Alternative Measures of Risk: An Analysis of Sow Farrowings in the United States

by

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Matt Holt and Giancarlo Moschini[1]

Introduction

Since Baron (1970) and Sandmo (1971), the analysis of price risk effects on producer behavior has continued to be an important area of research. The potential role of risk is especially relevant to agricultural production, where prices are typically more volatile than in other sectors, and where the competitive structure ensures that producers take as exogenous the equilibrium price distribution. If producers are risk averse, their behavior may be significantly affected by this price variability. Consequently, a considerable body of research has investigated the inclusion of risk terms in econometric commodity models (Just, 1974; Traill, 1978; Antonovitz and Green, 1990; Chavas and Holt, 1990).

Whether it is viable to introduce risk terms in econometric supply models, however, is not a settled issue. The crucial point is that to identify risk response within a commodity model, it is not sufficient that producers simply be risk averse, but rather it is also necessary that the risk variable to which they respond be time varying. Hence, an important issue in estimating risk response in commodity models concerns the explicit modeling of the time-pattern of (conditional) price variance. The estimation problem posed here is similar to that of financial econometric models with risk premia (French, Schwert, and Stambaugh, 1987; Bollerslev, Engle, and Wooldridge, 1988; Pagan and Ullah, 1988; Pagan and Schwert, 1990), and the possible solutions are similar. In particular, leading parametric specifications of conditional variance dynamics include Engle's (1982) Autoregressive Conditional Heteroskedasticity (ARCH) model and Bollerslev (1986) Generalized ARCH (or GARCH) model. The latter has recently been employed by Aradhyula and Holt (1989) in the context of a commodity model.

The purpose of this paper is to investigate alternative methods for modeling risk response in a commodity model. The specific application uses data on aggregate U.S. sow farrowing decisions, a setting where previous research (e.g., Hurt and Garcia, 1982; Tronstad and McNeill, 1989) has found output price risk to be important. The models of conditional variance considered here represent recent developments in the econometrics literature for modeling risk response. Specifically, we model conditional price variance in a multivariate ARCH-M and GARCH-M framework using Bollerslev's (1990) estimation approach. Additionally, we employ a nonparametirc model of conditional variance recently suggested by Pagan and Schwert (1990).

The paper is organized as follows. First, an overview of the model is presented, followed by a discussion of alternative conditional variance estimators. Empirical results, using U.S. aggregate quarterly data for the period 1958-1990, are then presented. The paper concludes with a summary of results and implications.

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The Model

The empirical model estimated, by necessity, is an approximation to the true model. Consequently, little generality is lost by assuming a specific case of risk averse behavior. Furthermore, useful insights may be gained for the evaluation of empirical results. Specifically, assume producers have a constant absolute risk aversion (CARA) utility function, and that price risk is (conditionally) normal. It is well known that in this case utility maximization is equivalent to maximizing a (linear) mean-variance criterion. The producers problem can then be represented as:

$$(1) \qquad \text{Max}_y \; [\bar{p}y - C(y,\underline{w}) - \tfrac{1}{2}\lambda y^2 v^2]$$

where y is output, $(\bar{p}, v^2)$ are mean and variance of the price distribution, $C(y,\underline{w})$ is the (indirect) cost function with $\underline{w}$ representing input prices, and λ is the constant coefficient of absolute risk aversion. The first order condition for this problem is:

$$(2) \qquad \bar{p} - C_y(y,\underline{w}) - \lambda y v^2 = 0 \, .$$

Hence, production occurs at point where marginal cost C_y is lower than expected price, the difference being the marginal risk premium $\lambda y v^2$. The solution to (2) gives the optimal supply response $y^* = y(\bar{p},\underline{w},v^2)$.

One interesting implication of the above model is that bounds can be placed on producer response to price risk as represented by the variance of price, v^2. Define the (mean) price supply elasticity as $\eta_p \equiv (\partial y/\partial \bar{p})(\bar{p}/y)$, and the supply elasticity with respect to price variance as $\eta_v \equiv (\partial y/\partial v^2)/(v^2/y)$. Then, implicit differentiation of the first order condition (2) yields:

$$(3) \qquad \eta_p = \frac{\bar{p}}{yC_{yy} + \lambda y v^2}$$

$$(4) \qquad \eta_v = - \frac{\lambda y v^2}{yC_{yy} + \lambda y v^2}$$

where C_{yy} denotes the slope of marginal costs. If marginal costs are nondecreasing in output,[2] then $-1 \leq \eta_v \leq 0$ under risk aversion. More importantly, one gets:

$$(5) \qquad - \frac{\eta_v}{\eta_p} = \frac{\lambda y v^2}{\bar{p}}$$

Hence, the (negative of the) ratio between variance elasticity and price elasticity gives the marginal risk premium as a proportion of the expected price, i.e., the percentage departure from marginal cost pricing. This relationship can be useful for assessing empirical estimates of response to price risk.

The solution to equation (2) gives optimal output y^* as a function of expected output price, price variance, and input prices. For the purpose of

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[2] This is not required under risk aversion because the second order condition of problem (1) only requires $C_{yy} + \lambda\sigma^2 > 0$.

modeling sow farrowings, assume y^* represents the desired level of production, and that actual production can differ from the desired level in the short-run because of adjustment costs [Nerlove (1958) stock adjustment model]. Letting t index time periods, then actual supply y_t is related to desired supply y_t^* by:

$$(6) \qquad y_t = y_{t-1} + \theta\, (y_t^* - y_{t-1})$$

where θ is the coefficient of adjustment satisfying $0 \leq \theta \leq 1$. Hence, the supply equation is dynamic, and can be written as $y_t = (1-\theta)\, y_{t-1} + \theta\, y(\overline{p}_t, v_t^2, \underline{w_t})$. To apply this framework we need to specify how expectations of the moments $\overline{p}_t$ and v_t^2 are formed.

Expectations that drive supply decisions are conditional on information available when supply decisions are made. To specify the information set properly, it is important to understand the technological lags governing production. Because the application that follows uses quarterly data, we take a period as meaning a quarter. Given the required gestation period, and allowing for some time to implement production decisions, the number of sows farrowing at time t, say S_t, is the result of decisions made in period (t-2). Moreover, sow farrowings are of interest because they are an indicator of future hog supplies (Tomek and Robinson, 1990, chapter 4). The pig crop resulting from sow farrowings at time t will come to market two quarters hence. At time (t-2) then, production decisions were made that will be realized at (t+2), a four quarter production lag.

To simplify notation, define $y_t = S_{t-2}$, i.e. (planned) output for time t equals the number of sow farrowings at (t-2), and this output was planned at (t-4). For the purpose of specifying the supply equation, we have:

$$(7) \qquad \overline{p}_t = E[p_t | I_{t-4}] \quad \text{and} \quad v_t^2 = E[(p_t - \overline{p}_t)^2 | I_{t-4}]$$

where E denotes the expectation operator and I_{t-4} denotes the information set used by hog producers in period (t-4) when making breeding decisions.

To make the above model operational, it is necessary to specify the dynamic process governing both formation and the conditional variance. We assume hog the price equation can be represented by an autoregressive process of order q:

$$(8) \qquad p_t = b_0 + \sum_{i=1}^{q} b_i\, p_{t-i} + e_t$$

where $e_t | I_{t-1} \sim N(0, \sigma_t^2)$. That is, we explicitly assume the innovation in (8) is conditionally heteroskedastic. Given this price equation, the relevant conditional price expectation is expressed in terms of the following equations:

$$(9.1) \qquad E[p_t | I_{t-4}] = b_0 + b_1 E[p_{t-1} | I_{t-4}] + b_2 E[p_{t-2} | I_{t-4}] + b_3 E[p_{t-3} | I_{t-4}] + b_4 p_{t-4} + b_5 p_{t-5} + b_6 p_{t-6}$$

$$(9.2) \qquad E[p_{t-1} | I_{t-4}] = b_0 + b_1 E[p_{t-2} | I_{t-4}] + b_2 E[p_{t-3} | I_{t-4}] + b_3 p_{t-4} + b_4 p_{t-5} + b_5 p_{t-6} + b_6 p_{t-7}$$

$$(9.3) \qquad E[p_{t-2} | I_{t-4}] = b_0 + b_1 E[p_{t-3} | I_{t-4}] + b_2 p_{t-4} + b_3 p_{t-5} + b_4 p_{t-6} + b_5 p_{t-7} + b_6 p_{t-8}$$

$$(9.4) \qquad E[p_{t-3} | I_{t-4}] = b_0 + b_1 p_{t-4} + b_2 p_{t-5} + b_3 p_{t-6} + b_4 p_{t-7} + b_5 p_{t-8} + b_6 p_{t-9}$$

Also, the relevant conditional variance can be expressed as:

$$E[(p_t - \overline{p}_t)^2 | I_{t-4}] = E[e_t^2 | I_{t-4}] + b_1^2\, E[e_{t-1}^2 | I_{t-4}] + (b_1^2 + b_2)^2\, E[e_{t-2}^2 | I_{t-4}] + (b_1^3 + 2b_1b_2 + b_3)^2\, E[e_{t-2}^2 | I_{t-4}] \quad (10)$$

Given an explicit model for the dynamics of the squared innovations e_t^2 (see below), the conditional expectations on the right-hand-side of (10) can be evaluated in a manner similar to that of the set of equations (9).

Finally, for the purpose of estimation the supply equation is specified in linear form:

$$y_t = a_0 + a_1 D_{75} + a_2 Q_2 + a_3 Q_3 + a_4 Q_4 + a_5 Q_2D_{75} + a_6 Q_3D_{75} + a_7 Q_4D_{75} + a_9 \overline{p}_t + a_{10} v_t^2 + a_{11} y_{t-1} + u_t \quad (11)$$

where Q_i (i=2,3,4) denote seasonal dummy variables, D_{75} is a dummy variable taking value 0 up to the last quarter of 1974 and value 1 thereafter, a_k (k=1,...,11) are parameters to be estimated, and u_t is the error term. The seasonal dummies are warranted by the use of quarterly data, and the use of the variable D_{75} is suggested by the apparent change in the structure of seasonality over the sample period. Moreover, if producers use (8) when determining conditional price and variance expectations, then the parameters in the AR(q) model, along with those associated with the process generating the conditional variance of e_t, will be shared by the supply equation (6). The implication is that there may be gains in estimation efficiency associated with modeling the supply equations and the price equation simultaneously. In the next section we review alternative methods for modeling conditional variances.

Conditional Variance Models

To implement the supply model with conditional expectations of the first and second moments of price, it is necessary to specify the process generating the time-varying conditional variances, σ_t^2. Previous research into the effects of risk on commodity supply response has largely used <u>ad hoc</u> procedures to measure conditional variances. Specifically, time-varying conditional variance terms are typically constructed as a weighted moving average of squared deviations between past expected and realized prices. That is, $\overline{p}_t$ and σ_t^2 are generated according to:

$$\sigma_t^2 = \sum_{i=1}^{m} \alpha_i\, (p_{t-i} - \overline{p}_{t-i})^2 \quad \text{and} \quad \overline{p}_t = \sum_{i=1}^{m} \beta_i\, p_{t-i} \quad (12)$$

and where the weights α_i and β_i sum to one.[3]

As Pagan and Ullah (1988) indicate, the specification in (12) may give misleading results. Also, by definition the unconditional variance of p_t implied by (12) is not defined, a condition which is at best an empirical proposition. Lastly, specification of the weights α_i and β_i is often entirely arbitrary, with no attempt to empirically estimate and/or test them for statistical significance. Hence, it is desirable to seek suitable alternative parameterizations.

In recent years, there have been numerous methodological developments that

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[3] For examples of applications of the moving average approach to commodity models see, among others, Behrman (1968), Anderson and Garcia (1989), Tronstad and McNeill (1989), and Chavas and Holt (1990).

facilitate the econometric investigation of risk. Engle (1982) and Bollerslev (1986) proposed a new class of time-series models referred to, respectively, as ARCH and GARCH processes. As with the weighted moving average model (12), the ARCH model specifies that the conditional variance today, σ_t^2, is a function of past squared innovations of p_t. That is,

(13) $$\sigma_t^2 = \omega_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_p e_{t-p}^2,$$
$$\omega_0 > 0, \alpha_1, \ldots, \alpha_p \geq 0$$

is an ARCH(p) model where e_{t-i} are the innovations of the equation of motion of prices [in our case, the AR(q) model in (8)]. The important difference between (13) and (12) is that the "weights" associated with (13) are not arbitrarily imposed and that, under suitable conditions, the unconditional variance, σ^2, is well defined.[4]

The GARCH model proposed by Bollerslev generalizes the ARCH model in (13) by allowing past values of the conditional variance to affect the current level of the conditional variance. Specifically, the GARCH(q,p) model is given by:

(14) $$\sigma_t^2 = \omega_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_p e_{t-p}^2 + \beta_1 \sigma_t^2 + \ldots + \beta_q \sigma_{t-q}^2$$
$$\omega_0 > 0, \quad \alpha_1, \ldots, \alpha_p \geq 0, \quad \text{and} \quad \beta_1, \ldots, \beta_q \geq 0$$

Of interest in the present case is the GARCH(1,1) model where p=q=1. For q=0, the specification in (14) reduces to the ARCH(p) model in (13). In addition, if $\alpha_1 + \ldots + \alpha_p + \beta_1 + \ldots + \beta_q < 1$, then the unconditional variance of p_t is defined and is given by $\omega_0/(1 - \alpha_1 - \ldots - \alpha_p - \beta_1 - \ldots - \beta_q)$. In either the ARCH or GARCH specifications, if $e_t \sim N(0, \sigma_t^2)$ then the parameters of the conditional variance model can be estimated using maximum likelihood.

Numerous extensions to the basic ARCH and GARCH models have been considered. Of interest here is the ARCH-M (ARCH-in-mean) model, proposed originally by Engle, Lilien and Robins (1987) (ELR). The ARCH-M (GARCH-M) model allows the conditional variance to affect the level of the conditional mean, $\bar{p}_t$, or, alternatively, the realization of p_t. Specifically, in the context of the AR(q) model in (8), we would have:

(15) $$p_t = b_0 + \sum_{i=1}^{q} b_i p_{t-i} + \delta\sigma_t^2 + e_t$$

where σ_t^2 is specified using either (13) or (14). Here δ measures the response of p_t to risk, i.e., the risk premium. However, this is not the motive of interest here. What we wish to measure is the effect of price risk on supply decisions, and this calls for a multivariate approach.

ELR's univariate ARCH-M (GARCH-M) model has been extended to a multivariate ARCH-M (GARCH-M) setup by Bollerslev, Engle, and Wooldridge (1988); Engle, Ng, and Rothschild (1990); Bollerslev (1990); and others. In this context, we can think of generalizing the univariate process in (15) to a vector-valued process with conditional variance-covariance matrix H_t, and where the conditional means could, in principle, be a function of <u>all</u> conditional variance and covariance terms. Letting $\underline{Y}_t$ be an N-dimensional vector and H_t a positive definite conditional covariance matrix, the multivariate GARCH-M model is:

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[4] Specifically, $\sigma^2 = \omega_0/(1 - \alpha_1 - \ldots - \alpha_p)$ is the unconditional variance implied by the ARCH(p) model if $\alpha_1 + \ldots + \alpha_p < 1$. If the restriction $\alpha_1 + \ldots + \alpha_p < 1$ is violated, then the unconditional variance of p_t is not defined (as in the weighted moving average model).

(16.1) $\underline{Y}_t = F(X_t) + \underline{\delta}H_t + \underline{e}_t$

(16.2) $vech(H_t) = \underline{\omega}_0 + \sum_{i=1}^{p} \underline{\alpha}_i vech(\underline{e}_{t-i}\underline{e}'_{t-i}) + \sum_{i=1}^{q} \underline{\beta}_i vech(H_{t-i}),$

(16.3) $\underline{e}_t \sim N(\underline{0}, H_t),$

where $F(X_t)$ is a vector-valued function (typically a linear function) which models the mean response as a function of a set of variables X_t, including lagged values of Y_t, vech(.) denotes the column stacking operator of the lower portion of a symmetric matrix, $\underline{e}_t$ is an Nx1 innovation vector, $\underline{\omega}_0$ is a 0.5N(N+1) parameter matrix, and $\underline{\alpha}_i$, i=1,...,q, and $\underline{\beta}_i$, i=1,...,p, are 0.5N(N+1)x0.5N(N+1) parameter matrices.

In the context of the sow farrowings model, vector $\underline{Y}_t$ represents realizations of hog price and supply (sow farrowings in t-2). Both conditional price expectations and conditional price risk could then be specified to condition current sow farrowings, and the vector $\underline{\delta}$ would reduce to a scalar. As Engle and Bollerslev (1986) note the setup in (16), in the context of a commodity supply model, is similar to the structural rational expectations models considered by Wallis (1980), except that rational expectations are now taken with respect to the first two conditional moments of price.

Several approaches have been advanced for estimating the parameters of a multivariate GARCH-M (ARCH-M) model. A computationally convenient method was presented recently by Bollerslev (1990). The parameterization considered by Bollerslev (1990) allows for time-varying conditional variances and covariances, but assumes constant conditional correlations. Let σ_{ijt} denote the ijth element of H_t. Then the conditional correlation, evaluated at time t-1, between the ith and jth element of $\underline{Y}_t$ is defined in the usual way by $\rho_{ijt} = \sigma_{ijt}/(\sigma_i^2\sigma_j^2)^{1/2}$ where $-1 \leq \rho_{ijt} \leq 1$ for all t. Although ρ_{ijt} can in general be time-varying, it may be useful to assume $\rho_{ijt} = \rho_{ij}$ for all t, i.e., the conditional correlations are constant. It follows then that $\sigma_{ijt} = \rho_{ij}(\sigma_{iit}^2\sigma_{jjt}^2)^{1/2}$, j = 1,...,N; i=j+1,...,N.

As Bollerslev (1990) indicates, an appealing feature of the multivariate ARCH (GARCH) model with constant conditional correlations is the simplifications introduced into the estimation. Specifically, the conditional correlation matrix can be partitioned as $H_t = D_t\Psi D_t$ where D_t denotes a NxN stochastic diagonal matrix with elements $\sigma_{1t},\ldots,\sigma_{Nt}$ and Ψ is an NxN time-invariant positive definite matrix with typical element ρ_{ij}.

The above assumptions greatly reduce the computational complexities associated with obtaining maximum likelihood estimates of the model. By direct substitution, the likelihood function, apart form a constant term, is:

(17) $L(\underline{\theta}) = -(T/2)\log|\Psi| - \sum_{t=1}^{T} \log|D_t| - 0.5 \sum_{t=1}^{T} \tilde{\underline{e}}'_t \, \Psi \, \tilde{\underline{e}}_t$

where $\tilde{\underline{e}}_t = (D_t^{-1} \underline{e}_t)$ is a Nx1 vector of standardized residuals and $\underline{\theta}$ is a parameter vector. Although the likelihood function in (17) is still highly nonlinear in the parameters, note that only one NxN matrix inversion is called for during each iteration of the maximization process. Moreover, note that $\log|D_t| = \Sigma \log \sigma_{it}$. Below we use the Broyden's algorithm along with numerical first derivatives in the maximization of (17) when estimating bivariate ARCH-M and GARCH-M models of hog price and sow farrowings.

An alternative approach to modeling conditional variance, based on nonparametric methods, was suggested by Pagan and Ullah (1988). Several nonparametric specifications are possible (Pagan and Schwert, 1990). Here we consider the nonparametric kernel method, which offers an appealing density function estimation procedure that can be applied to the estimation of conditional moments (Ullah, 1988). In particular, given as sample of T realizations $(\hat{e}_t^2, z_t)$, where the conditioning variable(s) z may be lagged value(s) of $\hat{e}^2$'s, the conditional expectation $E[e_t^2|z_t]$ can be estimated by:

$$\hat{E}[e_t^2|z_t] = \sum_{i=1}^{T} \hat{e}_i^2 \; r_i(z_t) \tag{18}$$

Thus, the estimator $\hat{E}(e_t^2|z_t)$ is a weighted average of the sample's $\hat{e}^2$'s, where the weights $r_i(z_t)$'s depend on the evaluation point z_t via a "kernel" function and assign the heaviest weight to the observations closest to the evaluation point. To implement this nonparametric estimator, one must choose a kernel function and the window width. The latter is the most important choice because the window width controls the amount of smoothing imposed on the data. Here we follow a common practice and use the normal density for the kernel function, and use the window width specification adopted, among others, by Pagan and Schwert, 1990; Pagan and Hong, 1988; and Moschini, 1990.

When the data is sparse and the sample is small, the local averaging of the kernel estimator may give undue weight to the observation being predicted, thereby overstating the (within sample) predictability of the model. A possible way out is to use the leave-one-out estimator, defined as:

$$\hat{E}[e_t^2|z_t] = \sum_{i\neq t}^{T} \hat{e}_i^2 \; r_i(z_t) \tag{19}$$

with the weights r_i's also properly adjusted by leaving out the t^{th} observation.

Estimation Results

The above methods are used to estimate risk-responsive aggregate models of sow farrowing decisions in the U.S. Quarterly time-series data for the period 1958-90 are used. Specifically, sow farrowings, y_t, denotes the total number of sows that have farrowed (in one hundred thousands) in the U.S. within a given quarter. The hog price variable, p_t, (in dollars per cwt.) is taken to be the seven-market price of barrows and gilts, and the corn price variable is the average price paid (in dollars per bu.) by farmers for no. 2 corn. The sow farrowings data were obtained from various issues of Hogs and Pigs; slaughter prices for barrows and gilts were obtained from various issues of Livestock and Meat Statistics; and corn prices were collected from various issues of Agricultural Prices. Additionally, all prices were deflated by the wholesale price index.

The estimation results are presented in Tables 1 and 2. Table 1 contains estimation results for the autoregressive models of slaughter price for barrows and gilts. Table 2 contains parameter estimates for the risk-responsive sow farrowings equations for alternative specifications of the conditional variance process. Preliminary estimates revealed that an AR(6) specification of the price equation captured the essential dynamic adjustments occurring in slaughter hog prices. OLS estimates of the AR(6) model are reported in column one of Table 1. Several diagnostic tests of the residuals of the price equation, including the Breusch-Godfrey test and the Ljung-Box Q test indicate the innovations are free of any additional autocorrelation at lags four and ten, respectively. Moreover, the model provides a reasonable fit to the data, as indicated by the high R^2.

A different picture arises, however, when we consider the McLeod and Li Q^2 test using the squared innovations from the estimated OLS price equation. The resulting Q^2 test statistic at ten degrees of freedom is 37.36, a value which is well above the critical value of 18.31 from the asymptotic χ^2 distribution at the 5 percent level with 10 degrees of freedom. As Bollerslev (1986) suggests, the absence of serial correlation in the conditional first moment of hog prices coupled with the presence of serial correlation in the squared residuals is one of the implications of conditional autoregressive heteroskedasticity.

In what follows, we use the fitted values and the residuals from the OLS estimates of the AR(6) price model in Table 1 to generate several measures of expected price and price risk for inclusion in the sow farrowings supply model. Specifically, we follow Pagan and Schwert (1990) in using the squared residuals from the OLS AR(6) equation to estimate a conditional variance equation of the form:

$$(20) \qquad e_t^2 = \omega_0 + \alpha_1 W_t \quad \text{with} \quad W_t \equiv \sum_{i=1}^{6} w_i e_{t-i}^2$$

where the parameters ω_0 and α_1 are estimated using OLS with the weights w_i set equal to (10/26, 6/26, 4/26, 3/26, 2/26, 1/26). Using the predictions generated from equation (20), along with the predictions of the conditional mean from the AR(6) model, we estimate the parameters of the sow farrowings equation. We refer to this approach as the two-step method, and the associated parameter estimates are reported in Table 2 under the column headed Single Equation ARCH. Another two-step approach that we consider utilizes the kernel estimator of the conditional variance. The results reported in Table 2 utilize the leave-one-out estimator given in (19), with $z_t \equiv W_{t-4}$.

Overall, the 2-step models fit the data well as indicated by the reasonable R^2. Moreover, the signs of the estimated coefficients agree with a priori expectations in that sow farrowings show a positive response to expected price and a negative response to the conditional price variance and the lagged price of corn. All coefficients associated with economic variables are also statistically significant at conventional levels for the ARCH model, but the risk term is not significant in the kernel model.

Table 2 also reports the results of the price and supply equations estimated as a system of (restricted) multivariate ARCH-M and GARCH-M models. The results of these two models are strikingly similar to those of the two-step ARCH model reported in the first column of Table 2.

Because the supply equation estimates producers' response to the expected price and the conditional variance, it is worth asking how well these predictions perform. Table 3 reports the predictive power of the estimated expected price and conditional variance. The results in Table 3 pertain to linear regressions of observed prices on estimated expected prices, and of observed squared deviations from (estimated) expected prices on estimated conditional variance. Because of the nature of the information set discussed earlier, we are interest in the predictive power for 4 periods ahead. However, for comparison, we report also the results on predictive power for 1-period ahead. Going from 1 period to 4 periods ahead substantially deteriorates estimates of the conditional mean, with the R^2 falling from about 0.8 to about 0.3, although the estimator remains unbiased.

Not surprisingly, predicting the conditional variance is much more difficult. For ARCH and GARCH models, the 1-period ahead prediction have an R^2 of about 0.10. The kernel method does better, but when the (more reliable) leave-one-out method is used the kernel method does considerably worse than the ARCH and GARCH models. At 4 periods ahead, the predictive power of estimated conditional variances essentially vanishes, especially for the two-step approach. Hence, as we move the relevant planning horizon further into the future, it is clear that the estimated conditional variance quickly approaches the unconditional variance estimate.

While the finding that predicting conditional variances is difficult in the present context is not a surprising one, we are left with the apparent puzzle of a significant risk response to risk terms that appear substantially useless in predicting price volatility. The resolution of this question probably lies in the fact that our simplified supply equation is misspecified, as indicated by the presence of autocorrelated errors suggested by the Breusch-Godfrey statistic. Resolving this specification problem may involve a more accurate representation supply decisions. Alternatively, one may simply correct for autocorrelation. When this is done by modeling autocorrelation as an autoregressive process of order 4, unreported results suggest that the statistical significance of the risk coefficients disappears.

Finally, Table 4 reports some elasticities of sow farrowings, all evaluated at the sample means. The estimated elasticitity with respect to expected price ranges from 0.15 to 0.19 in the short run, and from 0.48 to 0.63 in the long run. The elasticity with respect to lagged corn price are smaller in absolute value and of the opposite sign. The elasticities with respect to conditional price variance allow the calculation of the ratio reported in equation (5), which is found to take values of about 0.25. This would suggest a rather large departure from marginal cost pricing due to risk aversion. However, in view of the discussion above, the elasticities with respect to conditional variance are probably not very reliable, and any conclusion concerning the extent of risk effects does not appear warranted.

Conclusion

This paper has explored alternative specifications of risk response in a hog supply model expressed in terms of sow farrowing decisions. Because of the nature of the information set available to producers when making production decisions, predicting conditional variances is difficult. Hence, it could be argued that the risk to which producers probably respond is best represented by the unconditional variance of prices. While this leaves open the possibility of risk being important in determining production decisions, it makes the econometric estimation of risk response virtually impossible in the framework of this paper. If interest lies in improving the performance of econometric models of hog supply, our results would suggest that omitting an explicit model of risk response is not likely to result in model misspecification. If, on the other hand, interest lies in characterizing risk preferences of hog producers, and in estimating the extent of their adjustment to price risk, then our results suggest that the conventional specification of risk terms in a hog supply model may be inadequate to this task. Further research in developing an improved methodology is required.

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Table 1. Parameter Estimates for the AR(6), AR(6)-ARCH(6), and AR(6)-GARCH(1,1) Models.

		Two-Equation System	
Parameter	AR(6)	AR(6)-ARCH(6)	AR(6)-GARCH(1,1)
Conditional Mean:			
b_0	1.874 (0.962)	1.167 (0.806)	1.166 (0.803)
b_1	1.049 (0.080)	1.001 (0.079)	1.004 (0.076)
b_2	-0.161 (0.105)	-0.104 (0.095)	-0.109 (0.094)
b_3	-0.058 (0.103)	-0.064 (0.091)	-0.065 (0.090)
b_4	0.294 (0.103)	0.257 (0.095)	0.261 (0.093)
b_5	-0.718 (0.108)	-0.627 (0.090)	-0.629 (0.086)
b_6	0.493 (0.078)	0.468 (0.065)	0.468 (0.061)
Conditional Variance:			
ω_0	-	1.912 (0.509)	0.720 (0.320)
α_1	-	0.499 (0.167)	0.196 (0.083)
β_1	-	-	0.616 (0.120)
R^2	0.806	0.803	0.803
BG(4)	3.776		
Q(10)	15.293		
$Q^2(10)$	37.358		

Note: Standard errors appear in parentheses. BG(4) is the Breusch-Godfrey test statistic; Q(10) is the Ljung-Box portmanteau Q-statistic; and $Q^2(10)$ is the McLeod and Li portmanteau Q^2-statistic.

Table 2. Estimates of Sow Farrowing Supply Models under Alternative Risk Specifications, 1958-90.

	Single Equation		Two-Equation System	
Parameter	ARCH	Kernel leave-one-out	ARCH	GARCH
constant	-1.902 (3.177)	-0.318 (3.285)	0.138 (3.040)	0.132 (3.084)
Q2	6.700 (0.976)	6.667 (0.997)	6.523 (0.907)	6.476 (0.907)
Q3	5.241 (1.056)	5.018 (1.086)	5.368 (0.985)	5.319 (0.988)
Q4	23.565 (1.215)	23.264 (1.237)	22.888 (1.155)	22.851 (1.167)
Q2·D75	-5.603 (1.081)	-5.593 (1.101)	-5.232 (1.002)	-5.192 (0.966)
Q3·D75	-6.459 (1.116)	-6.094 (1.149)	-6.238 (1.028)	-6.204 (0.994)
Q4·D75	-16.800 (1.127)	-16.620 (1.145)	-16.305 (1.053)	-16.266 (1.028)
D75	6.680 (0.878)	6.348 (0.890)	6.162 (0.852)	6.156 (0.837)
$\bar{p}_t$	0.300 (0.088)	0.247 (0.087)	0.255 (0.090)	0.263 (0.084)
v_t^2	-0.098 (0.040)	-0.042 (0.033)	-0.096 (0.058)	-0.102 (0.063)
p_{t-4}^c	-0.197 (0.091)	-0.282 (0.083)	-0.200 (0.090)	-0.195 (0.098)
y_{t-1}	0.706 (0.061)	0.700 (0.062)	0.680 (0.058)	0.677 (0.059)
R^2	0.892	0.889	0.893	0.893
BG(4)	36.3	43.3		

Note: Standard errors are in parentheses.

Table 3. Comparison of Within-Sample Predictive Power for the Conditional Mean of Real Hog Prices

Equation	One Period Ahead			Four Periods Ahead		
	Intercept	Slope	R^2	Intercept	Slope	R^2
Conditional Mean						
Single Equation	0.000 (0.836)	1.000 (0.043)	0.806	0.535 (2.467)	0.973 (0.129)	0.305
ARCH(6) System	0.121 (0.838)	1.000 (0.043)	0.803	2.960 (2.204)	0.865 (0.118)	0.294
GARCH(1,1) System	0.118 (0.838)	1.000 (0.043)	0.803	2.944 (2.207)	0.866 (0.118)	0.294
Conditional Variance						
Two-step ARCH(6)	0.000 (1.224)	1.000 (0.278)	0.090	9.937 (6.231)	0.281 (0.422)	0.003
Two-step Kernel	-0.354 (1.067)	1.125 (0.241)	0.144	-4.437 (7.572)	1.296 (0.514)	0.047
Two-step Kernel leave-one-out	1.994 (1.138)	0.505 (0.261)	0.023	15.049 (6.692)	-0.088 (0.442)	0.0003
System ARCH(6)	0.116 (1.185)	0.982 (0.264)	0.096	0.630 (1.581)	0.255 (0.113)	0.038
System GARCH(1,1)	0.148 (1.190)	0.970 (0.265)	0.094	0.739 (1.162)	0.245 (0.115)	0.034

Note: Standard errors are in parentheses. All results were obtained by regressing the realized price (squared innovations for the conditional variance) against the expected price (the estimated conditional variances) based on information in periods t-1 and t-4, respectively.

Table 4. Elasticities of Sow Farrowings Evaluated at Data Means, 1958-90.

Model	Expected Price of Hogs		Conditional Variance of Hog Price		Lagged Price of Corn	
	Short Run	Long Run	Short Run	Long Run	Short Run	Long Run
<u>Two-Step Equations</u>:						
ARCH(6)	0.185	0.629	-0.044	-0.150	-0.068	-0.231
Kernel leave-one-out	0.152	0.507	-0.019	-0.063	-0.098	-0.327
<u>System</u>:						
ARCH(6)	0.153	0.479	-0.040	-0.126	-0.069	-0.217
GARCH(1,1)	0.158	0.490	-0.043	-0.134	-0.068	-0.210