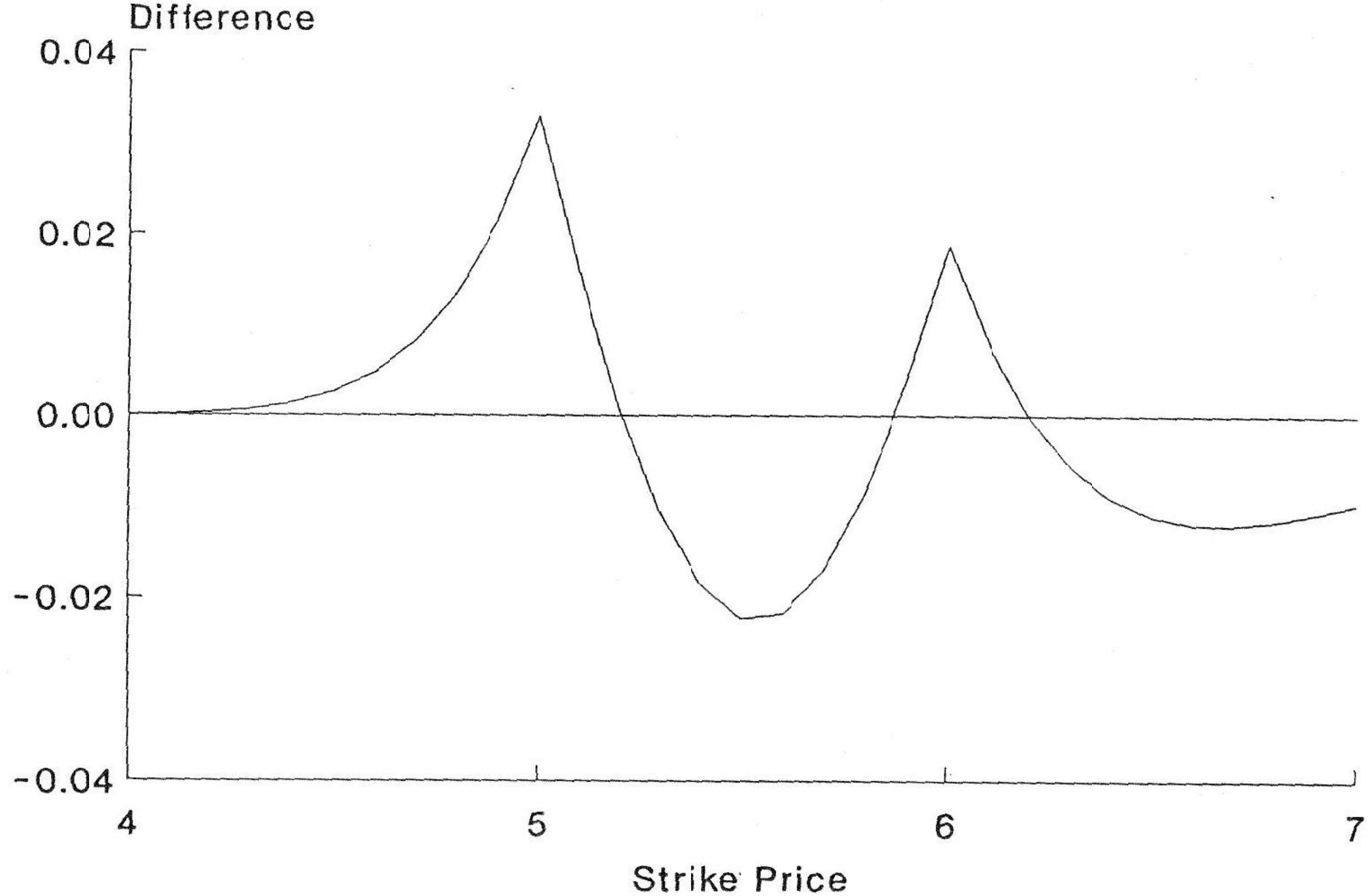
Support = $5.00
Release = $6.00
Futures = $5.59

Figure 1c. Call Option Premia:
CRE Model vs. Black Scholes



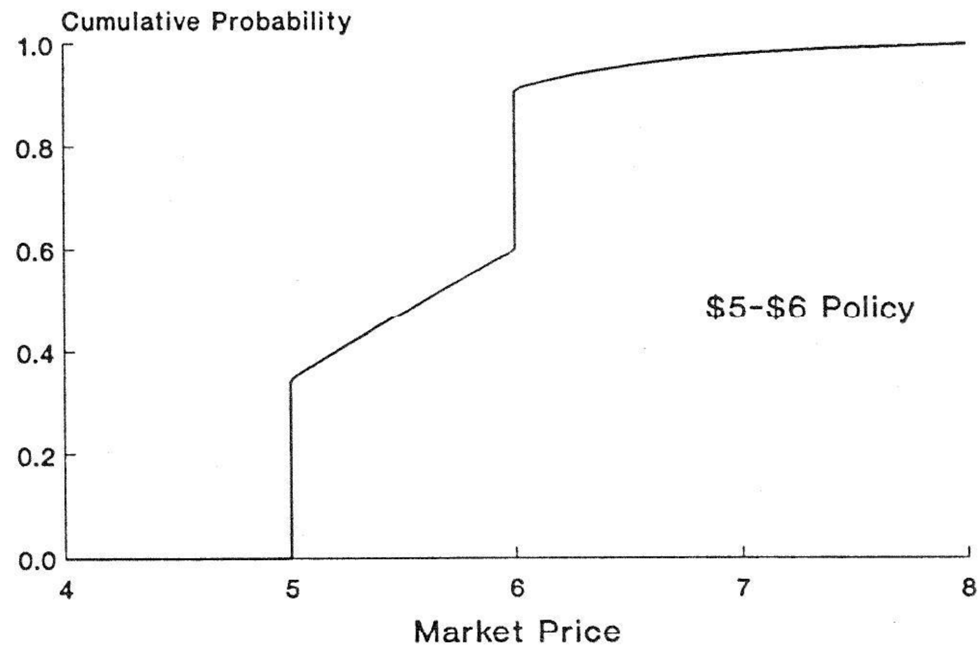
Support = $5.00
Release = $6.00
Futures = $5.59

Figure 1d. B-S Premium Less CRE Premium

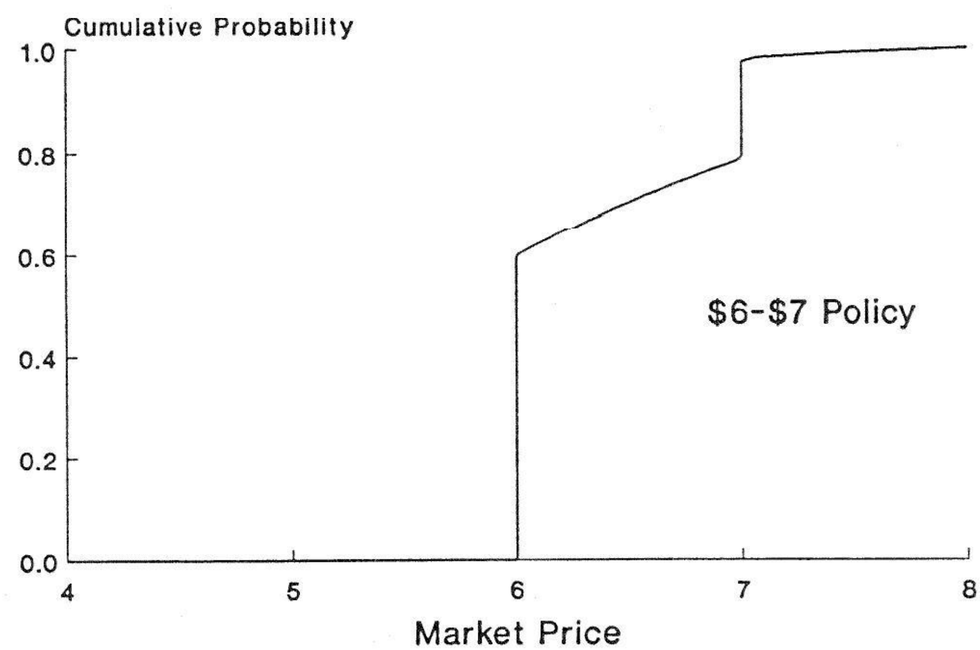


Support = $5.00
Release = $6.00
Futures = $5.59

Figure 2a. Distribution of Price under Different Support-Release Price Policies



Futures • $5.59



Futures • $6.32

Figure 2b. Put Option Premia for Different Support-Release Price Policies

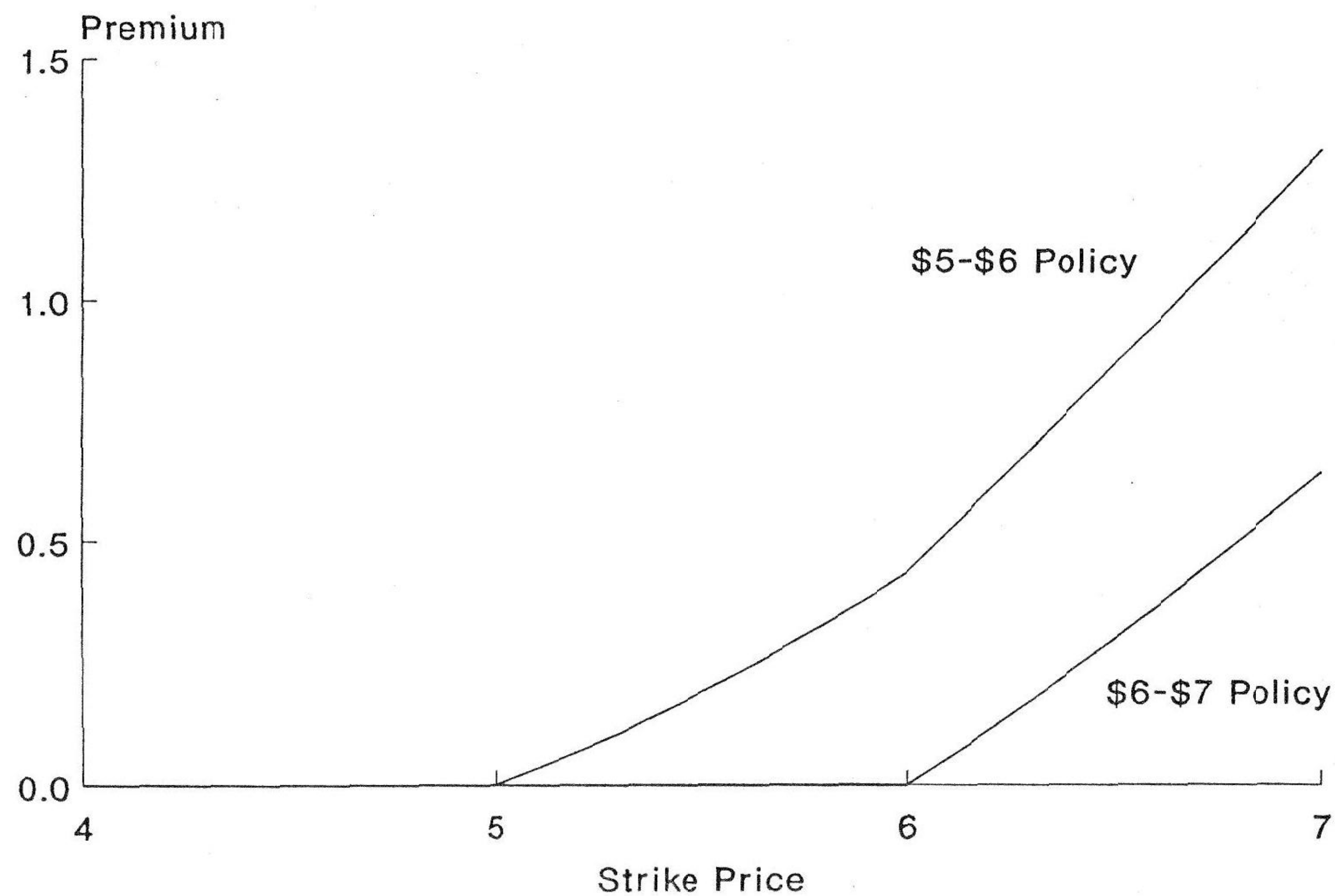


Figure 2c. Call Option Premia for Different Support-Release Price Policies

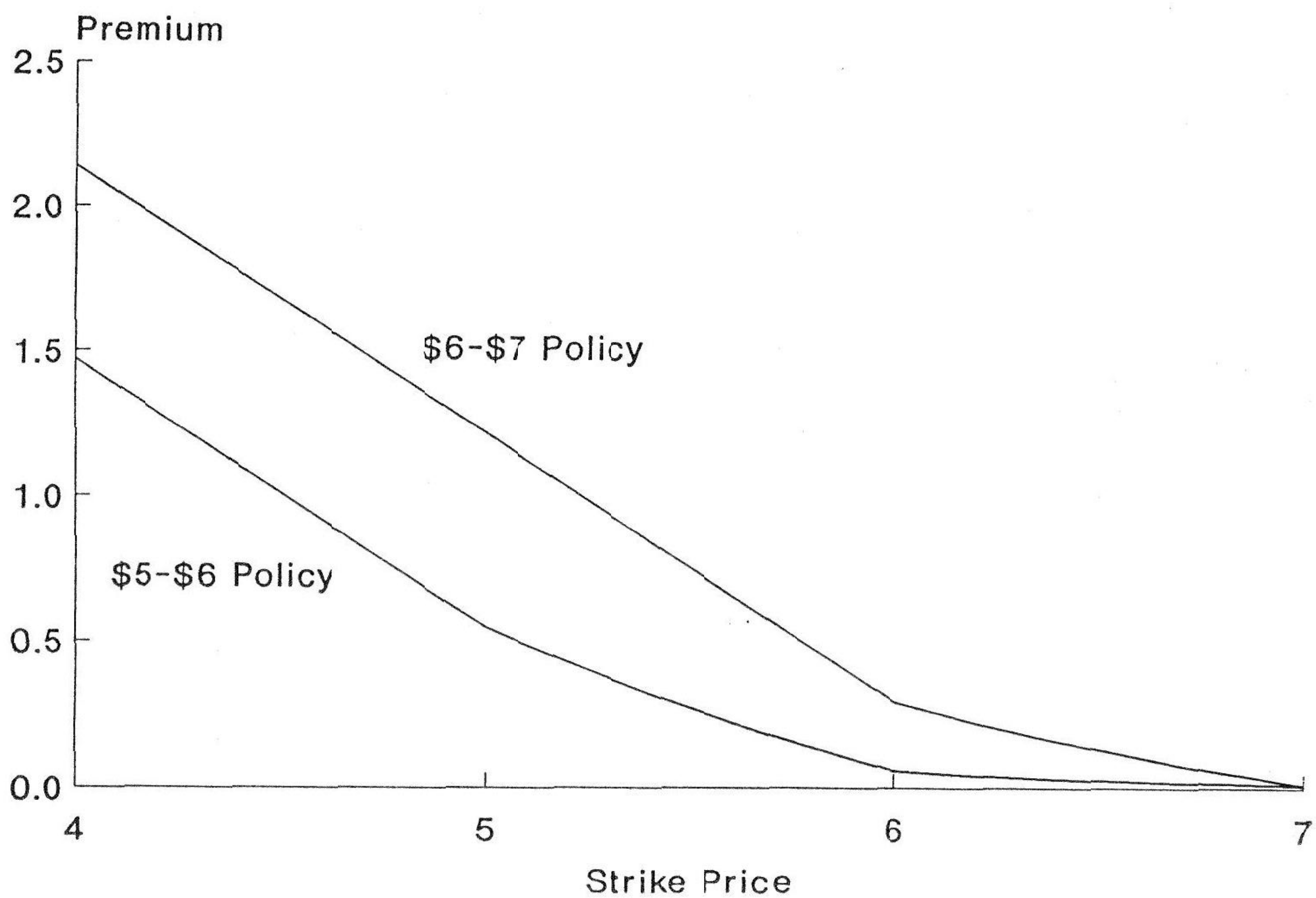
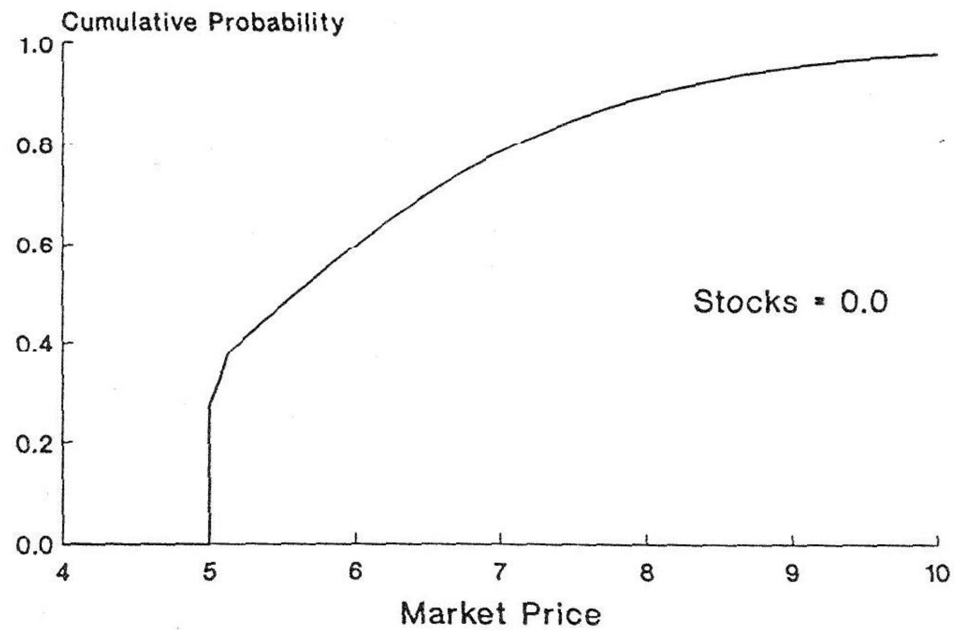
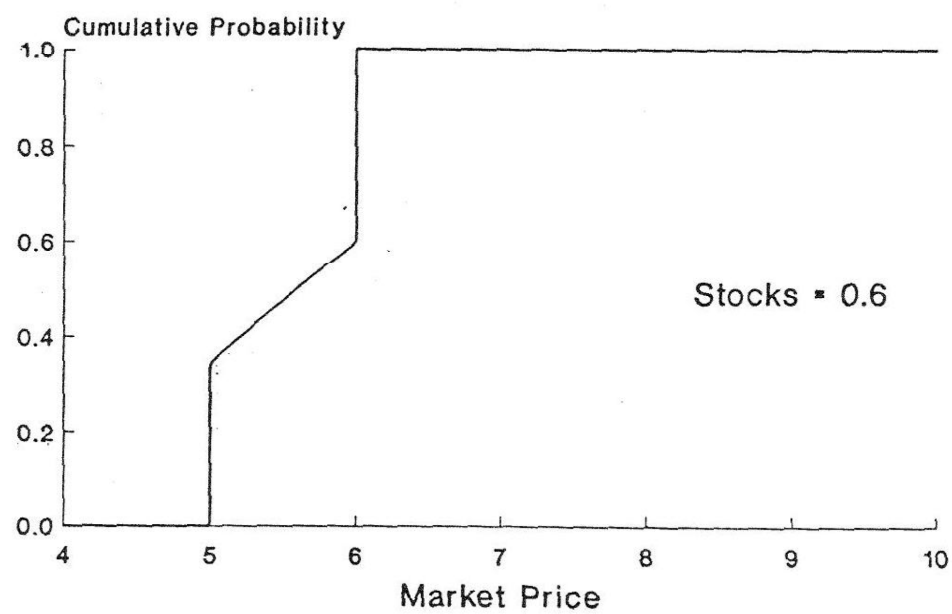


Figure 3a. Distribution of Price at Different Levels of Government Stocks



Futures = $6.12



Futures = $5.53

Figure 3b. Put Option Premia for Different Levels of Government Stocks

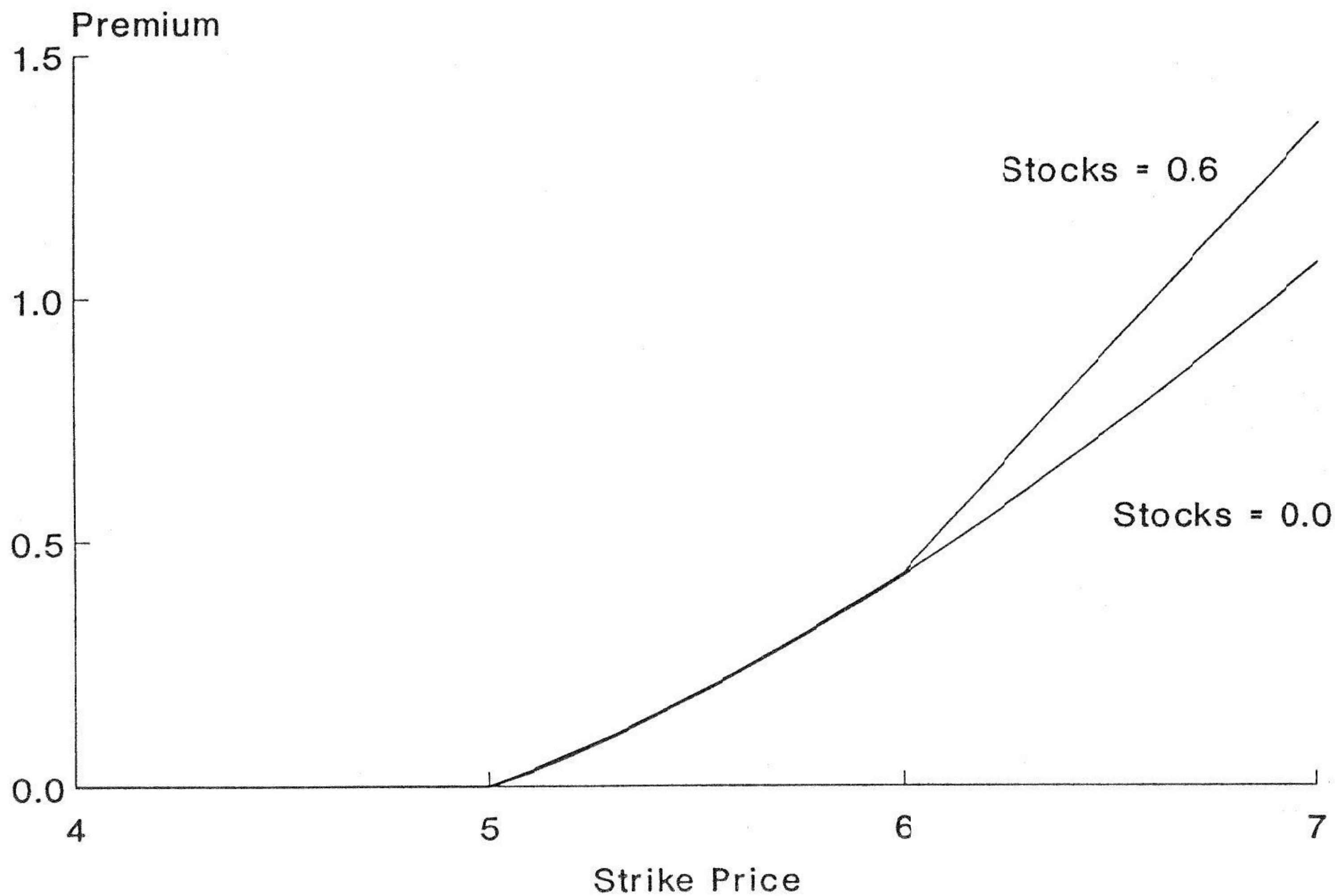


Figure 3c. Call Option Premia for Different Levels of Government Stocks

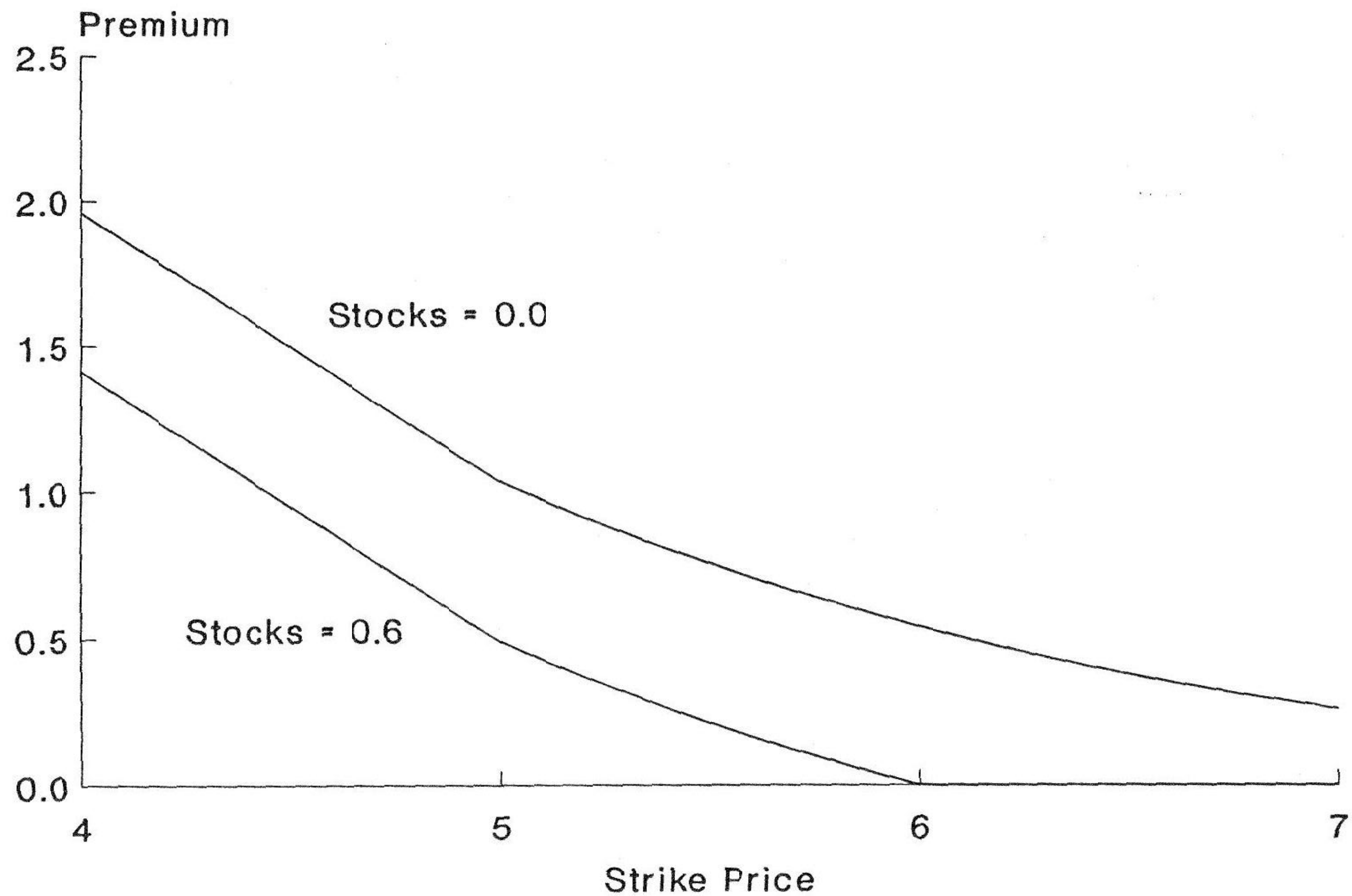
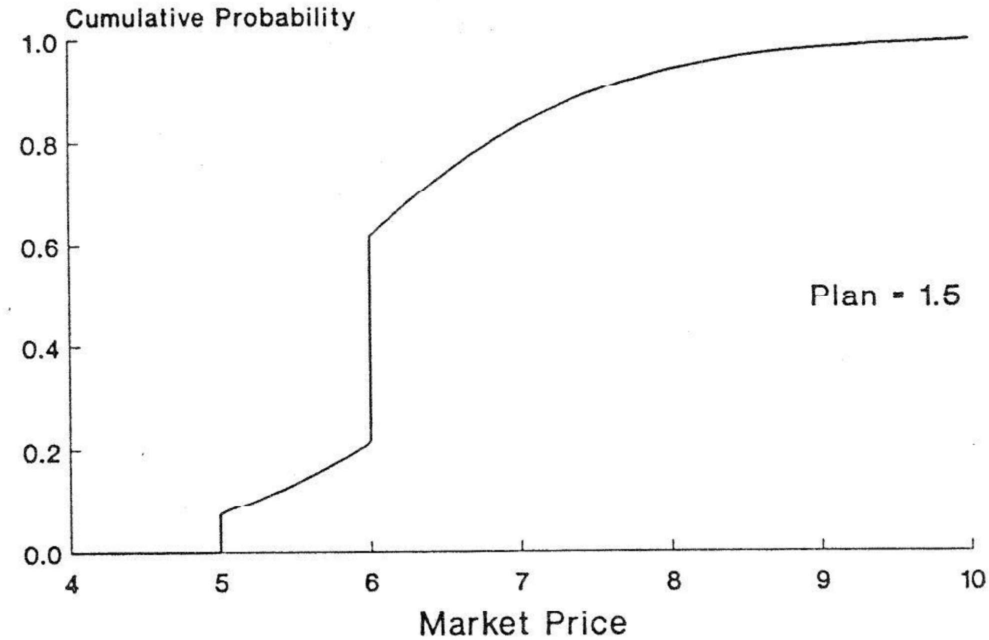
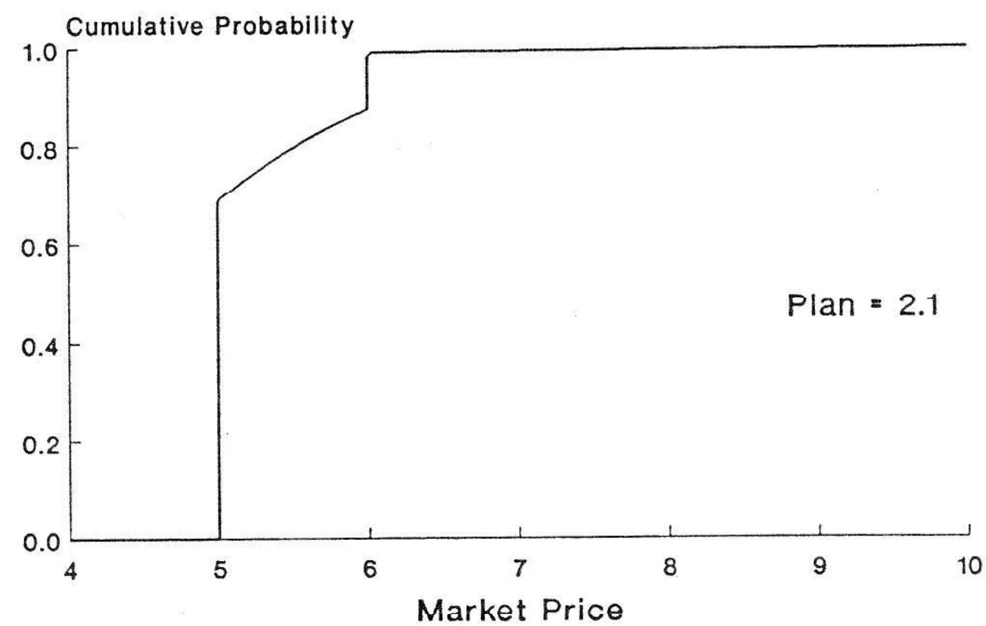


Figure 4a. Distribution of Price for Different Levels of Planned Production



Futures - $6.29



Futures - $5.22

Figure 4b. Put Option Premia for Different Levels of Planned Production

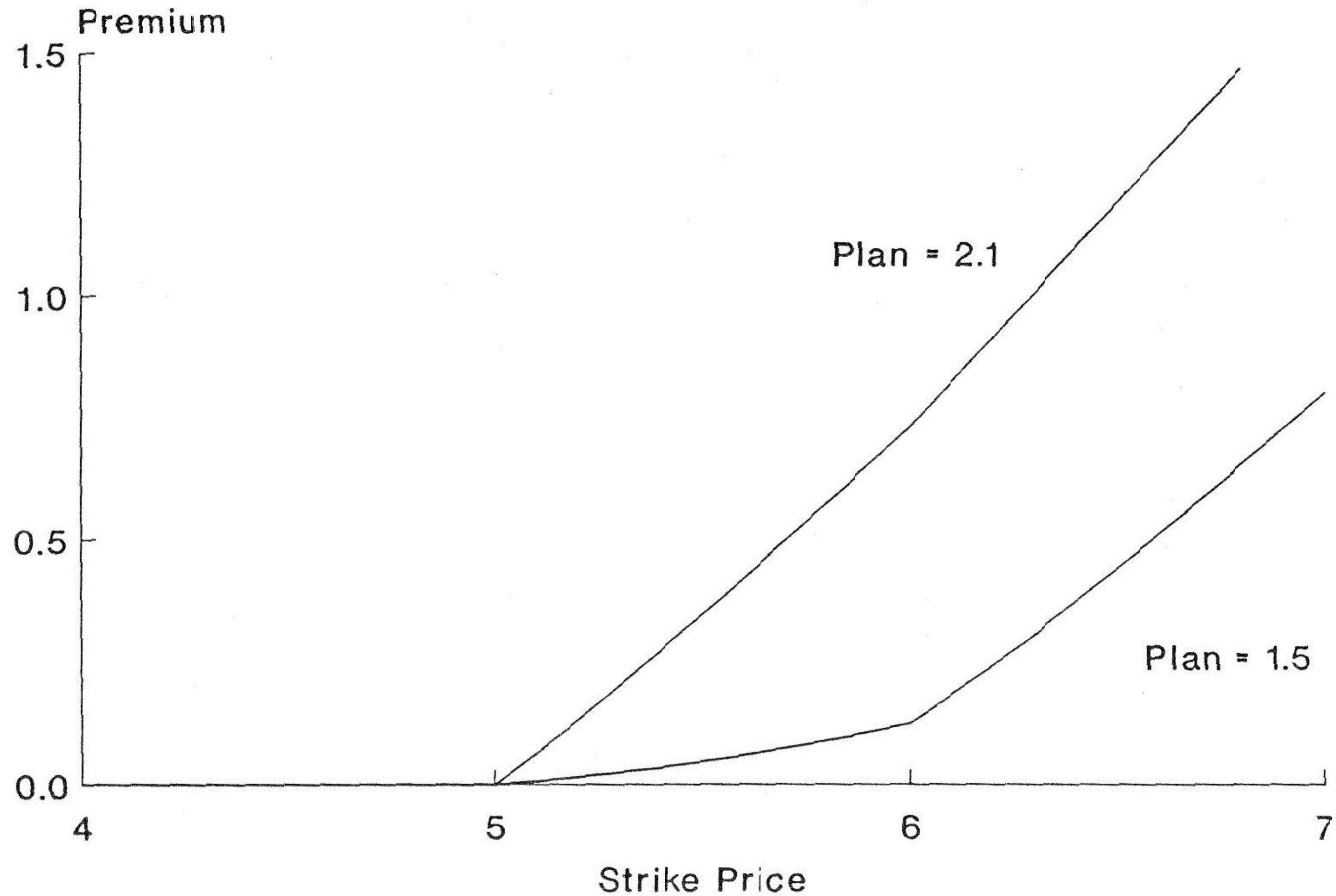


Figure 4c. Call Option Premia for Different Levels of Planned Production