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Sensitivity of Risk Premium Analyses to Violations of Distributional Assumptions

T. Randall Fortenbery and Hector O. Zapata

Introduction

The search for a futures market risk premium has led to increased sophistication in analytical techniques, and a recognition that the risk and return characteristics of futures contracts should be measured in a portfolio context, as opposed to individually. Most risk premium work has been conducted with empirical models that assume normality in the underlying distribution of futures and portfolio returns. The assumption of normality has not always been empirically tested, and the sensitivity of specific results to a violation of this assumption have been lacking.

The purpose of this paper is to use futures contracts and investment horizons employed by three well-known risk premium analyses, estimate futures contract risk using the methodologies of the above-mentioned studies, test the appropriateness of the normality assumption using the Bera-Jarque Lagrange Test for Normality, and discuss the implications of violations of the normality assumption and data transformations to the specific results.

The studies considered are those of Dusak, Bodie and Rosansky, and Lee and Leuthold. These studies generally attempted to measure risk and risk premiums via the Capital Asset Pricing Model (CAPM). The standard form of this model was simultaneously developed by Sharpe, Lintner, and Mossin (Elton and Gruber). It can be expressed as: $E(R_i) - R_f = [E(R_W) - R_f]\beta_i$ where R_i is the return to some asset i, E(.) is an expectations operator, R_f is the risk-free rate of return, R_W is return on total wealth, and β_i represents the risk of asset i relative to that of total wealth. The CAPM is generally estimated as $R_i = \alpha + \beta R_M$, where R_i is the excess return to an individual investment instrument, and R_M is the excess return to a perfectly diversified market portfolio. In a futures market risk premium context, systematic risk is said to exist if the Beta coefficient is statistically significant. Once systematic risk is detected, then any excess return to the investment can be interpreted as a risk premium (Dusak).[1]

Previous Work

Dusak used the CAPM to measure the effect on systematic portfolio risk associated with adding a futures contract to a well-diversified investment portfolio. Her study was unique in that it was the first risk premium study which argued that futures contracts should be evaluated in a portfolio context. She argued that the Keynesian notion of risk defined as the variability of an individual investment's price was not an appropriate definition of risk. Since futures contracts are no different in principle than markets for any other risky assets, and thus are candidates for an investor's portfolio, the appropriate definition and measure of the risk of futures contracts is the effect they would have on the systematic risk of a portfolio when added to it.

Dusak defined the return to a futures position to be the percentage change in futures price over some given investment interval. The commodities she studied were wheat, corn, and soybeans from May 1952 through November 1967. Returns were computed for two-week holding periods, and no account was made for transactions costs. Dusak used the Standard and Poor's Index of 500 Common Stocks as a proxy for return on total wealth (i.e., the market portfolio), and the measure from which futures contracts were evaluated. She used the 15-day Treasury Bill rate as her measure of the risk-free rate of return.

Dusak found that portfolio risk for all three futures contracts was near zero. In addition, risk premiums were essentially non-existent.

Bodie and Rosansky, also using the CAPM, compared the returns of an all stock portfolio, an all futures portfolio, and portfolios consisting of both stocks and futures for the period 1950 through 1976. Using 23 individual commodities they calculated quarterly returns (i.e., futures contracts were held for three months) using two separate definitions of the rate of return. Rate of return was first defined as the percentage change in futures price over the three-month holding period. Their second definition of return included the interest which could have been earned by posting Treasury bills for the required margin of a futures investment.

Returns to futures were considered on an individual commodity basis as well as a portfolio basis. The portfolio approach assumed that each of the 23 commodities would be invested in equally. Since only ten of the 23 commodities were traded in 1950, the portfolio grew with time.

When Bodie and Rosansky compared the returns of their futures portfolio with the excess returns from an all stock portfolio (as defined by the Standard and Poor's 500 discounted by the 90-day Treasury bill rate) they found that the means and standard deviations for both investment alternatives were almost equal. In addition, they found that the frequency distribution of commodity returns was more positively skewed than that of stocks. Based on this they concluded that futures may actually be a less risky investment than stocks. In contrast to Dusak, Bodie and Rosansky derived positive expected returns to futures, and as such concluded that risk premiums do exist in futures markets.

Their use of CAPM differed from that of Dusak. Since they found mean returns to futures and stocks to be nearly equal, they tested for $\beta = 1$. While they argued that risk premiums exist in futures markets, they were unable to verify the existence of systematic risk in the CAPM framework.

Lee and Leuthold evaluated 42 individual futures contracts for the period 1972 to 1977. Using the Sharpe single index model (market model)[2] they tested for the relationship between investment horizon and risk and return characteristics.

Lee and Leuthold regressed the rate of return (defined as the percentage change in price) of the 42 futures contracts on the rate of return to a stock portfolio. The stock portfolio was proxied by the Standard and Poor's 500 stock index.

The results of Lee and Leuthold's work led them to conclude that risk premiums in futures markets do not exist. They found the Beta coefficients from their model did not differ significantly from zero for investment horizons ranging from one to 22 days.

Unlike Dusak, and Bodie and Rosansky, Lee and Leuthold did explicitly test for normality in the underlying distribution of returns. Their tests consisted of a kurtosis and a skewness test. They found that skewness was significantly different from zero in sixteen percent of their returns series, and kurtosis was significantly different from zero in eighteen percent of their returns series. This led them to conclude that standard statistical tests could be conducted, and that the Sharpe single index model was an appropriate empirical test.

Assumptions of CAPM

Elton and Gruber have outlined the basic assumptions of the CAPM as follows:

1. there are no transactions costs,
2. all assets are infinitely divisible,
3. there is no personal income tax,
4. an individual's transactions do not affect prices,
5. returns are normally distributed, or investors' exhibit quadratic utility functions,
6. unlimited short sales are allowed,
7. unlimited borrowing and lending at the riskless rate are allowed,
8. all investors have identical expectations,
9. all assets are marketable.

A more complete discussion of CAPM, including its assumptions, is available from Jensen.

For the purposes of this analysis, we focus on assumption 5, and empirically test the extent to which futures data violate the assumption, the implications of the data transformations on the assumption, and the sensitivity of the results to a violation of the normality assumption.

Data

The data period in this study corresponds to that of Lee and Leuthold; 1972 through 1977. The specific futures contracts considered are the November soybean and December corn futures. These contracts were included in each of the previous studies. The commodity futures index used by Lee and Leuthold is also considered.[3] In addition, the Standard and Poor's 500 stock index is used as the independent variable in all applications. This corresponds to the market portfolio proxy in the previous studies. When analyzing horizons of Dusak, and Bodie and Rosansky we use monthly average T-bill rates as measured on CRSP tapes as our risk free measure. This differs slightly from the measures of Dusak and Bodie and Rosansky. However, they both found the quality of their results to be unaffected by their risk-free instrument, and thus we do not believe this to be a critical detraction.

Investment horizons and measures of return corresponding to those used by Dusak; Bodie and Rosansky; and Lee and Leuthold are simulated, and returns to each investment are calculated for each investment horizon. Using these returns series, results are generated measuring systematic risk with regression techniques employed by the other analysts.

CAPM Results

The results of the CAPM model using Dusak's measure of excess returns and investment horizon are comparable to Dusak's original results. Both the intercepts and the beta coefficients are insignificant at the 5 percent level. This suggests an absence of risk premiums in corn and soybean futures markets for the two-week investment horizon.

The horizon results of Lee and Leuthold are similar to those discussed above. Lee and Leuthold reported results by investment horizon and by year. We utilized the same format. We found about 17 percent of the beta coefficients to be significant at the 5 percent level. This is generally consistent with Lee and Leuthold, who also found a small percentage of significant betas.[4] While there appears to be no systematic pattern to the significant betas, we do find, like Lee and Leuthold, that the soybean contract is most frequently associated with a significant slope coefficient. We also found 12 percent of the intercepts were significant at the 5 percent level. All of these occurred in 1972 and 1973.

Results associated with the investment horizons used by Bodie and Rosansky were also generated. Again, most coefficients are insignificant at the 5 percent level.

Normality Tests

An assumption which has often gone untested in risk premium work is that of normality. This section of the paper applies the Bera-Jarque Lagrange Multiplier (LM) normality test to the returns data utilized above.

In the univariate case, the Bera-Jarque Lagrange Multiplier (LM) test for normality simultaneously tests for skewness and kurtosis. In addition, Bera and Jarque have shown the test to be more powerful than the kurtosis test, skewness test, the D'Agostino D^* test, the Shapiro and Wilk W test, and the Shapiro and Francia W' test. They also concluded that the LM test is more powerful than several other normality tests, including the Kolomogorov-Smirnov test and the Durbin test, because previous work had shown that they were not as powerful as the W test.

Bera and Jarque show that the LM test can be reduced to:

$$LM = N(\sqrt{b_1})^2/6 + (b_2-3)^2/24$$

where $\sqrt{b^1} = \mu 3/\mu 2^{3/2}$ and $b_2 = \mu 4/\mu 2^2$. Notice that $\sqrt{b_1}$ and b_2 are the standard test statistics for skewness and kurtosis, respectively. The statistic is asymptotically distributed as a chi-squared with two degrees of freedom.

The number of times normality was not rejected (from a total of 22 estimated LM values for each return by year) at the 5 percent level are given in Table 1. For a normal distribution, expected skewness is zero and kurtosis is 3. However, skewness and kurtosis of returns series can be affected by the length of the differencing interval over which returns are measured (see Law and Wingender for a more complete discussion). As such, Table 4 also presents the number of positive skewness and kurtosis estimates.

The results in Table 1 indicate that normality of returns is rejected more than half of the time for the 22 investment horizons during 1972-1977. A few exceptions are CFI in 1976 (13 out of 22 cannot be rejected for normality), soybeans in 1973 (12 out of 22) and 1974 (11 out of 22). Although not shown in Table 4, the mean of all returns during each year increased as the investment horizon increased from 1 to 22 days; the variance of returns increased also, but at a slower rate than the mean resulting in a decreasing coefficient of variation by investment horizon. Thus, the standard deviation of returns becomes a smaller percentage of the mean as the investment horizon lengthens.

Results suggest that returns for CFI, corn and soybeans tend to be positively skewed, particularly in the early 1970s. Returns for corn and soybeans, as expected, behaved very much like returns for CFI with respect to skewness preference. The T-Bill had no qualitative impact on either the skewness or kurtosis parameters of the S&P500. Kurtosis for CFI, corn and soybeans took values above and below three; positive kurtosis significantly larger than 3 tended to occur at early investment horizons.

Within the CAPM framework, systematic risk is estimated by the covariance between a given security and the market rate of return divided by the variance of the market rate of return (Dusak). Evidence against normality on either component of these variance-covariance measures can be interpreted as erratic systematic risk measurement. In other words, since the standard deviation of a given return can have an erratic behavior when the sample values do not come from a Gaussian distribution (see Fama and Roll for an analysis on this point), the resulting systematic risk behaves similarly. This is investigated in Table 2.

Across the top of Table 2, the estimated mean value of the 22 betas, the respective standard errors, skewness, kurtosis and LM values are presented. As expected, normality is rejected for all commodities and years when OLS estimates of the systematic risk are used. When correction for autocorrelation using the Cochrane-Orcutt method is made, normality is again rejected for the most part. From the magnitudes and signs of skewness and kurtosis, it is evident that kurtosis fails in almost every case, with skewness varying between positive and negative values. These results suggest that empirical estimates of systematic risk within the CAPM framework using

TABLE 1. NUMBER OF NORMALITY NON-REJECTIONS AT ALPHA = .05, LM TEST.

YEAR		CFI	SP500	CORN	SOYBEAN	SP500-TB*
1972	LM	6	4	9	7	2
	Sk	22	6	22	14	6
	k	21	1	19	13	1
1973	LM	9	7	10	12	7
	Sk	11	17	18	17	16
	k	16	7	18	19	8
1974	LM	1	8	4	7	8
	Sk	18	17	15	19	17
	k	7	16	10	19	16
1975	LM	5	3	6	2	3
	Sk	19	14	22	21	14
	k	16	10	15	14	10
1976	LM	13	7	4	6	7
	Sk	6	21	9	9	21
	k	17	15	11	16	15
1977	LM	1	5	6	11	6
	Sk	0	17	9	4	17
	k	6	17	8	17	17

NOTE: LM is the lagrange multiplier test for normality, sk is the skewness and k is the kurtosis parameters for normality. The values for Sk and k in the table are their positive counts.

* SP500 index deflated by the average T-bill rate.

TABLE 2. SUMMARY STATISTICS FOR PERCENT CHANGE BETAS, 1972-77.

	OLS					AUTO				
ASSET & YEAR	MEAN	STD ERROR	SKEW.	KURT.	LM	MEAN	STD ERROR	SKEW.	KURT.	LM
CFI72	0.17	0.02	-0.10	-0.16	-9.10	0.16	0.03	-1.00	2.40	-11.00
CORN72	0.03	0.05	-0.08	-0.01	-8.28	0.10	0.04	0.07	-0.22	-7.85
SOYBEANS72	0.32	0.05	-0.89	1.50	0.81	-0.32	-0.05	-0.93	1.59	-10.99
CFI73	-0.05	0.05	-0.56	-0.90	-12.83	-0.08	0.05	-0.50	-0.73	-10.30
CORN73	-0.08	0.10	0.80	1.59	0.53	-0.12	0.08	-0.21	-0.21	0.92
SOYBEANS73	-0.13	0.09	-0.50	-0.85	-12.67	-0.10	0.08	-0.50	-0.68	-10.32
CFI74	0.01	0.04	1.25	2.14	5.07	0.01	0.03	1.08	1.09	0.92
CORN74	-0.04	0.06	0.67	0.31	-5.00	0.01	0.05	0.78	0.02	-2.26
SOYBEANS74	-0.20	0.07	-0.10	-0.04	-8.44	-0.19	0.08	-0.12	-0.09	-8.86
CFI75	-0.42	0.06	-0.45	-0.41	-9.89	-0.19	0.08	-0.12	-0.09	-10.24
CORN75	-0.45	0.09	0.90	0.22	-4.16	-0.35	0.10	0.65	-0.54	-3.50
SOYBEANS75	-0.29	0.11	0.67	-0.23	-7.91	-0.25	0.11	0.51	-0.58	-4.72
CFI76	0.12	0.02	0.06	-0.53	-11.39	0.11	0.05	1.08	1.01	0.85
CORN76	0.30	0.03	0.08	-0.42	-10.69	0.50	0.11	1.70	3.31	9.00
SOYBEANS76	0.32	0.04	0.36	-0.64	-11.63	0.46	0.10	1.00	1.05	0.03
CFI77	-0.10	0.05	-0.74	-0.55	-9.55	0.07	0.04	-0.07	0.04	-8.63
CORN77	-0.12	0.07	-0.60	-0.60	-10.58	0.02	0.07	-0.44	0.67	-10.14
SOYBEANS77	-0.04	0.08	-1.09	0.28	-2.42	0.09	0.12	0.59	1.86	-4.08

OLS are not only inefficient but also nonnormally distributed. Nonnormality is the result of both skewness and kurtosis failures. As such, inferences about systematic risk based on OLS or some form of an autocorrelation corrected model may not be warranted.

Implications for Risk Measure Interpretation

There is considerable literature discussing the implications of systematic risk measurements for futures contracts using CAPM when the proxy for the market portfolio is misspecified. Carter, Rausser, and Schmitz reevaluated Dusak's futures market risk premium measures based on the assumption that she had inappropriately proxied the market portfolio. Their work was followed by Baxter, Conine, and Tamarkin and by Marcus. It is clear from this body of research that, as one would expect, risk measures are sensitive to the market portfolio specification.

The results generated here suggest that violations of distributional assumptions may also result in erratic risk estimates. This is an important result. The literature directed at properly identifying the market portfolio consistently assumed that normality of returns was either a valid assumption, or that deviations from normality were not critical to risk measures. The effect was to attribute any differences in risk estimates entirely to differences in the way the market portfolio was proxied. This may be an overly simplistic interpretation of previous results.

The results generated here show that, for traditional returns measures in futures, normality is often not a valid assumption. Furthermore, failures of both kurtosis and skewness contribute to the nonnormality. Thus, returns distributions cannot even be assumed to be symmetric implying the first two moments of the distribution do not adequately measure the risk and return attributes of a futures contract. The implication is that even with an adequately defined market portfolio proxy, measures of future market risk premiums using the traditional CAPM approach may be suspect. Thus, while proper measurement of the market portfolio is important, a more fundamental question might be the extent to which measurement of systematic risk in futures is appropriate using CAPM. If the specific distributional assumption of the CAPM is not explicitly dealt with, risk measures may be unreliable.

Conclusions

Using data and empirical techniques employed in previous futures market risk premium analyses, this paper evaluated the appropriateness of distributional assumptions in model estimation. For the data used here, the assumption of normal returns distributions was violated more than half the time. Furthermore, violations of normality resulted from failures of both skewness and kurtosis. Thus, the less restrictive assumption of symmetric distributions was also found to be unsupported. Given the extent to which normality was rejected, it was argued that traditional measures of systematic risk in futures markets may be unreliable. While measures of sensitivity to violations of normality were not explored, it seems clear that the normality assumption is critical.

Based on these results, further research in measuring futures market risk is warranted. Areas for potential research include examinations of regression tendencies across beta estimates, measurements of futures returns which are consistent with the normality assumption, and sensitivity measures for violations of the normality assumption. These are important areas which could contribute to our general understanding of futures markets. They are, however, beyond the scope of this paper.

Footnotes

1. More explicitly, the CAPM implies that the mean excess return of a commodity (or its risk premium) is equal to its beta times the mean excess return on the market portfolio. Thus, testing $\beta = 0$ is a test of the systematic risk contribution of R_i to the portfolio R_m. In other words, it measures the systematic risk R_i would contribute to the portfolio R_m, and is not an explicit measure of R_i's risk premium. However, theory suggests that if $\beta = 0$ (i.e., there is no systematic risk between R_i and R_m) then a risk premium for R_i does not exist. Thus, while beta is not an explicit measure of the commodity risk premium in terms of its magnitude, it is nonetheless indicative of the existence of risk premiums.

2. The major difference between CAPM and the market model used by Lee and Leuthold is that the CAPM used returns in excess of the risk-free rate of return, while the market model considers gross returns. Lee and Leuthold chose the market model because of the difficulty in obtaining a proxy for the risk-free rate of return.

3. This is a daily index based on 27 commodities and constructed by the Commodity Research Bureau, Inc.

4. There are some minor differences in our results and those of Lee and Leuthold. However, these are likely explained by some differences in measurement techniques, and the way in which missing observations were treated.

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