Forecasting Prices in the Forward Cash Market for Corn and Soybeans

by

David D. Johnson

Suggested citation format:

Johnson, D. D. 1990. “Forecasting Prices in the Forward Cash Market for Corn and Soybeans.” Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

Forecasting Prices in the Forward Cash Market for Corn and Soybeans

David D. Johnson*

There is abundant literature on the relationship between cash and futures markets for grain. Much of the published empirical research has addressed issues of price determination or market efficiency, often in the context of an econometric model. For example, Canarella and Pollard examine whether futures prices are unbiased predictors of spot prices. They estimate a bivariate autoregression (of spot and futures prices) for a variety of agricultural commodities, and test the cross-equation restrictions implied by rational expectations. The authors conclude that futures prices are efficient and unbiased predictors of the spot prices at contract maturity.

Garbade and Silber examine the relative importance of spot and futures markets as centers of "price discovery". They develop a model of arbitrage between cash and futures markets, consistent with a simple form of price dynamics.[1] Futures and spot prices are represented statistically as a bivariate random walk; coefficients from the forecast equations indicate the relative importance of the two markets in price formation. Garbade and Silber present empirical results for a variety of agricultural commodities, suggesting that spot markets play a relatively minor role in the price discovery process.

These studies and many others[2] confine their attention to the relationship between futures and spot prices. However, the cash market for grain is not limited to spot transactions. On any given day, elevators and other grain merchandisers quote cash prices for an array of delivery dates. Prices are generally quoted in terms of premiums (or discounts) relative to a futures contract. For example, a forward bid for corn, specifying delivery in August, will be quoted in terms of a premium (in cents per bushel) relative to the September corn futures price. Forward cash contracts are essential marketing tools, and as such they represent an additional center of price discovery. Tests of market efficiency, framed in terms of econometric forecasts, can also be extended to the forward cash market. In an efficient market, one would not expect forward contract premiums to contain information relevant for the prediction of futures prices, or vice versa.

This paper is concerned with the predictability of forward contract premiums, and with possible dynamic interactions between forward premiums and futures. Broadly, the analysis addresses the following questions: 1) Can one contract price can be used to predict another?; and 2) Can an econometric model improve upon a naive, "no-change" price forecast for forward premiums or futures?

In the next section, results of some bivariate causality tests are presented. These indicate whether current and past observations

---

* Graduate student, Department of Agricultural and Applied Economics, University of Minnesota. I am indebted to Robert P. King and Christopher Sims for comments and suggestions.

of one price have predictive power for a second price. The forecast performance of a bivariate autoregression (including futures and forward premiums) is also summarized. The second section introduces the concept of "cointegration"--a property exhibited by some economic time series, with special implications for market dynamics--and applies a test for cointegration to samples of futures and forward premiums. The paper concludes with an overview of results.

1 Causality Tests and Bivariate Forecasts

If agents in the cash grain and futures markets made efficient use of available information, contract prices would behave approximately as martingales. That is to say, contract prices would equal their own conditional expectations. In an earlier paper[3], implications of martingale behavior were explored with univariate statistical tests. This section applies bivariate tests, designed to show whether the conditional distribution of one price (estimated through a linear forecast) is modified by knowledge of a second price.

Let X and Y denote the prices attached to two different contracts. If knowledge of Y (ie, current and past observations) affects the conditional forecast of X, then Y is said to *cause* X in the sense of Granger. A simple test of causality is based on the estimated residuals from two regressions:

$$X(t) = a + \sum_{j=1}^{K} b_j X(t-j) + \epsilon_t \tag{1}$$

$$X(t) = c + \sum_{j=1}^{K} d_j X(t-j) + \sum_{j=1}^{K} f_j Y(t-j) + \upsilon_t \tag{2}$$

Let $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ denote the mean squared errors from regressions (1) and (2), respectively. The null hypothesis for the test is that Y does not cause X. Define the test statistic:

$$T = n \cdot (\hat{\sigma}_1^2 - \hat{\sigma}_2^2) / \hat{\sigma}_2^2$$

where n is the number of observations. Under the null, T is distributed asymptotically as χ^2 with K degrees of freedom.[4] An analogous statistic is used to test the null that X does not cause Y. Tests of causation in both directions are reported in the results below, for different pairings of forward contract premiums and futures.

Daily data on forward premiums were drawn from the *Kansas City Grain Market Review*, a daily trade publication. These represent early morning bids by exporters for forward delivery of grain to Gulf ports, expressed in cents above a specified futures price. The forward premiums are in effect when trading begins at the Chicago Board of Trade; hence the relevant futures price is the daily open. The sum of a premium and specified futures represents a "flat price" bid for forward delivery of grain.[5]

Forward premiums, like futures prices, tend to exhibit greater volatility as the delivery date approaches. Accordingly, it was necessary to condition the data, prior to testing, to ensure unbiased statistics. Trends in variance were estimated from univariate regression reesiduals; the data were then transformed to produce homoskedastic errors, and test statistics were computed using weighted least squares. Details concerning the tests, and results for individual samples, are available from the author.

The first test was applied to samples consisting of two forward premiums (for corn or soybeans) with delivery in consecutive months. For example, the premium for corn delivered in October 1985, was paired with the premium for corn delivered in November, 1985. Nearly forty samples of this kind, containing at least 50 daily observations, were constructed for each commodity. Regressions in the form of equations (1) and (2) were estimated for each sample using 4 lags,[6] with variables interchanged to test causation in both directions. Results of these tests are summarized in Table 1.

Table 1

Bivariate Causality Test: Forward Contract Premiums In Consecutive Delivery Months *

Commodity	Samples Tested	Rejections of $H_0: Y \not\Rightarrow X$	Rejections of $H_0: X \not\Rightarrow Y$
Corn	38	9	10
Soybeans	39	12	12

* X denotes the first forward premium; Y denotes the second forward premium (ie, for delivery in the next calendar month). In each case, a rejection of the null hypothesis suggests that one premium causes the other, in the sense of Granger. Rejections are based on an approximate 5 % significance level.

In roughly a quarter of the corn samples and a third of the soybean samples, causation from the first premium to the second could not be rejected. In these samples the first premium appears to have predictive power for the second. Results in the opposite direction (from the second premium to the first) were similar. In a large number of cases, the premium associated with one delivery month does not fully adjust[7] to information contained in the premium for an adjacent month.

Further tests were conducted to see whether premiums for two different commodities could exert causal influence. Forty-eight samples were constructed (with at least fifty observations) consisting of one corn premium and one soybean premium, each with the same forward delivery date. Under the null hypothesis for the test, the premium for one commodity cannot Granger-cause another. Results are summarized in Table 2.

Table 2

Bivariate Causality Test: Forward Contract Premiums For Corn and Soybeans, Same Delivery Month *

Samples Tested	Rejections of $H_0: S \not\Rightarrow C$	Rejections of $H_0: C \not\Rightarrow S$
48	11	15

* C denotes the corn premium; S denotes the soybean premium. In each case, a rejection of the null hypothesis suggests that one premium causes the other, in the sense of Granger. Rejections are based on an approximate 5 % significance level.

Again, the number of rejections casts some doubt on the null hypothesis of pricing efficiency. In a large number of cases, the corn premium has predictive power for the soybean premium, and vice versa. That there should be some influence of one commodity premium on another is understandable, given their shared dependence on barge transportation.[8] However, the transmission of price effects between commodities would be virtually instantaneous if the market were fully efficient.

Finally, causality tests was applied to samples consisting of forward premiums and futures. In each sample, the premium was paired with the daily open of the relevant futures contract. Fifty-three such samples were constructed for corn, and forty-nine for soybeans; results are presented in Table 3. In a large number of cases, the forward premium has predictive power for the futures price. More striking is the evidence of causality in the other direction: in 23 of 49 soybean samples, and in 15 of 53 corn samples, the futures price has predictive power for the premium. While neither causal ordering is pervasive, the evidence does suggest the occurrence of lagged adjustment, in each market, to price information from the other market.

Table 3

Bivariate Causality Test: Forward Contract Premium and Futures Contract Price *

Commodity	Samples Tested	Rejections of Ho: P $\not\Rightarrow$ F	Rejections of Ho: F $\not\Rightarrow$ P
Corn	53	11	15
Soybeans	49	13	23

* P denotes the forward premium; F denotes the futures price. In each case, a rejection of the null hypothesis suggests that one variable causes the other, in the sense of Granger. Rejections are based on an approximate 5 % significance level.

The practical implications of these results are best examined with out-of-sample forecasts. In particular, one would not expect an econometric price forecast to perform any better than a naive, "no-change" forecast for either futures or forward premiums--provided that markets are informationally efficient. To test this empirically, bivariate autoregressions were estimated for each commodity and used to generate out-of-sample forecasts. The forecast equations were of the form:

$$F(t) = a + \sum_{j=1}^{4} b_j\, F(t-j) + \sum_{j=1}^{4} c_j\, P(t-j) + \upsilon_t \qquad (3)$$

$$P(t) = d + \sum_{j=1}^{4} e_j\, F(t-j) + \sum_{j=1}^{4} f_j\, P(t-j) + \epsilon_t \qquad (4)$$

where F(t) is the futures open price, and P(t) the forward premium. Samples with at least 75 observations (after accounting for lags) were selected for testing. In each sample, the first (n-25) observations were used to obtain initial parameter estimates; these estimates were updated with each successive observation and used to compute forecast values.

The econometric forecasts are readily compared to a "naive" price forecast. That comparison is summarized by Theil's U statistic, which is defined (for a k-step ahead forecast):

$$U_k = \frac{\Sigma\,(X_{t+k} - A_{t+k})^2}{\Sigma\,(A_{t+k} - A_t)^2}$$

Here X(t+k) denotes a forecast value, based on information available through period t; while A(t) and A(t+k) are actual values of the variable of interest. A statistic greater than one indicates that the naive forecast is superior to the econometric model (in terms of squared errors); a statistic less than one indicates that the naive forecast is inferior to the econometric model.

Thirty samples were tested for each commodity; a summary of forecast performance is presented in Table 4.

Table 4
Summary of Forecast Performance of a Bivariate Model:
Futures Price and Forward Premium *

		Percentage of samples in which Theil U Statistic < 1: Forecast Intervals					
Commodity	Contract	1-step	2-step	3-step	5-step	7-step	9-step
Corn	F	13.3	26.7	30.0	36.7	36.7	40.0
Corn	P	20.0	26.7	26.7	33.3	40.0	40.0
Soybeans	F	10.0	6.7	10.0	13.3	26.7	33.3
Soybeans	P	26.7	26.7	33.3	43.3	43.3	43.3

* F denotes futures contract, and P denotes premium. Thirty samples were tested for each commodity. The performance of the econometric model is deemed superior to the naive model if Theil's U statistic is less than one, for given forecast interval.

In most cases, the bivariate econometric model was out-performed by the naive, no-change price forecast. With each increase in the forecast interval, the bivariate model showed some improvement relative to the naive forecast; however, the results do not inspire confidence in the bivariate model as a forecasting tool. While the causality tests indicated some lagged responses of forward premiums to futures prices (and vice versa), it is not clear that these can be exploited in a practical setting.

2 Testing for Cointegration of Forward Premiums and Futures

The concept of cointegration, introduced by Engle and Granger, has relevance for the study of dynamic interaction between forward premiums and futures. In this section a test for cointegration is presented as an alternative test of pricing efficiency.

Cointegration concerns the joint behavior of a set of variables governed by arbitrage or equilibrium conditions. A variable X(t) is said to be *integrated* of order i, denoted I(i), if it must be differenced i times to induce stationarity. If a pair of variables

X(t) and Y(t) are each integrated of order 1, a linear combination of them

$$Z(t) = X(t) + \alpha\, Y(t)$$

will also be I(1) in general. However, among some sets of variables, there may be linear combinations that are stationary without differencing. If there exists a value α such that Z(t) is I(0), then the variables X(t) and Y(t) are said to be *cointegrated*.

An interesting attribute of cointegrated variables is that they may be represented by an error-correction model. Let W(t) denote a nx1 vector of variables that are individually I(1), and let α denote a nx1 vector of constants satisfying

$$\alpha' W(t) = Z(t) \sim I(0)$$

The error-correction model has the form

$$A(L)\cdot(1-L)\cdot W(t) = -\gamma\cdot Z(t-1) + U(t)$$

where A(L) is a matrix polynomial in the lag operator, γ is a nx1 vector of constants, and u(t) is a stationary nx1 disturbance term.[9] The variable Z(t-1) can be interpreted as a deviation from an equilibrium condition; the elements of γ indicate how each variable in W(t) responds to this lagged deviation.[10]

Tests of cointegration have been applied in a wide variety of (mostly macroeconomic) contexts. A few studies have tested for cointegration of commodity prices--notably Ardeni, and Granger and Escribano. Ardeni finds that prices of individual commodities in different countries (expressed in common currency) do not exhibit cointegration--an apparent contradiction of the "law of one price".[11] Granger and Escribano test for (and reject) cointegration of gold and silver prices. They interpret the absence of cointegration between prices as a confirmation of market efficiency.

The arguments that motivate tests of cointegration in these two studies are quite different. Ardeni expects to find cointegration, because arbitrage should ensure that prices in spatially separate markets move together. Granger and Escribano argue that gold and silver prices--or indeed, prices of any two assets that might be investment substitutes--cannot exhibit any long-term, stable relationship. That is because the error-correction mechanism, implied by cointegration, gives rise to exploitable price forecasts.

In the case of forward premiums and futures, both lines of argument carry some force. On one hand, it seems plausible that price spreads in forward cash markets would reflect concurrent spreads in futures; otherwise, price spreads in the two markets would imply different returns to storage. This would suggest a fairly stable relationship between prices in the two markets, generated by a form of arbitrage between forward cash positions and futures. On the other hand, the existence of a stable relationship means that market participants should be able to identify temporary deviations from equilibrium. The ability to forecast prices (based on these short-lived deviations) is hard to reconcile with pricing efficiency in the cash grain and futures markets.

To make these ideas concrete, consider the relationship between two pairs of corn prices. Let C(Jt) denote a flat-price forward bid, observed on date t for delivery in July. Similarly, let C(At) denote a flat-price forward bid for delivery in August. These forward bids are quoted in terms of premiums vis-a-vis two futures contracts, maturing in July and September, respectively. The flat-price bids equal the sum of futures and premiums:

$$C_{Jt} = F_{Jt} + P_{Jt} \qquad\qquad C_{At} = F_{St} + P_{At}$$

Assume that, in their univariate representations, premiums and futures require first differencing to induce stationarity; and further, that the spread between forward bids reflects the spread between futures prices, except for a stationary, zero-mean error process:

$$C_{At} - C_{Jt} = \gamma\cdot(F_{St} - F_{Jt}) + Z_t$$

where γ is a constant and Z(t) is the error.[12] This illustrates a possible form of cointegration between futures and forward premiums; in general, the linear dependence between variables might take a different form. After substitutions and rearranging, the relationship between the two pairs of prices can be expressed as regression equation:

$$P_{Jt} = \alpha_0 + \alpha_1\cdot P_{At} + \alpha_2\cdot F_{Jt} + \alpha_3\cdot F_{St} + Z_t \qquad (3)$$

which is arbitrarily normalized on the first forward premium.[13] This form of "cointegrating regression" provides the basis for the tests presented below. Cointegration is said to exist if the process Z(t) (ie, the regression residual) is stationary without differencing.

Each sample for the test consists of two pairs of prices: two forward premiums (for consecutive delivery months), and two futures contracts. The delivery date for the first futures contract coincides with that of the first forward premium, as in the above example. The time interval between futures contracts includes the interval between forward delivery months, so that price spreads in the two markets are comparable in terms of the implied returns to storage. Thirteen samples were considered for each commodity.

For each sample, it was first necessary to verify that individual price series contained unit roots in their autoregressive representations. The presence of a unit root implies that differencing is necessary to induce stationarity. A convenient test, due to Dickey and Fuller, was used to test for unit roots in the price series.[14] The test is based on a regression of the form:

$$\Delta X(t) = \beta_0 + \beta_1 X(t-1) \sum_{i=2}^{k} \beta_i\cdot\Delta X(t-i) + u(t) \qquad (4)$$

where X(t) is the variable of interest, and first differences are denoted by $\Delta X(t)$. Under the null hypothesis, $X(t)$ has a unit root; the test is based on the t-value associated with the estimated coefficient β_1. The distribution of this t-value has a nonstandard distribution under the null, which Dickey has tabulated.[15] With a 5 percent significance level chosen for the test (and 4 lags in the regression) the null was tested against the stationary alternative for each premium and futures price. In no case was the null rejected.[16]

The next step was to estimate the cointegrating regression (equation 3), and test the residuals for stationarity. Under the null hypothesis for this test, the autoregressive representation of the residuals contains a unit root. A test of this hypothesis is based on the following regression:

$$\Delta Z(t) = \varphi_0 Z(t-1) + \sum_{i=1}^{4} \varphi_i \, \Delta Z(t-i) \tag{5}$$

where $Z(t)$ is the regression residual, and ΔZ denotes a first difference.[17] A test of the null hypothesis (of non-cointegration) is based on the t-value associated with φ_0. Critical values for this statistic (with four variables in the cointegrating regression) are tabulated by Engle and Yoo. Results of the test are summarized in Table 5.

Table 5

Test of Non-Cointegration: Samples Including 2 Forward Contract Premiums and 2 Futures Prices *

Commodity	Samples Tested	Rejections of H_0: $Z(t) \sim I(1)$
Corn	13	0
Soybeans	13	1

* The null hypothesis for the test is that $Z(t)$, the residual from regression equation (3), is integrated of order 1. Rejection of the null hypothesis indicates that the four price series are cointegrated.

In only one sample do the results support cointegration of forward premiums and futures. In all other samples, the test statistics do not permit rejection of the null, which holds that the residual series is nonstationary.

It is difficult to draw firm conclusions from these results. On one hand, the failure to reject non-cointegration suggests that there may be no stable, linear relationship between prices--i.e., no "arbitrage" or "equilibrium" condition like that postulated in error-correction models that could serve as a basis for price forecasts. While this supports the view that forward cash and futures markets make efficient use of price information, it also suggests, against intuition, that price spreads in the two markets are disconnected--i.e., can wander arbitrarily far apart.

On the other hand, the failure to reject (in all but one case) does not imply that the null is necessarily true. It is possible that the price series are indeed cointegrated, but that longer samples (or more powerful tests) are required to detect it. An examination of the characteristic roots associated with each series Z(t) lends some support to this idea. In all cases, the estimated roots were less than one in absolute value, as required for stationarity. (See appendix table.) Moreover, in 5 of the soybean samples and 8 of the corn samples, the maximal root was less than 0.8 in magnitude, indicating fairly rapid reversion to the mean. From this perspective, the evidence does not disprove cointegration of forward premiums and futures--but also does not offer strong support.

Further testing, involving additional data series, may be necessary to resolve these issues more satisfactorily. In particular, it seems possible that spatial price differences (which change over time) could account for an apparent absence of cointegration between prices in the forward cash and futures market. If forward barge rates were included in the information set, different test results would obtain.[18]

3 Summary of Results and Implications

This paper has addressed issues related to pricing efficiency in the forward market for corn and soybeans. Particular attention was paid to relationships between forward premiums and futures--an aspect of price determination that has received little attention in the published literature.

Results from bivariate causality tests indicate that, in a large number of cases, the information contained in one forward premium is relevant for the prediction of a second premium. Thus, contract premiums for adjacent delivery months have predictive power for each other in about a quarter of the corn samples, and in about a third of the soybean samples. Premiums for different commodities (but the same delivery date) also have predictive power in many cases. In these samples, premiums adjust to new price information with a lag, rather than instantaneously as one would expect in an efficient market.

When applied to forward premiums and futures, the test yields some evidence of causality in each direction. The influence of futures on premiums is more pronounced. In nearly a third of the corn samples, and nearly half of the soybean samples, futures prices have predictive power for forward premiums. This may be due to an informational asymmetry: if exporters' early-morning bids are made in advance of the market open, they are known to futures traders; the futures price must then reflect more information than the forward premium. This argument obviously cannot explain causation in the other direction (ie, from premiums to futures). In about a fifth of the corn samples, and a fourth of the soybean samples, forward premiums have predictive power for the associated futures contract.

It is natural to ask whether econometric price forecasts are useful in practice. Although further tests might be warranted, the results presented here do not encourage that belief. The out-of-sample performance of a bivariate autoregression (on futures and premiums) was generally inferior to that of a naive, no-change price forecast. Moreover, tests of non-cointegration suggest that there may be no stable relationship between pairs of forward premiums and futures prices. While these results are not conclusive, they do cast some doubt on the possibility of arbitrage between the forward cash market and futures. Differences in the implied returns to storage may elicit some portfolio adjustments by grain merchandisers, but there is little evidence of a price-correction mechanism linking the two markets.

References

Ardeni, P.G. "Does the Law of One Price Really Hold for Commodity Prices?" *American Journal of Agricultural Economics*. 71(1989):661-669.

Canarella, G., and S.K. Pollard. "Efficiency of Commodity Futures: A Vector Autoregression Analysis." *The Journal of Futures Markets*. 5(1985):57-76.

Dickey, D.A., and W. Fuller. "Likelihood Ratio Statistics for Autoregressive Series with a Unit Root." *Econometrica*. 49(1981):1057-1072.

Engle, R.F., and C.W.J. Granger. "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica*. 55(1987):251-276.

Engel, R.F., and B.S. Yoo. "Forecasting and Testing in Co-integrated Systems." *Journal of Econometrics*. 35(1987):143-159.

Fuller, W.A. *Introduction to Statistical Time Series*. New York: John Wiley and Sons, 1976.

Garbade, K.D., and W.L. Silber. "Price Movements and Price Discovery in Futures and Cash Markets." *Review of Economics and Statistics*. 65(1983):289-297.

Granger, C.W.J., and A. Escribano. "The Long-Run Relationship Between Prices in an Efficient Market." Discussion Paper (draft), Department of Economics, University of California-San Diego, September 1988.

Granger, C.W.J., and P. Newbold. *Forecasting Economic Time Series*. San Francisco: Academic Press, 1987.

Johnson, D. "The Stochastic Behavior of Forward Contract Premiums for Corn and Soybeans," Center for the Study of Futures Markets, Columbia Business School, Working Paper #202, January 1990.

Kamara, A. "Issues in Futures Markets: A Survey." *The Journal of Futures Markets*. 2(1982):261-294.

Sims, C.A., and J. Uhlig. "Understanding Unit Rooters: A Helicopter Tour." Discussion Paper, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis, June 1988.

Footnotes

1 The forecasting model estimated by Garbade and Silber is of the for

$$\begin{bmatrix} C_t \\ F_t \end{bmatrix} = \begin{bmatrix} \alpha_c \\ \alpha_f \end{bmatrix} + \begin{bmatrix} 1 - \beta_c & \beta_c \\ \beta_f & 1 - \beta_f \end{bmatrix} \begin{bmatrix} C_{t-1} \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^c \\ \epsilon_t^f \end{bmatrix}$$

where C is the spot price and F is the futures, expressed (after discounting) in terms of its cash equivalent. If the ratio $\beta_c/(\beta_c + \beta_f)$ is close to one, the spot market is essentially a "satellite" of the futures market, with little independent influence in the process of price formation. See pages 293-294.

2 See Kamara for a useful review of the literature.

3 See Johnson.

4 See Granger and Newbold, pp. 220-221 and 259-260.

5 For example, consider a forward premium for soybeans (eg, quoted in July) for Gulf delivery in October. October premiums use the November soybean futures as a reference; hence the the flat-price forward bid will be the November futures price plus the October premium.

6 Based on a Box-Pierce Q test, four lags were sufficient to produce uncorrelated residuals in the univariate regressions.

7 Because many of the samples are for overlapping periods, they are not strictly independent. The overall number of rejections should thus be interpreted with some caution. When the results are disaggregated by calendar month of delivery (to ensure independence), they do show a higher proportion of rejections than would be expected from repeated random sampling.

8 For example, changes in the corn premium may affect the demand for barge transportation to the Gulf; changes in barge rates will then become reflected in soybean premiums.

9 For simplicity, we are assuming that there is a unique cointegrating vector α. More generally, it is possible that several cointegrating vectors will exit. In that case, α will be a nxr matrix (r<n), Z(t) will be rx1, and γ will be nxr.

10 If the error correction term were omitted (and with suitable assumptions about the errors), the model would be simply be a vector autoregression in first differences of W(t). In the presence of cointegration, such a model (omitting the correction term) would be misspecified. Moreover, there may be efficiency gains from generating forecasts with an error-correction model, rather than a VAR in levels of the data, since the latter ignores cross-equation restrictions implied by cointegration.

11 Ardeni tests a number of bivariate samples, consisting of export or import prices for individual agricultural commodities. He argues that arbitrage should force the convergence of commodity prices in two countries, at least in the long term. Formally, he posits the equilibrium condition:

$$P = P' \cdot E = \hat{P}$$

where P is the price in country 1; P′ is the price in country 2 (expressed in own currency); E is the exchange rate; and $\hat{P}$ is the country-2 price expressed in the currency of country 1. Expressed in log form, this is the "cointegrating equation" estimated by Ardeni. His test results suggest that the law of one price may not hold, even in the long run, for a variety of traded primary commodities.

12 Since the time interval between forward delivery periods does not coincide with the interval between futures contracts, γ need not equal one for spreads to imply the same returns to storage.

13 Engel and Granger suggest that the choice of normalization for the cointegrating regression matters little in practice.

14 See Dickey and Fuller [1981] for a comparison of several alternative tests of the unit root hypothesis. The test applied to forward premiums and futures is sometimes referred to in the literature as an "augmented Dickey-Fuller" test, to distinguish it from a test involving only one lagged difference in the regression.

15 Critical values for the test are reproduced in Fuller [1976], p. 373.

16 For a Bayesian critique of classical tests of the unit root hypothesis, see Sims and Uhlrig. They point out that tests of the hypothesis have low power against stationary alternatives.

17 Equation 5 was estimated by weighted least squares to avoid bias due to changing variance of ΔZ. Under the null for the test, ΔZ is stationary with fixed variance. Weights were calculated by fitting a fourth-order autoregression to ΔZ, then estimating standard errors as a function of time.

18 Unfortunately, forward transportation rates for delivery to the Gulf are not readily available on a daily basis. The St. Louis Merchants Exchange has a daily call session for barge transportation, but bids and offers (by point of origin on the Mississippi) are highly irregular.

Appendix

Test of Non-Cointegration: Two Forward Premiums and Two Futures Contracts*

Corn

Contracts in Sample				Observations	t-value	maximal root
P 85-03	P 85-04	F 85-03	F 85-05	100	-2.011	0.916
P 86-03	P 86-04	F 86-03	F 86-05	79	-2.924	0.749
P 88-03	P 88-04	F 88-03	F 88-05	65	-1.533	0.856
P 89-03	P 89-04	F 89-03	F 89-05	71	-3.485	0.683
P 85-05	P 85-06	F 85-05	F 85-07	97	-2.111	0.868
P 86-05	P 86-06	F 86-05	F 86-07	73	-2.374	0.761
P 87-05	P 87-06	F 87-05	F 87-07	57	-3.032	0.795
P 85-07	P 85-08	F 85-07	F 85-09	81	-2.982	0.709
P 86-07	P 86-08	F 86-07	F 86-09	71	-2.070	0.736
P 86-09	P 86-10	F 86-09	F 86-12	67	-2.436	0.781
P 87-09	P 97-10	F 87-09	F 87-12	69	-2.891	0.610
P 87-12	P 88-01	F 87-12	F 88-03	58	-2.905	0.809
P 88-12	P 89-01	F 88-12	F 89-03	52	-2.480	0.803

Soybeans

Contracts in Sample				Observations	t-value	maximal root
P 85-01	P 85-02	F 85-01	F 85-03	84	-4.058 †	0.742
P 86-01	P 86-02	F 86-01	F 86-03	115	-2.084	0.887
P 87-01	P 87-02	F 87-01	F 87-03	51	-2.027	0.739
P 89-01	P 89-02	F 89-01	F 89-03	52	-1.587	0.852
P 87-03	P 87-04	F 87-03	F 87-05	52	-2.744	0.628
P 88-03	P 88-04	F 88-03	F 88-05	55	-0.602	0.976
P 89-03	P 89-04	F 89-03	F 89-05	71	-1.827	0.881
P 85-05	P 85-06	F 85-05	F 85-07	87	-2.839	0.790
P 87-05	P 87-06	F 87-05	F 87-07	60	-2.175	0.818
P 85-07	P 85-08	F 85-07	F 85-08	74	-2.398	0.828
P 84-11	P 84-12	F 84-11	F 85-01	55	-1.200	0.869
P 85-11	P 85-12	F 85-11	F 86-01	123	-1.800	0.918
P 86-11	P 86-12	F 86-11	F 87-01	71	-3.353	0.529

* The contracts in each sample are identified by delivery date. P denotes a forward premium; F denotes a futures contract. The t-value applies to the coefficient φ_0 in the regression:

$$\Delta Z(t) = \varphi_0 Z(t-1) + \sum_{i=1}^{4} \varphi_i \Delta Z(t-i) + u(t)$$

where Z(t) is the residual from the cointegrating regression, and ΔZ is a first difference.

† signifies rejection of the null hypothesis (non-cointegration) based on 5 % critical values tabulated by Engle and Yoo.