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Optimal Decision Rules for Selling Farmer-Owned Wheat from Storage in the Pacific Northwest

by

Todd Lone and Leroy Blakeslee

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A stochastic dynamic programming model is developed to determine optimal decision rules for marketing white wheat from storage to the final sale. The model uses forecasts of the probability density functions for future prices; these functions are being updated each week based on current observed prices. Maximization of the expected utility of income from sequential decisions is used to capture the effect of risk aversion. Sensitivity of the optimal decision policy to risk aversion, opportunity cost of money, and storage costs is examined. The application of the model and its use in practical decision-making is discussed.

The Problem

Each year in the Pacific Northwest large quantities of farmer-owned wheat are placed in storage with no firm commitment for sale to a buyer. The farmer must market quantities of wheat to one or more buyers some time during the storage season. Farmers who market wheat face a complex decision problem involving various components, some of which are known while others are uncertain. Among these components, those which are of primary concern to the farmer are storage costs and price of wheat in the future. Storage costs accumulate and can encompass a wide variety of charges for which the farmer has detailed information when entering into a storage contract.

The component of the marketing decision for wheat which is the most perplexing to the farmer is wheat prices in the future. Farmers may gather all of the relevant data available to them and make a forecast of future prices. New price information that becomes available over time provides a basis for updating their price forecasts. Typically a sequential decision process is carried out by the farmer. Based on current price information, a decision is made to either sell all or a portion of the wheat, or to hold the wheat in expectation of a more favorable price in the future. If some of the wheat is held, a similar decision is made a short time later using updated information on prices and other data.

The objectives of this study are twofold. The primary objective is to identify optimal decision rules for use by farmers in the sequential decision process described above. In this case, optimality is defined in terms of maximizing expected utility of income from sequential decisions. The second objective is to examine the sensitivity of the optimal decision policy with respect to risk aversion and storage costs. The basic structure used for analysis is a stochastic dynamic programming model.

A number of studies have used dynamic programming to develop decision rules for marketing agricultural products. Ernst Berg maximized the certainty

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equivalent of a risky prospect to determine optimal marketing strategies for wheat. However, Berg's work assumed that probability density functions for prices in each decision period were known at the start of the storage season. Lambert and McCarl maximized expected utility within a discrete stochastic programming framework to determine optimal marketing strategies for white wheat. Comparisons between sequential and nonsequential formulations of the model were made for various utility functions and income. Each accounted for the effect of risk aversion. Tronstad used stochastic dynamic programming to determine optimal wheat marketing decision rules involving cash and futures markets. His study incorporated many concerns facing farmers such as cash flow requirements, tax considerations, and government farm program provisions. However, Tronstad's study does not account for risk behavior, instead he assumes risk neutrality. In a similar study, Mjelde, Taylor, and Cramer derived expected profit-maximizing decision rules for storing and marketing wheat and corn by taking account of farm program provisions. Their study used a price information vector within the model but made no explicit mention of risk. Finally, a study by Yager, Greer, and Burt incorporated production and future price expectations to derive cash market decision rules for cull beef cows. This study used a joint probability distribution of price and weight gain in determining marketing decisions within a stochastic dynamic programming framework.

Model Specification

The grain marketing problem in this study can be described at any point in time by two variables: the inventory level and current price. Inventory in the next period will be determined with certainty by the current decisions. Price in the next period is unknown and must be regarded as a random variable. However, a conditional probability density function for next periods price is formulated, based on the current price. To ascertain when and how much to sell, the decision maker considers current values of both variables as well as the conditional probability distribution for future values of price. This procedure is repeated at specific intervals of time throughout the storage season. In problems such as this, stochastic dynamic programming has proven to be a computationally efficient method of finding the optimal solution. However, to apply stochastic dynamic programming to a multi-period optimization problem, it is essential to properly specify stages, state variables, decision alternatives, and the relationship between states at different stages.

Stages

The multi-period nature of the problem necessitates a division into time intervals or stages. In this study decisions are made on a weekly basis beginning September 1 and extending through March 31. September 1 was selected as the initial stage since the majority of grain is harvested and placed in storage by this date. A total of 31 stages are contained in the model. When using the conventional backward recursion method of dynamic programming, stage 31, the week of March 31, would be the first week considered and stage 1, the week of September 1, would be the last in the solution sequence.

States

State variables describe the decision environment at each stage of the decision problem. In this study the state variables are inventory level and wheat price. Within the model these variables are divided into discrete units. The assumption of a 1,000 acre farm with a 60 bushel per acre yield results in total production of 60,000 bushels of wheat to be marketed. An additional assumption that total production is disposed of in 5,000 bushel increments produces 13 possible inventory states which range from 0 to 12[2]. Wheat price possibilities are approximated by assuming that only 102 values may be observed. These are the midpoints of 100 5-cent intervals between $1.50 per bushel and $6.50 per bushel, plus values of $1.475 per bushel and $6.525 per bushel. Probabilities are attached to these values by integrating appropriate normal densities over the relevant 5-cent ranges plus the two open-ended ranges for prices above $6.50 and below $1.50. Therefore, a total of 1326 possible price/inventory states exist for each stage. Each of the 1326 states will have an optimal decision associated with it at each stage of the problem.

Decision Alternatives

The decisions available at any given stage and state are, in part, dictated by prior decisions. Current decisions partly control the state of the process in the succeeding stage. The decisions in this study include only selling and holding of unhedged inventories. The selling decision can include any amount of the existing inventory, in discrete increments. Therefore, at each price/inventory state a decision is made regarding the disposal of the inventory. For example, if the current price/inventory state is represented by $3.50 and 10 units, any one of 11 decisions may be made. Each involves selling some or none of the 10 inventory units at $3.50 per bushel and storing the remainder.

Cash flow and tax considerations influencing the timing of sales are not included. The government loan program was excluded as a marketing alternative, as were futures markets. None of the harvest is committed to the government loan program. The aforementioned attributes of the actual farmers decision problem were not included in the formulation of this model so as to maintain a computationally manageable model. This analysis may be expanded to include these attributes at a later time.

Price Transition Probabilities

A one period autoregressive equation was estimated in order to forecast the probability density functions for future prices. For week t the price prediction equation can be written as:

(1) $P_t = a + bP_{t-1} + u_t, \quad t=1,2,\ldots,31.$

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[2]Given the mechanics of the dynamic programming algorithm, the solution actually provides optimal decision rules for marketing any initial inventory up to 60,000 bushels, provided that sales are in 5,000 bushel increments.

where t-1 corresponds to September 1. The disturbance term u_t was assumed to be normally distributed with mean zero and variance σ^2. The price data used to estimate this family of probability density functions were for number one soft white wheat at the Portland terminal market. These prices were collected on a weekly basis using the Wednesday reported price for the crop years 1974/75 through 1988/89.

The estimated price prediction equation for soft white wheat was:

$$(2) \quad PW_t = \underset{(2.255)}{0.05533} + \underset{(156.42)}{0.98625}\, PW_{t-1}$$

Standard error of regression = 0.08447, R^2 = 0.98182

where PW_t is the weekly price of white wheat. The t-ratios appear in parentheses. A t-test of the price coefficient leads to the conclusion that it is significantly less than one, indicating a dynamically stable equation. By successive substitutions, this linear difference equation could be used to predict a sequence of prices for all future times, given any initial price. The sequence would monotonically approach an asymptotic value of $4.0238 per bushel. Thus, higher prices are predicted each week when initial PW is below $4.0238 and lower predictions follow when initial PW is above this value. The price equation was kept simple in order to avoid the dynamic programming curse of dimensionality; this ensures a computationally feasible model for a personal computer.

Probability density functions are integrated as described above to create a matrix of Markovian transition probabilities which give the probability of being in price state j next week given price state i this week. The calculation of these finite state transition probabilities is integrated into the dynamic programming algorithm. This is done to facilitate possible use of a price forecasting equation reflecting seasonality. In such a case, different transition probability matrices would be computed for each week. To date, our estimation efforts have uncovered no convincing evidence of a credible seasonal pattern in price movements.

Utility Functional Form

In order to decide on the best action, the decision maker needs some way of ordering his/her choices under conditions of uncertainty. Daniel Bernoulli recognized that expected monetary payoffs do not always reveal the decision makers preferences when evaluating risk choices; consequentially he formulated the expected utility principle. The principle states that people assign a value (expected utility), invariant with respect to a transformation of origin and unit of scale, to each risky outcome and choose that risky prospect which maximizes their expected utility. In this context a risky prospect has a probability distribution of outcomes associated with it.

For this study all returns to the decision maker are in the form of income earned through the sale of wheat. To determine the expected utility of outcomes, the decision maker is actually evaluating the expected utility of income streams. The decision maker's attitude toward risk is reflected in the shape of the utility function. A utility function used by Freund and several others in similar studies is the negative exponential:

(3) $U(y) = 1 - e^{-ay}$, where y = income and $a > 0$.

This function assumes a risk averse decision maker. The positive constant 'a' is Pratt's absolute risk aversion coefficient. The larger 'a' becomes, the more risk averse (conservative) the decision maker. By assuming income to be normally distributed, it can be shown that the expected value of (3) is maximized by choosing 'y' so that the expression in (4) is maximized.

(4) $V = E(y) - (a/2)\sigma_y^2$.

Equation (4) is the objective function for the dynamic programming model. The equation is specified recursively in solving a multi-period problem with the dynamic programming technique. Under the backward solution method, only values associated with optimal solutions for the succeeding stage are considered while determining the current solution. This process, based on Bellman's Principle of Optimality, guarantees that the overall solution is optimal. Bellman's Principle states that for an optimal policy, whatever the initial state and decision, all remaining decisions must constitute an optimal policy with respect to the state resulting from the initial decision. In this study an optimal policy takes the form of a selling/holding decision rule defined in terms of the price/inventory state for each stage.

Dynamic Programming Algorithm

At every stage and for every state the dynamic programming algorithm chooses optimal decisions according to equation (5).

$$(5)\quad \underset{d^t}{\text{Max}}\; V(d^t|S_q^t,P_i^t) = \underset{d^t}{\text{Max}}\; \{CR^t(d^t|S_q^t,P_i^t) + rE[DR^t(d^t|S_q^t,P_i^t)] - kr^2\text{Var}[DR^t(d^t|S_q^t,P_i^t)]\}$$

In this expression, $V(d^t|S_q^t,P_i^t)$ is expected income in stage t when inventory is S_q^t and price is P_i^t, and when decision d^t (sell d^t units of inventory at price P_i^t) is taken. Thus, d^t represents <u>any</u> feasible decision. $CR^t(\cdot)$ is current return in stage t. It reflects current sales revenue, shrinkage of inventory, and 1 week's storage cost on retained inventory. From the perspective of stage t, it is non-stochastic. $DR^t(\cdot)$ is the delayed return from later marketing of the retained inventory under d^t. It is a random variable from the perspective of stage t. Its expected value, discounted to stage t, when added to $CR^t(\cdot)$, defines expected income in stage t, discounted to stage t. Since only $DR^t(\cdot)$ is random, its variance is the variance of total income from the perspective of stage t. This variance, multiplied by the risk aversion coefficient k (one-half the Pratt coefficient), and the square of the discount factor (r^2), is subtracted from expected income to arrive at a risk-adjusted expected income associated with decision d^t. A value of d^t that maximizes this expression maximizes expected utility under a negative exponential utility function.

Recursion is introduced by defining $E[DR^t(\cdot)]$ and $\text{Var}[DR^t(\cdot)]$ in terms of characteristics of <u>optimal</u> solutions for states in stage t+1 that are reached as a result of taking decision d^t, equations (6), (7) and (8).

$$(6)\quad E[DR^t(d^t|S_q^t,P_i^t)] = \sum_j \text{Prob}(P_j^{t+1}|P_i^t)(CR^{t+1}(D^{t+1}|S_q^t-d^t,P_j^{t+1}) + E[DR^{t+1}(D^{t+1}|S_q^t-d^t,P_j^{t+1})])$$

$$(7)\quad Var[DR^t(d^t|S_q^t,P_i^t)] = E[DR^t(d^t|S_q^t,P_i^t)]^2 - (E[DR^t(d^t|S_q^t,P_i^t)])^2$$

$$(8)\quad E[DR^t(d^t|S_q^t,P_i^t)]^2 = \sum_j \text{Prob}(P_j^{t+1}|P_i^t)(E[DR^{t+1}(D^{t+1}|S_q^t-d^t,P_j^{t+1})]^2 + [CR^{t+1}(D^{t+1}|S_q^t-d^t,P_j^{t+1}) + E[DR^{t+1}(D^{t+1}|S_q^t-d^t,P_j^{t+1})]]^2 - [E[DR^{t+1}|S_q^t-d^t,P_j^{t+1}]]^2)$$

The term D^{t+1} represents the decision in stage t+1 that is optimal for the price/inventory state associated with the term in which it appears. Thus, equation (6) defines expected delayed returns as a probability-weighted sum of optimal expected total returns for states in stage t+1 that are reached as a result of taking decision d^t. The probability weights relate to the probabilities of price changes between the 2 stages.

Equation (7) defines the variance of delayed returns as the second moment about zero for delayed returns, minus the square of the first moment. The second moment, in turn, is defined in terms of the second moment of optimal delayed returns for stage t+1 states that are reached via d^t, adjusted by 2 terms. These have the effect of allowing for the fact that though $CR^{t+1}(D^{t+1},S_q^t-d^t,P_j^{t+1})$ is non-random from the perspective of stage t+1, it is random from the perspective of stage t. This is shown in equation (8).

Optimality is defined in terms of maximizing the expected utility of income from these sequential decisions. In this context, both the expectation and variance of uncertain incomes affect decision selection. Because of the variance of delayed returns, it may be optimal to market portions of the inventory at different times as a strategy for adapting to risk. The risk aversion coefficients used in this study are those elicited from previous studies of farms with comparable total net revenue. These risk values are unique to each individual and they can vary with time.

The dynamic programming algorithm was coded in the programming language, GAUSS. The central feature of GAUSS is that its basic data element is a matrix rather than a scalar. An extensive array of operators with matrices as their arguments is available, and they execute very efficiently. Solutions were calculated on a 25 Mhz 386-based micro computer in about 3 minutes each.

Marketing Results Under Optimal Decisions

In addition to optimal decision rules, the algorithm provides directly 2 values that are of central interest: the expected income and variance of income associated with optimal decisions for each initial price/inventory state. Of course, their validity is conditional on the validity of the price forecasting model and other input variables as is the case with any approach of this kind. Beyond these, the set of optimal decisions and the price transition probability matrices can be used to calculate the probability of being in any state at any stage, expected cumulative sales through each week of the storage season, and the average length of storage that results from applying the optimal rules.

Suppose the decision maker has q units of initial storage. We seek first a set of matrices PS^t such that the element in row i, column j is the probability that storage will be in state i and price will be in state j at the beginning of stage t. Let D^t be a matrix having elements D^t_{js} which are the optimal selling decisions in stage t when price is in state j and storage is in state s. Then for t=2, PS^2 will have columns which are unit vectors with the "one" for column j being in row $q-D^1_{jq}$. That is, every probability will be either zero or one.

Let P^t be a matrix whose i,j-th element is the probability that price is in state j in stage t+1, given that price is in state i in stage 1. It is calculated as in equation (9);

$$(9) \qquad P^t = \prod_{i=1}^{t} P_i$$

where P_t is a matrix of probabilities that price transits from state i to state j between stages t and t+1. From this and matrix D^t, a set of lower-triangular qxq matrices PQ^{it} can be found. Each has (r,s) elements giving the probability that storage will be in state s in stage t+1 given that storage was in state r in stage t, and that price was in state i in stage 1. These are found as in equation (10);

$$(10) \qquad PQ^{it}_{rs} = \sum_{j \in \Theta_{rs}} P^t_{ij}$$

Here, Θ_{trs} is the set of j indices for which $r-D^t_{jr}=S$. This set indexes those probabilities associated with stage t prices such that when those prices occur together with storage in state r, the optimal decision will be to move to storage state s in stage t+1.

For t>2, PS^t may be determined recursively. Column i is computed in terms of a particular matrix PQ^{it} and column i of PS^{t-1}, equation (11).

$$(11) \qquad PS^{t+1}_{.i} = (PQ^{it})' PS^t_{.i} \quad ,i=1,2,\ldots,q$$

The set of PS^t matrices provides a vehicle for computing expected cumulative sales in each stage, given an initial price. Column i of PS^t gives the probabilities of being in each storage state in stage t, given price in state i at stage 1. Thus, expected cumulative sales through week t, given initial price in state i, is calculated as in equation (12).

$$(12) \qquad ECS_{ti} = \sum_{r=0}^{q} (q-r) PS^t_{ri}$$

Finally, the fraction of sales in week t, given initial price in state i may be found as in equation (13).

$$(13) \qquad f_{ti} = \begin{cases} ECS_{ti}/q; & t=1 \\ (ECS_{ti} - ECS_{(t-1)i})/q; & t=2,3,\ldots,n \end{cases}$$

These fractions may be used to calculate the expected time that a unit of inventory remains in storage before it is sold, given initial price in state i. This is shown in equation (14).

$$(14) \quad \bar{t}_i = \sum_{t=1}^{n} f_{ti} \cdot (t-1)$$

Model Results

Prior to discussing the empirical results, an exposition of results in the simple cases can be explored. If we consider the situation with a unity discount factor, no storage costs, and risk neutrality, the price prediction equation becomes the driving force of the solution. Under these circumstances the model should indicate a delay in sales in the early stages when prices are low since the price equation predicts higher values in the future. The situation is reversed at higher price levels where the model predicts lower values for the next period and thus necessitates a sell now decision. In both instances the sell or hold decision is for the entire inventory level since the variance associated with the decision is neglected.

Typical values for the risk aversion coefficient, storage costs, and discount factor are used in the initial model formulation. It is instructive to consider the effects of varying each parameter separately, while holding the other two constant.

If storage costs are allowed to vary there should be an inverse relationship to the time of sale. If the predicted increase in prices is sufficient to cover storage costs a hold-for-later sale decision will be made. When storage costs are equal to zero the gap between the predicted price increase and storage costs is at its maximum and the decision to sell should be delayed. However, if storage costs exceed the predicted increase in prices then a sell-now decision will follow. The decisions made at the upper price levels will be unchanged by the value of storage costs. All inventory will be sold immediately.

Similar effects should occur when a discount factor reflecting the time value of money is varied. Because the weekly discount factor is small, its influence on optimal decisions may be small. Nevertheless, its impact on the hold versus sell decision should replicate the outcomes as described above. As the interest rate is increased, the opportunity cost of holding into the future increases necessitating an earlier sell decision. The holding period on inventory will be extended when the interest rate is decreased.

Increasing the risk aversion coefficient, while holding other model parameters constant, allows the variance of income to have a larger impact on the optimal decision. Lower values, associated with decreasingly risk averse decision makers, result in a decreasing portion of income variance for each decision being subtracted from expected income. With all else equal, low risk penalties will extend the holding period and raise expected income, and high penalties will shorten inventory holding periods and lower expected income, provided that not all inventory is sold immediately.

When storage costs, discount factor, and risk coefficient are varied simultaneously, it becomes difficult to predict the total effect on the optimal solution.

Before examining the initial model solution, hereafter referred to as the base model solution, three prices of interest will be defined. The marginal hold-all price is defined as the highest initial price at which all inventory is held for at least one week. Analogously, the marginal sell-all price is defined as the lowest initial price at which all inventory is sold in the first stage. For all prices within these limits the model returns mixed selling strategies for the first week. For comparisons between model formulations a mid-price defined as the price midway between the marginal hold-all and sell-all prices has been calculated. The expected cumulative sales for these three price levels are graphed. In addition, results are presented for the lowest and highest price levels that were considered.

The initial model solution is based on a risk aversion coefficient of 0.0002, storage costs of $.022 per month, and an interest rate of 6 percent per year. The storage costs and interest rate are values presently charged for commercial storage and the current rate of return on a money market fund, respectively. The risk aversion coefficient is within the range of Pratt risk aversion coefficients commonly reported in other studies for farms having similar total revenue.

Results for the base model solution are reported in Table 1 and the corresponding graph appears in Figure 1. The wheat price range for which mixed selling strategies occur is between $1.58 and $3.03 per bushel with a mid-price of $2.33. The expected returns' columns of Table 1 indicate the amount anticipated by the producer on a total and per bushel basis. These figures represent the expected returns from selling and holding throughout the storage season, assuming that the same decision criterion is applied in each week and that weekly price forecasts are made with the model discussed earlier. Associated with expected returns are the standard deviations of those returns. It becomes quite evident that at higher prices more inventory will be sold earlier in the storage season. This can be seen by the decrease in the mean length of storage at higher prices, thus reflecting a higher probability of returns coming from early sell decisions. Therefore, the standard deviation of returns decreases at higher prices.

Figure 1 depicts the expected cumulative sales, revealing that two-thirds of the inventory is sold in the first 8 weeks for the marginal hold-all price. The figure also illustrates ten units of inventory will be sold in the first week of the storage season for the mid-price.

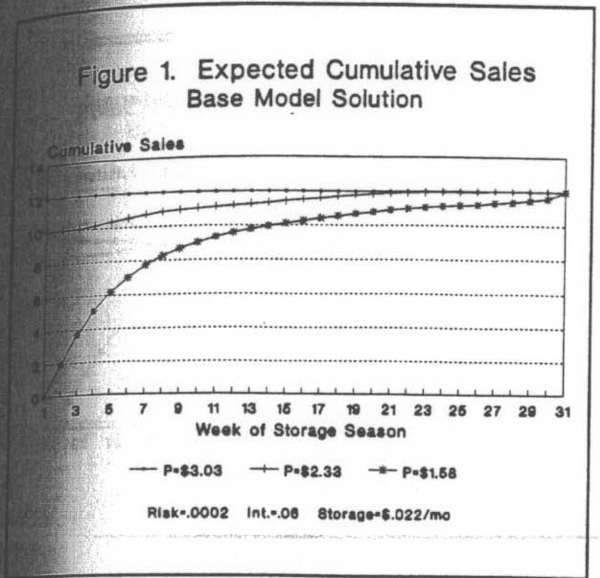
When the risk aversion coefficient used in the model was decreased, the earlier hypothesized impact was realized. The marginal hold-all and sell-all prices increased, necessitating an increase in the mid-price. The mean length of storage given in Table 2 and the graph in Figure 2 indicate the grain was held a longer time based on the probability of achieving a higher price in the future. Not until the eleventh week was two-thirds of the inventory sold at the marginal hold-all price. The expected returns generated from this strategy were higher as were their corresponding standard deviations. In fact the standard deviations were the second highest of all models examined. This result was expected since holding for a longer period is associated with increased uncertainty, higher expected prices, and delayed returns. Higher risk aversion coefficients had the opposite effect of decreasing mean storage lengths and the expected returns from those decisions.

Sensitivity of the model with respect to storage costs is reflected in Figures 3 and 4 and their respective tables. Again the hypothesized relationship was observed. As storage costs were reduced to one half and zero of the original value specified in the base model, the mean length of storage increased. The marginal prices all increased in value indicating a willingness to delay sale in anticipation of higher prices in the future. The range of prices in which mixed strategies occur also increased as storage costs were reduced. This was undoubtedly due to a widening gap between the predicted increase in price and storage costs as storage costs were reduced. The results illustrated in Tables 3 and 4 indicate that expected returns and the standard deviations of returns increase with the decision to hold longer. The uncertainty of higher prices in the future accounted for the increase in the standard deviation of returns.

Varying the interest rate meant that the discount factor in the model was altered. Two cases were investigated: a zero interest rate indicating a unity discount factor and a higher interest rate of 9 percent. The end result of these changes was consistent with economic theory. Increasing the interest rate increased the opportunity cost of holding grain for extended periods. Table 6 and Figure 6 reveal that grain was held for a shorter period of time and the expected returns from such a decision reflected the lower prices received with such a strategy. However, following a strategy of selling earlier reduced the standard deviation of expected returns. The opposite effect was achieved with no discounting (zero interest rate). In this case the range of prices in which mixed strategies were obtained was slightly larger than in the base solution. The mean length of storage increased since the time value of money was assumed to be zero. The expected returns increased since it was now worthwhile to wait for expected higher prices. However, as in all delayed returns the standard deviation associated with them increased, since higher prices were now used in the calculation of expected returns.

The final two variations on the base model allowed two of the parameters to vary simultaneously. Discounting and storage costs were eliminated from the analysis; the results are reported in Table 7 and appear graphically in Figure 7. This scenario produced the widest range of prices in which mixed strategies occurred. It also produced the highest marginal sell-all price. This was anticipated given that storage and discounting costs were eliminated. Expected returns were the second highest of all the models examined and the standard deviations of these returns were higher than most solutions. Again this was due to relatively long mean storage lengths when compared to other solutions and the uncertainty associated with higher expected prices.

The last scenario coupled low risk aversion with zero storage costs. An examination of Table 8 reveals that the longest mean storage lengths were achieved in this model. In fact, Figure 8 shows that this combination produced the most even selling strategy throughout the storage season and thus reinforces the results of Table 8. For most prices this model produced the largest expected returns generated by all models. The large storage length values were associated with large standard deviations of returns, reflecting



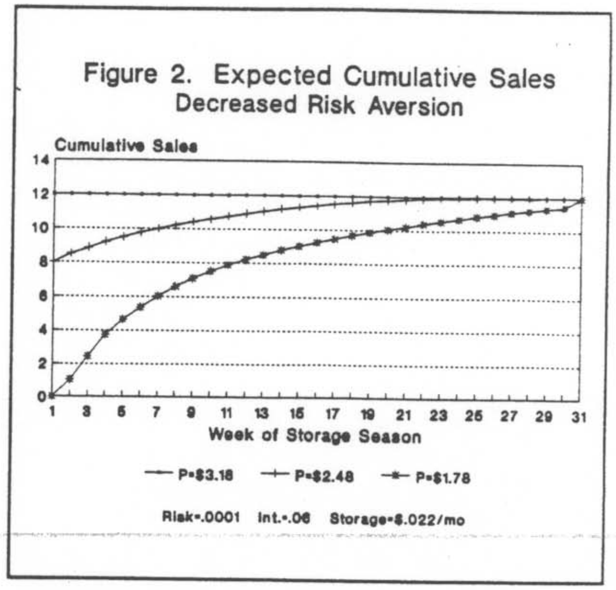
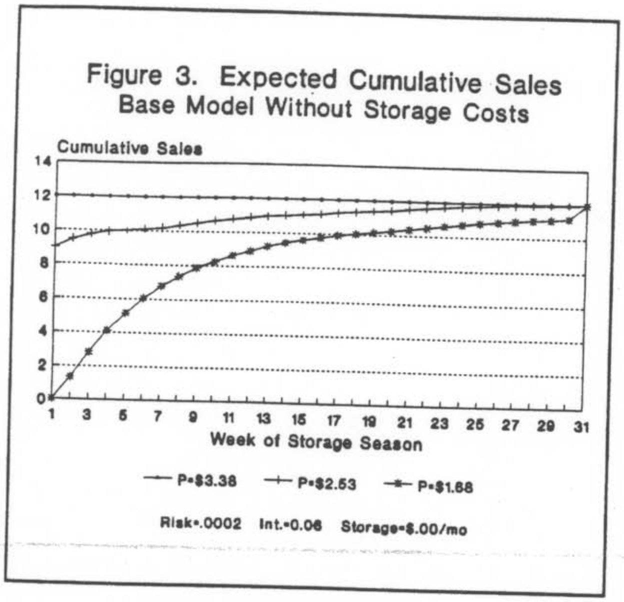


TABLE 1: Expected Returns and Storage Length, Base Model Solution

Price Level	Price Value ($)	Expected Returns Total ($)	Expected Returns Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	79,399	1.32	5,184	8.60
Marginal Hold All Price	1.58	80,894	1.35	5,142	7.38
Mid-Price	2.33	113,731	1.90	2,038	1.72
Marginal Sell All Price	3.03	153,728	2.56	0	0
High Price	6.53	362,678	6.04	0	0

TABLE 2: Expected Returns and Storage, Decreased Risk Aversion

Price Level	Price Value ($)	Expected Returns Total ($)	Expected Returns Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	86,376	1.44	8,634	13.46
Marginal Hold All Price	1.78	93,320	1.56	7,989	9.66
Mid-Price	2.48	123,345	2.06	3,423	2.66
Marginal Sell All Price	3.18	162,683	2.71	0	0
High Price	6.53	362,678	6.04	0	0



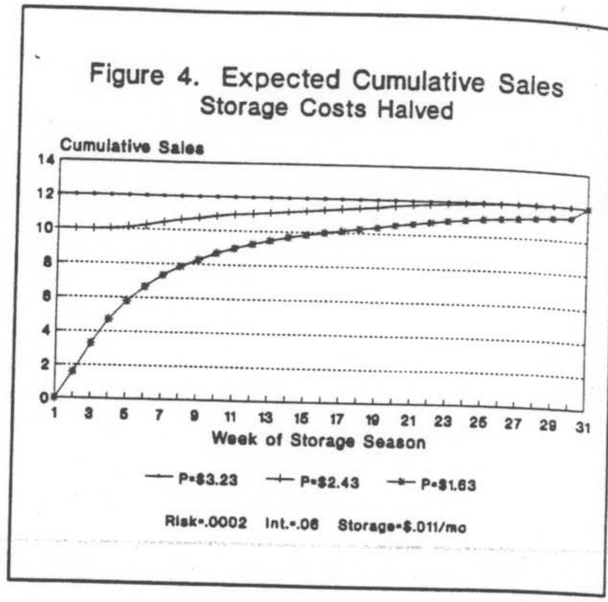


TABLE 3: Expected Returns and Storage Length, Base Model Without Storage Costs

Price Level	Price Value ($)	Expected Returns Total ($)	Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	87,144	1.45	6,189	11.19
Marginal Hold All Price	1.68	90,446	1.51	6,009	8.84
Mid-Price	2.53	126,682	2.11	2,556	2.53
Marginal Sell All Price	3.38	174,623	2.91	0	0
High Price	6.53	362,678	6.04	0	0

TABLE 4: Expected Returns and Storage Length, Storage Costs Halved

Price Level	Price Value ($)	Expected Returns Total ($)	Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	83,028	1.38	5,674	9.87
Marginal Hold All Price	1.63	85,435	1.42	5,591	8.09
Mid-Price	2.43	120,085	2.00	2,213	2.06
Marginal Sell All Price	3.23	165,668	2.76	0	0
High Price	6.53	362,678	6.04	0	0

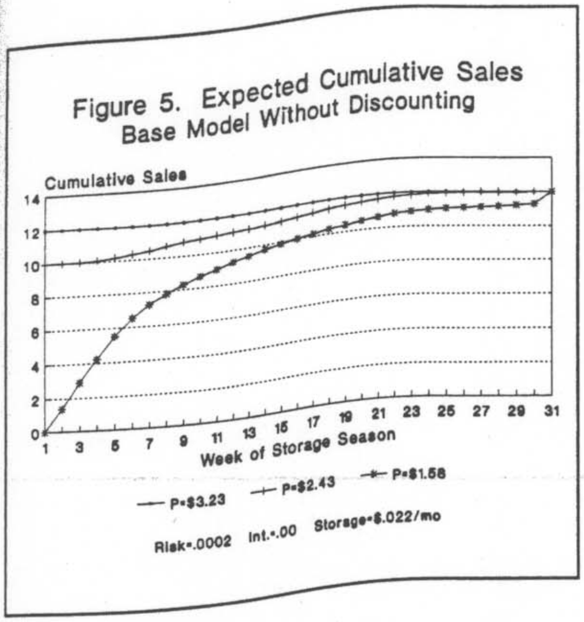


Figure 5. Expected Cumulative Sales Base Model Without Discounting

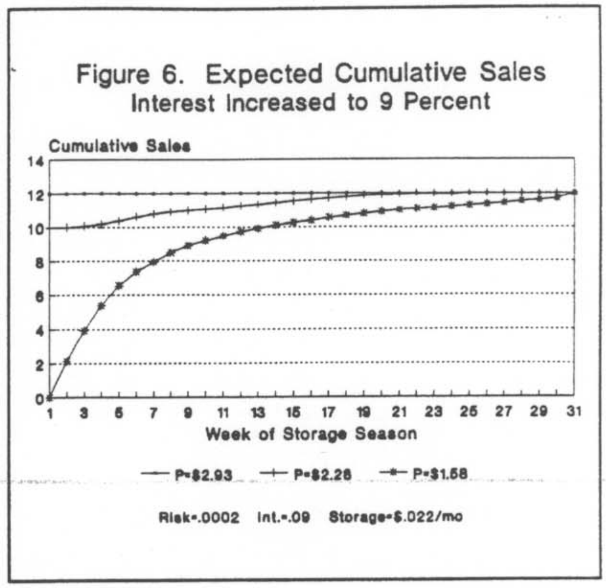


Figure 6. Expected Cumulative Sales Interest Increased to 9 Percent

TABLE 5: Expected Returns and Storage Length, Base Model Without Discounting

Price Level	Price Value ($)	Expected Returns Total ($)	Expected Returns Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	81,505	1.36	5,536	9.35
Marginal Hold All Price	1.58	82,989	1.38	5,525	8.21
Mid-Price	2.43	120,039	2.00	2,237	2.05
Marginal Sell All Price	3.23	165,668	2.76	0	0
High Price	6.53	362,678	6.04	0	0

TABLE 6: Expected Returns and Storage Length, Interest Increased to 9 Percent

Price Level	Price Value ($)	Expected Returns Total ($)	Expected Returns Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	78,259	1.30	4,984	8.16
Marginal Hold All Price	1.58	79,866	1.33	5,003	6.98
Mid-Price	2.28	110,629	1.84	1,975	1.62
Marginal Sell All Price	2.93	147,758	2.46	0	0
High Price	6.53	362,678	6.04	0	0

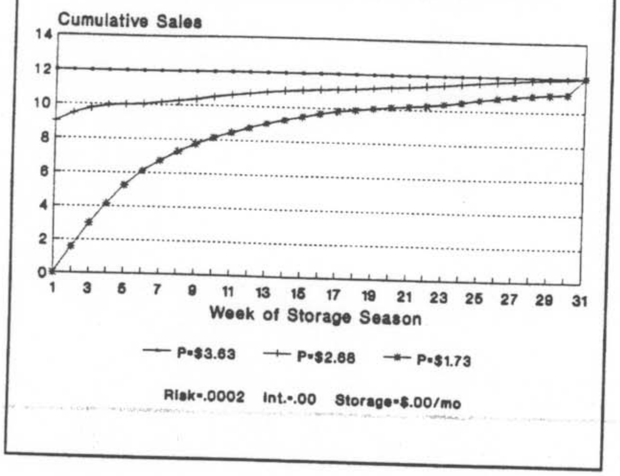


Figure 7. Expected Cumulative Sales
No Discounting nor Storage Costs

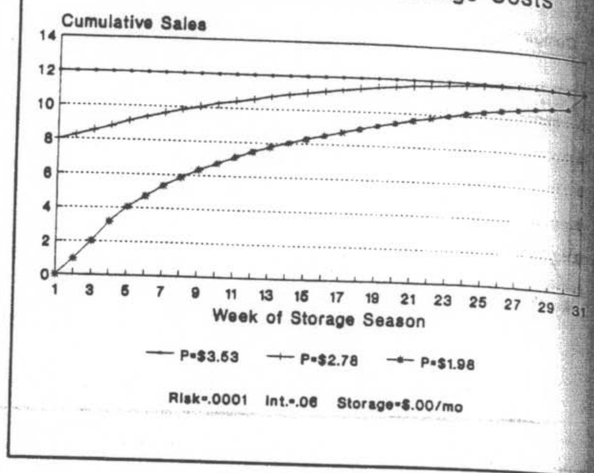


Figure 8. Expected Cumulative Sales
Low Risk Aversion & No Storage Costs

TABLE 7: Expected Returns and Storage Length, No Discounting nor Storage Costs

Price Level	Price Value ($)	Expected Returns: Total ($)	Expected Returns: Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	89,785	1.50	6,570	11.94
Marginal Hold All Price	1.73	94,138	1.57	6,298	9.08
Mid-Price	2.68	135,881	2.26	2,703	2.81
Marginal Sell All Price	3.63	189,548	3.16	0	0
High Price	6.53	362,678	6.04	0	0

TABLE 8: Expected Returns and Storage Length, Low Risk Aversion & No Storage Costs

Price Level	Price Value ($)	Expected Returns: Total ($)	Expected Returns: Per Bushel ($)	Std. Dev. of Returns ($)	Mean Length of Storage (Weeks)
Low Price	1.48	95,447	1.59	10,099	16.74
Marginal Hold All Price	1.98	107,985	1.80	8,815	11.15
Mid-Price	2.78	141,858	2.36	3,784	3.24
Marginal Sell All Price	3.53	183,578	3.06	0	0
High Price	6.53	362,678	6.04	0	0

their accompanying high degrees of price uncertainty. This model also had the highest marginal hold-all price, indicating the propensity to accept the risks of achieving a higher price in a future period.

Conclusion

A model for marketing soft white wheat during the storage season, while taking account of future price uncertainty, was developed in this study. Marketing strategies which maximize the expected utility of income generated throughout the storage season were obtained. The impact on the optimal marketing strategies of varying the level of three primary parameters was investigated; the parameters were the risk aversion coefficient, storage costs, and a discount factor.

General results indicate that specific directional changes in one parameter or two parameters in combination will impact expected returns and mean length of storage. Reductions in the risk aversion coefficient, storage costs, and discount factor, whether separately or in combination, will increase expected returns and the length of storage. Increases in these parameters likewise have the opposite effect. Storage costs and risk aversion coefficients tended to have the greatest impact on the optimal marketing strategies.

The potential usefulness of directly solving sequential decision problems under the premise of expected utility maximization should not be underestimated. Many rules of thumb and postulates exist about how and when producers should sell their grain. These rules are many times a seat of the pants recommendation based on tradition, time of year, perceived price trends, etc. The sell versus hold decision depends on many factors such as the decision makers attitude toward risk, potential storage costs, discounting of returns, and the beliefs of what will happen to prices in the future. The approach used here demonstrates how several of these factors can be taken into account within a formal optimizing framework. The optimal decision rules derived from the model often include selling portions of the inventory at different times during the storage season as a risk management strategy. This corresponds to the actual behavior of many producers. Clearly, there are other grain marketing options that have not been considered here, and other factors that drive farmers' marketing decisions. However, improved precision and consistency in accounting for the influence of the many factors that determine optimal selling decisions can only improve the business performance of grain farmers and the welfare of farm operators. Given recent changes in government farm programs which place more of the burden of obtaining a favorable price on the wheat producer, any possible increase in returns, commensurate with the attendant risk, is of interest to the producer.

References

Berg, Ernst. "A Sequential Decision Model to Determine Optimal Farm-Level Grain Marketing Policies." Eur. Rev. Agr. Econ. 14(1987):91-116.

Lambert, David K., and Bruce A. McCarl. "Sequential Modeling of White Wheat Marketing Strategies." N. Cent. J. Agr. Econ. 11(1989):105-15.

Mjelde, James W., C. Robert Taylor, and Gail L. Cramer. "Optimal Marketing Strategies for Wheat and Corn Producers in the 1982 Farm Program." N. Cent. J. Agr. Econ. 7(1985):51-60.

Tronstad, Russell. "Optimal Wheat Marketing/Storage/Hedging Strategies." Paper presented at Applications of Dynamic Programming to Agricultural Decision Problems Conference, Auburn University, 1989.

Yager, William A., R. Clyde Greer, and Oscar R. Burt. "Optimal Policies for Marketing Cull Beef Cows." Amer. J. Agr. Econ. 62(1980):456-67.