The Inverse Almost Ideal Demand System

by

James Eales and Laurian Unnevehr

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INTRODUCTION

The Almost Ideal Demand System (AIDS) model of Deaton and Muellbauer (1980a, 1980b) has enjoyed great popularity in applied demand analysis for a number of reasons. First, it is derived from a specific cost function and thus corresponds to a well defined preference structure, which is convenient for welfare analysis. These so-called PIGLOG preferences also have the property of consistent aggregation from the micro to the market level, while allowing nonlinear Engel curves. Second, the functional form of the preferences is "flexible" in that it can be thought of as a local second order approximation to an unknown preference structure. Third, demand restrictions depend only on estimated parameters and so are easily imposed and/or tested. Finally, in their original paper, Deaton and Muellbauer suggested a linear approximation of the nonlinear AIDS model, which works well if prices are collinear. This last advantage of the AIDS model has been quite important, as the Linear Approximate/Almost Ideal Demand System (LA/AIDS) is frequently what is estimated in practice. A few of the examples of applications of AIDS to agricultural commodities are: Blanciforti, Green, and King; Chalfant and Alston; Eales and Unnevehr; Moschini and Meilke; Alston, et al.; Haden; Hayes, Wahl, and Williams.

Although the AIDS model has worked well in applications, there are commodities for which the assumption of predetermined prices at the market level may be untenable. For example, applications of the AIDS model to the demand for perishable commodities, which are produced subject to biological lags, using monthly or quarterly market-level time-series data, may not be viable. For such situations, it has been typical in applications to specify an inverse demand system, often in an ad hoc manner.[1] Such demand models are sometimes embedded within a larger market model (for examples, see: Freebairn and Rausser; Arzac and Wilkinson). The assumption is that quantity is predetermined by production at the market level, and since it is not storable, price must adjust so the available quantity is consumed.

Not all previous studies which have employed inverse demand structures have proceeded in an ad hoc manner. Heien, and Chambers and McConnell developed separable inverse demand systems and applied them to food commodities. Barten and Bettendorf developed an inverse Rotterdam system and applied it to the demand for fish. Christensen, et al. develop the direct translog demand system (as well as the indirect system). Both they and Jorgenson and Lau use the direct translog demand system to test demand restrictions. Huang used the theoretical development of Anderson and the distance function to generate a system of inverse demands, which were applied to composite food and nonfood commodities. In the next section an Inverse Almost Ideal Demand System (IAIDS) is developed, which carries over many of the advantages of the AIDS model. To aid in the interpretation of results, the scale compensation, suggested by Anderson, is reviewed. Calculation of flexibilities and their

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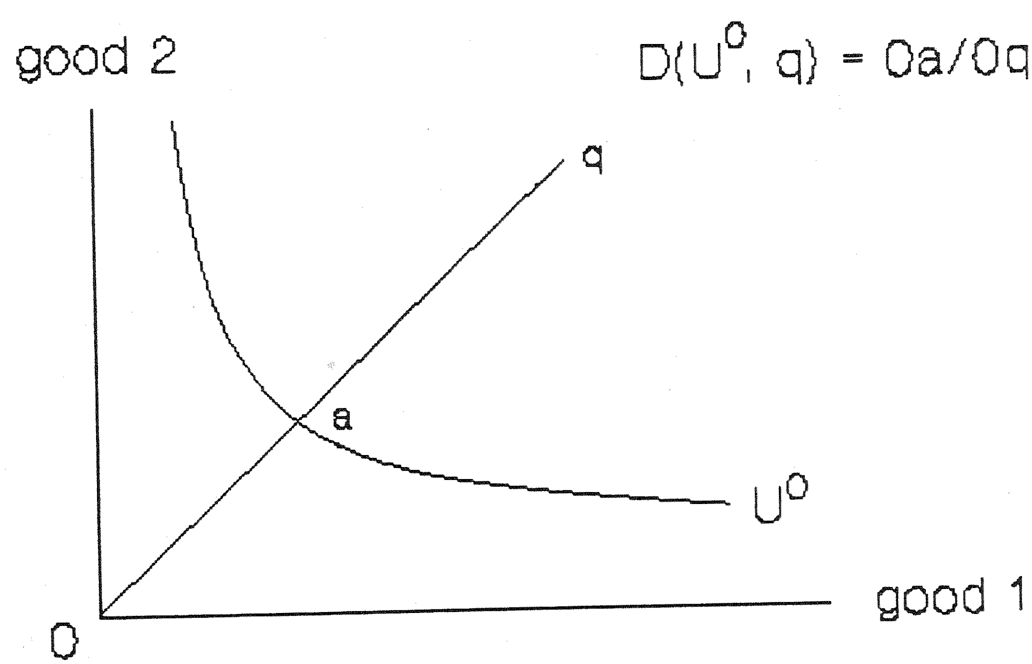
interpretation is then discussed. The IAIDS model is then applied to quarterly US meat demand for illustrative purposes. The application shows that in this case, a linearized version of the IAIDS model performs well. Finally, results are summarized and conclusions presented.

THE IAIDS MODEL

As noted in the introduction, while the AIDS model has been employed successfully in a number of applications, there exist commodities for which the assumption of fixed prices at the market level, may be difficult to accept for a given interval of observation. In such situations it would seem appropriate then to use an inverse form of the AIDS model. The key to development of such a model is to begin with an alternative representation of preferences, dual to the cost function. This representation is known as the distance function.[2] Briefly, it characterizes the amount by which all quantities consumed must be changed proportionally to attain a particular utility level. That is, it gives the proportional "distance" along a ray through the origin that quantities must be reduced or inflated to reach a particular indifference surface. The distance function is defined implicitly by $U\{q/D(u,q)\} = u$. This is illustrated for a 2 good example in Figure 1.

Figure 1. The Distance Function



Thus, to derive an Inverse AIDS model, start by specifying an appropriate distance function. It possesses the same properties as the cost function; i.e. it is linear-homogeneous, concave, and non-decreasing in quantities (as opposed to prices); and it is decreasing in utility, rather than increasing (Diewert). It also has a derivative property similar to the cost function (Deaton, 1979; p 394). That is, at the optimum, differentiation with respect to the quantity of a particular good yields the compensated demand for that good. Thus, in a manner similar to Deaton and Muellbauer's derivation of the AIDS model (1980b) a logarithmic distance function may be specified:

$$\ln D(U,q) = (1-U) \ln a(q) + U \ln b(q) \tag{1}$$

Because the distance function has the same requirements as the cost function to be consistent with theory (with quantities substituted for prices), ln a(q) and ln b(q) may be specified in a manner analogous to that employed in the AIDS development.[3]

$$\ln a(q) = \alpha_0 + \sum \alpha_j \ln q_j + 0.5 \sum_i \sum_j \gamma_{ij}^* \ln q_i \ln q_j \tag{2}$$

$$\ln b(q) = \beta_0 \prod_j q_j^{\beta_j} + \ln a(q)$$

This parameterization yields:[4]

$$\ln D(U,q) = \alpha_0 + \sum_j \alpha_j \ln q_j + 0.5 \sum_i \sum_j \gamma_{ij}^* \ln q_i \ln q_j + U \ \beta_0 \prod_j q_j^{\beta_j} \tag{3}$$

Differentiation yields compensated <u>inverse</u> demands:[5]

$$\partial \ln D / \partial \ln q_i = w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i U \ \beta_0 \prod_j q_j^{\beta_j} \tag{4}$$

where: $\gamma_{ij} = 0.5 (\gamma_{ji}^* + \gamma_{ij}^*)$

Inversion of the distance function at the optimum yields the direct utility function which may be used to uncompensate the inverse demands.[6]

$$U(q) = -\ln a(q) / \{ \ln b(q) - \ln a(q) \} \tag{5}$$

This yields a system of inverse demand functions which will be called the <u>Inverse Almost Ideal Demand System</u> or IAIDS:[7]

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q \tag{7}$$

with ln Q given by:

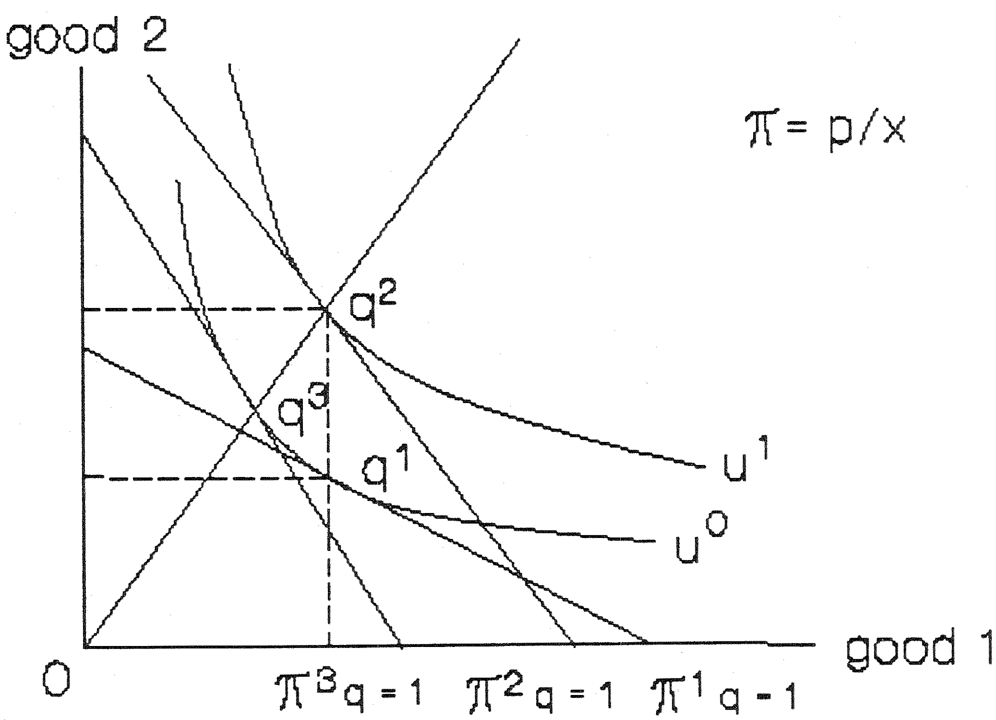
$$\ln Q = \alpha_0 + \sum \alpha_j \ln q_j + 0.5 \sum_i \sum_j \gamma_{ij} \ln q_i \ln q_j \tag{8}$$

As with the AIDS model, the typical demand restrictions involve only the fixed, unknown coefficients and so may be easily tested or imposed. These restrictions are: $\sum_i \alpha_i = 0, \sum_i \gamma_{ij} = 0, \sum_i \beta_i = 0$ (adding up); $\sum_j \gamma_{ij} = 0$ (homogeneity); $\gamma_{ij} = \gamma_{ji}$ (symmetry).

As it stands, Equation 7 requires nonlinear estimation. It would be convenient if one could substitute an approximation to the quantity index (Equation 8) which did not depend on unknown parameters, such as Stone's quantity index. Obviously, the justification employed for the AIDS model, that prices in time-series data are collinear, is not appropriate for the IAIDS model. Quantities do not move together. Thus, the adequacy of the approximation is an empirical question and no generalization can be made for the LA/IAIDS model.[8] This is addressed further in an example below.

Demand estimation yields elasticities. On the inverse side, the sensitivities are typically measured by flexibilties (Houck). An important consideration in the interpretation of inverse demand results is the appropriate analog of the expenditure elasticity. Anderson proposed a compensation technique for inverse demands which has much to recommend it. His method was one of compensation according to "scale." It is illustrated in Figure 2. A movement from bundle q^1 to q^2, is broken up into two pieces. The movement along the indifference surface, representing utility level U^0 to bundle q^3 gives the substitution effect, while the movement along the ray through the origin from q^3 to q^2 is the scale effect. The distance function measures distance along rays through the origin. Thus, it is appealing to compensate along such rays, i.e. according to the "scale" of consumption, just as with the cost function it is appealing to compensate with income or expenditure.

Figure 2. Inverse Demand Compensation



This notion of compensation in inverse demands has a number of other attractive implications. For example, to be consistent with theory ordinary demand curves must satisfy the homogeneity, Cournot, and Engel aggregation relations. For inverse demands, Anderson shows that similar aggregation relations hold. That is, if f_{ij}, f_i, and w_i represent cross-price flexibilities, scale flexibilities, and expenditure shares, respectively; then flexibilties must satisfy the following aggregation relations: $\sum_j f_{ij} = f_i$ (homogeneity); $\sum_i w_i f_{ij} = -w_j$ (Cournot); $\sum_i w_i f_i = -1$ (Engel).[9]

Interpretation of flexibilities can be made in a manner similar to elasticities. Demand for a commodity is said to be inflexible if a 1% increase in consumption of that commodity leads to a less than 1% decrease in the marginal value of that commodity in consumption; that is, its normalized price. Commodities are termed gross q-substitutes if their cross price flexibility is negative, gross q-complements if it is positive (Hicks).

An interpretation of scale flexibilities can be made by considering the case of homothetic preferences, i.e. all expenditure elasticities equal to one. In this case, expanding consumption of all commodities by 1%, that is moving out along a ray through the origin, requires no change in relative prices to support the new equilibrium consumption bundle. But, expenditures must increase by 1% to achieve this new bundle. Thus, the normalized prices, price divided by expenditures, will decrease by approximately 1%, and all scale flexibilities must be −1. Necessities and luxuries can then be defined in reference to the base case of homothetic preferences. Scale flexibilities are less than -1 for necessities and greater than -1 for luxuries. At the margin, normalized price is proportional to marginal utility. Therefore, as consumption of all goods increases 1 %, the marginal utility of necessities declines more than proportionately (scale flexibility < -1) and the marginal utility of luxuries declines less than proportionately (scale flexibility > -1).

Given the scale decomposition of inverse demands, the formulae for the calculation of Marshallian cross-price, expenditure, and compensated or Hicksian elasticities are almost identical to those required for the cross-price, scale, and compensated flexibilities. For comparison, these formulae are given in Table 1.[10]

AN APPLICATION TO US MEAT DEMAND

Food commodities provide many examples of perishable goods for which supply may be fixed during short intervals. The demand for meat is examined here as an example. The high cost of cold storage and the physiological lags in production prevent meat supply from adjusting to price changes within a calendar quarter, for example. Thus, a quarterly model of US meat demand is used to illustrate the application of IAIDS. Data on quarterly per capita consumption and prices of beef, pork, and chicken, and population are from USDA sources. Monthly personal consumption expenditures and the consumer price index for urban consumers from US Department of Commerce publications are averaged over quarters. Data and exact sources are listed in an appendix.

The system consists of demands for: beef, pork, chicken, and all other goods. The other goods equation is dropped in estimation due to singularity of the cross-equation covariance matrix.[11] The remaining three equations are estimated in two ways. Preliminary investigation and previous findings suggest that dynamics are important in meat demand. Deaton and

Table 1. **COMPARISON OF THE FORMULAE FOR AIDS ELASTICITIES AND IAIDS FLEXIBILITIES**

AIDS Elasticities and IAIDS Flexibilities

	AIDS	IAIDS
Own & Cross Price	$\varepsilon_{ij} = -\delta_{ij} - \{ \gamma_{ij} - \beta_i (w_j - \beta_j \ln (x/P)) \} / w_i$	$f_{ij} = -\delta_{ij} + \{ \gamma_{ij} + \beta_i (w_j - \beta_j \ln Q) \} / w_i$
Expenditure or Scale	$\varepsilon_i = 1 + \beta_i / w_i$	$f_i = -1 + \beta_i / w_i$
Compensated	$\varepsilon_{ij}^* = \varepsilon_{ij} + w_j \varepsilon_i$	$f_{ij}^* = f_{ij} - w_j f_i$

In the table εs are elasticities; fs are flexibilities; ws are expenditure shares; δ_{ij} is the Kronecker delta; x is total expenditure; and P and Q are the price and quantity indices; and αs, βs, and γs are parameters of the appropriate models.

Muellbauer proposed a first difference form for the AIDS model in their original article. A similar approach is taken here.[12]

To capture seasonal effects, likely to be important in meat demand, seasonal dummies were added to Equation 7, after first differences were taken, resulting in the following model:

$$\Delta w_i = (\textstyle\sum_k \theta_{ki} D_k) + \sum_j \gamma_{ij} \Delta \ln q_j + \beta_i \{ \sum_j (\sum_k \theta_{kj} D_k) \Delta \ln q_j + 0.5 \sum_i \sum_j \gamma_{ij} \Delta(\ln q_i \ln q_j) \} \tag{9}$$

where the D_k are 4 quarterly dummies and the θ_{kj} are the associated coefficients in each of the share equations. First, using nonlinear least squares, the IAIDS system in first difference form is estimated, with homogeneity and symmetry imposed. Second, Stone's quantity index is substituted for the IAIDS quantity index and the model is re-estimated using iterative SURE. The coefficient estimates and summary statistics are presented in Table 2.

In the top half of the table the NL/IAIDS coefficients and their standard errors (multiplied by 100) are given. They were obtained using the NL command of the SHAZAM program, which employs a Davidon-Fletcher-Powell algorithm. Starting values were the LA/IAIDS estimates, although other starting values converged to these estimates as well. The explanatory power of the model is reasonable, especially for a specification in first difference form. The pork equation shows some evidence of autocorrelation, though the Durbin-Watson statistic has an unknown distribution in this instance. Sixteen of the twenty-four estimated coefficients are more than twice their asymptotic standard errors.

Comparison of these estimates with those of the LA/IAIDS reveals that the linear approximation performs quite well. The own-quantity and scale coefficients are all within 10% of their nonlinear counterparts, except the own-quantity effect for all other goods. The root mean square percentage error between the quantity index from the NL/IAIDS and Stone's quantity index is only 0.4% over the period, which helps account for the close correspondence between coefficients of the two models.

A comparison of the flexibilities from the NL/IAIDS and LA/IAIDS is offered in Table 3.[13] In the table, flexibilities (calculated using the formulae in Table 1 at the sample means) are given for the NL/IAIDS and LA/IAIDS models, respectively. While the match between the two is not exact, the parallel is striking. None differ by more than 6%.

In both cases, own-price, cross-price, and scale flexibilities are negative, as expected.[14] All the meats are q-substitutes (Hicks). All the own-price flexibilities are slightly less than 1 in absolute value. For example, a 1% increase in beef quantity consumed is associated with a 0.95% decline in the price of beef. Scale flexibilities indicate that as the scale of consumption increases by 1%, i.e. all quantities consumed increase by 1%, the marginal value of meats in consumption declines by 1.9% for beef and 2.5% for poultry. Such magnitudes are plausible scale flexibilities for such food commodities. These results are comparable to those of Huang (1988). Own-price flexibilities for beef/veal, pork, and poultry in his inverse demand model were slightly larger than 1.0, in absolute value; scale flexibilities ranged from -1.3 to -2.0.[15]

Since both models have the same set of dependent variables, it is possible to test whether the NL/IAIDS model rejects the LA/IAIDS. This may be accomplished using the multivariate version of the non-nested testing framework developed by Davidson and MacKinnon.[16] In this case, the test measures the significance of the difference between the predictions of the two models in a seemingly unrelated regression of the residuals from the LA/IAIDS on the original explanatory variables and the difference in predicted values. If the coefficient associated with

Table 2. COMPARISON OF COEFFICIENTS: NL/IAIDS AND LA/IAIDS[1]

NL/IAIDS

	QTR 1	QTR 2	QTR 3	QTR 4	BEEF Q	PORK Q	CHK Q	OTHER Q	Q	R^2/DW
BEEF	0.010	0.024	0.053*	-0.092*	0.527*	-0.121	-0.313*	-0.093	2.152	0.341
	(0.022)	(0.021)	(0.021)	(0.024)	(0.246)	(0.117)	(0.084)	2	(1.229)	2.098
PORK	-0.048*	-0.041*	0.040*	0.052*	-0.121	0.310*	-0.319*	0.131	-1.913*	0.700
	(0.014)	(0.013)	(0.012)	(0.016)	(0.117)	(0.095)	(0.056)	2	(0.743)	1.589
CHK	-0.014*	0.014	0.020*	-0.024*	-0.313*	-0.319*	0.158	0.474	-0.711	0.706
	(0.007)	(0.010)	(0.007)	(0.011)	(0.084)	(0.056)	(0.088)	2	(0.399)	2.240

LA/IAIDS

	QTR 1	QTR 2	QTR 3	QTR 4	BEEF Q	PORK Q	CHK Q	OTHER Q	Q	R^2/DW
BEEF	0.010	0.021	0.056*	-0.092*	0.541	-0.087	-0.307*	-0.147	-2.242	0.341
	(0.023)	(0.021)	(0.022)	(0.024)	(0.272)	(0.137)	(0.089)	(0.395)	(1.266)	2.102
PORK	-0.048*	-0.044	0.040*	0.053*	-0.087	0.337*	-0.315*	0.066	-1.889*	0.700
	(0.014)	(0.013)	(0.013)	(0.016)	(0.137)	(0.109)	(0.060)	(0.230)	(0.764)	1.589
CHK	-0.014	0.014	0.021*	-0.024*	-0.307*	-0.315*	0.145	0.477*	-0.781	0.708
	(0.008)	(0.010)	(0.007)	(0.011)	(0.089)	(0.060)	(0.093)	(0.154)	(0.400)	2.239

* Indicates the ratio of the coefficient to its standard error exceeds 2 in absolute value.
1 Coefficients and their standard errors are multiplied by 100 to facilitate presentation.
2 Coefficients calculated using the homogeneity restriction.

Table 3. COMPARISON OF FLEXIBILITIES: NL/IAIDS AND LA/IAIDS

NL/IAIDS

	BEEF Q	PORK Q	CHK Q	OTHER Q	SCALE
BEEF P	-0.947	-0.173	-0.180	-0.595	-1.902
PORK P	-0.351	-0.990	-0.318	-0.759	-2.418
CHK P	-0.930	-0.899	-0.755	0.094	-2.490
OTHER P	0.008	0.009	0.008	-0.975	-0.950

LA/IAIDS

	BEEF Q	PORK Q	CHK Q	OTHER Q	SCALE
BEEF P	-0.954	-0.173	-0.180	-0.595	-1.902
PORK P	-0.332	-0.965	-0.321	-0.782	-2.400
CHK P	-0.955	-0.912	-0.799	0.029	-2.637
OTHER P	0.008	0.009	0.008	-0.974	-0.949

the difference in predicted values is significant, then the NL/IAIDS rejects the LA/IAIDS.[17] The coefficient, in this case, is -2.16 with a standard error of 4.10, so the LA/IAIDS is not rejected.

SUMMARY AND CONCLUSIONS

A new demand system was developed from the distance function representation of preferences. The functional form of the derived share equations is similar to the AIDS demand system of Deaton and Muellbauer, except the right-hand-side variables are quantities, instead of prices and expenditure. Thus, it is called the Inverse Almost Ideal Demand System or IAIDS.

As it is derived from a distance function, all of the desirable theoretical properties attributed to the AIDS model carry over to IAIDS, with the exception of the aggregation from the micro to the market level.

Anderson's scale compensation is employed as a tool to aid in interpretation of IAIDS results. It is shown that the functional forms of flexibilities from the IAIDS model are analogous to the elasticities of the AIDS model. Benchmarks for interpreting flexibilities are given. In particular, it is shown that if preferences are homothetic all scale flexibilities are -1, just as all expenditure elasticities are one. Luxuries and necessities can be classified in reference to this base.

The desirable empirical property of obtaining estimates which are reasonably close to those of the NL/IAIDS model from LA/IAIDS does appear to have worked well for the example of quarterly US meat demand. The multivariate P test of Davidson and MacKinnon was employed to test whether the NL/IAIDS model rejected its linear approximation, and it did not. Furthermore, all coefficients agreed to at least three decimal places and many to six. The Stone's quantity index was close to the NL/AIDS quantity index, never differing by more than 6% and the root mean square percentage error between the two series was 0.4%. Meat quantities are not highly correlated. Thus, the success of this linear approximation suggests that it may do well in a wide variety of cases.

Flexibilities were calculated from both the NL/IAIDS and LA/IAIDS estimates. The two sets of flexibilities showed a close correspondence. Both models produced reasonable estimates. The own-price flexibilities appear slightly larger than would be anticipated on the basis of past research, although these results are qualitatively similar to those found by Huang for similar commodities and time period.

Most meat demand studies have employed quantity-dependent demand systems, presumably because such systems are consistent with theory. Smallwood, et al. review this literature and of the 17 studies listed in their Table 3, 14 employ quantity-dependent demands. Yet the market for meat is likely to be characterized by fixed supply during monthly, quarterly, and possibly annual data intervals. The IAIDS provides an alternative that is both consistent with theory and with observations of the price discovery process. The IAIDS may have further application within market models of supply and demand that are frequently used for policy analysis. In contrast to ad hoc inverse demand systems, the IAIDS model provides theoretically justifiable measures of welfare changes.

ENDNOTES

1. Of course, the question of whether quantities or prices may appropriately taken as exogenous is an empirical one for many applications. It might be addressed in the form of hypothesis tests or in estimation by the use of instrumental variables (for example, see Thurman).

2. Accessible references on the uses of the distance function in demand analysis are: Deaton(1979) and Deaton and Muellbauer (1980a pp 53-7). The distance function has also been called the transformation, deflation, direct cost, or gauge function.

The algorithm for model development employed below is similar to that used by Huang (1988), who also utilizes the scale compensation of Anderson. He, however, uses a

Laspeyres index to approximate the "scale effect;" whereas, the quantity index associated with the scale effect in the NL/IAIDS model comes directly from the consumer's preference structure.

3. Interpretation of a(q) and b(q) are analogous to those of a(p) and b(p) for the AIDS model. If utility is scaled such that $0 \leq U \leq 1$, then a(q) gives the value by which q must be divided to reach subsistence (U = 0) and b(q) is the value by which q must be divided to achieve bliss (U = 1).

4. The following logarithmic distance function is not the dual of the AIDS logarithmic cost function. To our knowledge, no closed-form solution for the dual of the AIDS log–cost function exists.

5. The derivative property of the distance function is $\partial D(U, q) / \partial q_i = \pi_i = p_i / x$ (Deaton, p 394). This may be manipulated in a manner similar to that which gives $\partial \ln C(U, p) / \partial \ln p_i = w_i$, yielding:

$$(\partial D(U, q) / \partial q_i)(q_i / D(U, q)) = \partial \ln D(U, q) / \partial \ln q_i = p_i q_i / x = w_i.$$

6. If q* is the consumption bundle proportional to q which lies on indifference surface U, then D(U, q*) = 1. If equation 1 is evaluated at q* then:

$$\ln D(U,q^*) = 0 = (1-U) \ln a(q^*) + U \ln b(q^*) = \ln a(q^*) + U\{ \ln b(q^*) - \ln a(q^*) \}$$

and therefore:

$$U(q^*) = -\ln a(q^*) / \{ \ln b(q^*) - \ln a(q^*) \}.$$

See Deaton (1979; p 393).

7. It has recently come to our attention that a model similar to the one developed here has been developed independently by Moschini and Vissa.

8. This is, of course, true for the AIDS model as well. That is, the adequacy of the approximation is always an empirical question. However, often those who employ LA/AIDS do not check its validity for their particular application. Theoretically, the accuracy of the approximation is of little importance. However, from the practical perspective, the availability of a linear approximation is of some interest. This is especially true if the IAIDS model is to be embedded within a market model, where issues of concern include not only ease of estimation, but model solutions as well.

9. Anderson actually refers to the f_{ij} and f_i as quantity and scale elasticities (δ_{ij} and μ_i in his notation). Choice of the terms price and scale flexibilities is made for its consistency with previous work in the Agricultural Economics literature. There is a misprint in Anderson. The Engel aggregation for inverse demands says that the expenditure share weighted sum of the scale flexibilities is -1.

10. The IAIDS flexibilities are derived as follows: The ith share equation is

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln q_j + \beta_i \ln Q$$

differentiate with respect to the log of q_j:

$$\begin{aligned} \partial w_i / \partial \ln q_j &= \gamma_{ij} + \beta_i (\partial \ln Q / \partial \ln q_j) \\ &= \gamma_{ij} + \beta_i (\alpha_j + \textstyle\sum_k \gamma_{kj} \ln q_j) \\ &= \gamma_{ij} + \beta_i (w_j - \beta_j \ln Q) \end{aligned}$$

but:

$$\begin{aligned} \partial w_i / \partial \ln q_j &= \partial / \partial \ln q_j (p_i q_i / x) = \partial p_i / \partial \ln q_j (q_i / x) = \partial \ln p_i / \partial \ln q_j (p_i q_i / x) \\ &= f_{ij} w_i \end{aligned}$$

which implies for $i \neq j$:

$$f_{ij} = \{ \gamma_{ij} + \beta_i (\alpha_j - \beta_j \ln Q) \} / w_i .$$

For the own price flexibility there is a second term from: $\partial q_i / \partial \ln q_i (p_i / x) = (\partial q_i / \partial q_i) w_i$. So:

$$f_{ii} = -1 + \{ \gamma_{ii} + \beta_i (\alpha_i - \beta_i \ln Q) \} / w_i .$$

The scale flexibility for the IAIDS model can be derived from the homogeneity aggregation relation; i.e. the scale flexibility, $f_i = \partial \ln p_i (\lambda q) / \partial \ln \lambda$ for any scalar λ, is the sum over j of the f_{ij}:

$$\begin{aligned} f_i &= \textstyle\sum_j f_{ij} \\ &= \textstyle\sum_j [-\delta_{ij} + \{ \gamma_{ij} + \beta_i (\alpha_j - \beta_j \ln Q) \} / w_i] \\ &= \textstyle\sum_j -\delta_{ij} + \{ \sum_j \gamma_{ij} + \beta_i \sum_j \alpha_j - \beta_i \ln Q \sum_j \beta_j) \} / w_i \\ &= -1 \quad + \{ \ 0 \quad + \quad \beta_i \quad - \beta_i \ln Q \cdot 0) \} / w_i \\ &= -1 + \beta_i / w_i \end{aligned}$$

where δ_{ij} is the Kronecker delta. These are the formulae reported in Table 1.

11. The quantity of all other goods was calculated as follows: expenditures on beef, pork, and chicken were subtracted from personal consumption expenditures per capita; then this was divided by the consumer price index for urban consumers. Exact sources for the relevant variables are given in the appendix.

12. Barten and Bettendorf derive a linear equation similar to equation 9 for a monthly model and point out its similarity to an inverse form of the AIDS model. However, their form was derived from an inverse Rotterdam model and, as Deaton and Muellbauer point out, interpretation is very different between the two. Previous work, which employed similar dynamics when

estimating US meat demand, consists of Wohlgenant (1985; 1989), Moschini and Meilke (1984; 1989), Eales and Unnevehr (1988), Dahlgren (1987).

13. Green and Alston advocate an entirely different formula, for the calculation of the analogues of the LA/IAIDS flexibilities. We prefer the interpretation of Deaton and Muellbauer. In their American Economic Review article, they state: "However, it must be emphasized that (16) (the LA/AIDS) exists only as an approximation to (15) (the NL/AIDS) and will only be accurate in specific circumstances, albeit widely occurring ones in time-series estimation." (page 317, material in parentheses has been added) Our interpretation, Green and Alston not withstanding, is that the coefficients from the LA/IAIDS should be taken as estimates of the underlying NL/IAIDS coefficients and, therefore, it makes sense to use the formulae for flexibilities implied by the NL/IAIDS model.

14. The associated matrix of Antonelli substitution effects is symmetric and negative semi-definite when evaluated at the sample means. Given homogeneity and symmetry, which were imposed in the estimation, the distance function is a legitimate characterization of preferences.

15. In Huang's model, the scale flexibility is given as the coefficient of the Laspeyres quantity index and is $\partial \ln \pi_i / \partial \ln Q$ (where $\pi_i = p_i / x$), approximately. In the NL/IAIDS, the scale flexibility differs from $\partial \ln \pi_i / \partial \ln Q$ ($= \beta_i / w_i$) by -1.

16. For a full description of the test procedure and its properties see Davidson and MacKinnon, for Monte Carlo evidence on the performance of the test see Chalfant and Finkelshtain.

17. In usual nonnested testing situations it is typical to reverse the roles of the null and alternative hypotheses, since neither preferred a priori. Such is not the case in the present circumstance. The NL/IAIDS is the theoretically correct model. However, in keeping with previous applications, the role of the NL/IAIDS and LA/IAIDS were reversed and the multivariate P test performed. The estimated coefficient was 3.68 with a standard error of 4.02. Both tests were conducted with demand restrictions imposed.

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APPENDIX

Data and sources employed in the example of U.S. Meat demand are given in the table below. The variables and their definitions are:

CKQ	young chicken broiler consumption in pounds per capita
BFQ	beef consumption in pounds per capita
PKQ	pork consumption in pounds per capita
CKP	broiler price in cents/pound
BFP	choice beef price in cents/pound
PKP	pork price in cents/pound
CPI-U	consumer price index-urban; 1982 = 1.00
PCEX	personal consumption expenditures; total in dollars per capita
POP	us population in millions

DATA SOURCES

Variables	66,1-79,4	80,1-87,4	88,1-88,4
BFQ	2	2	4
PKQ	2	2	4
CKQ	3	3	4
BFP	1*	2	4
PKP	1*	2	4
CKP	3	3	4
CPI-U	5*	5*	5*
PCEX	5*	5*	5*

* Monthly data was averaged over quarters.
1 Livestock and Meat Situation. USDA, ERS, Nov, 1980.
2 Livestock and Meat Statistics, 1984-1988. USDA, ERS, Stat Bull #784. Sep, 1989.
3 US Egg and Poultry Statistical Series, 1960-1987. USDA, ERS, Stat Bull #775. Apr, 1989.
4 Livestock and Poultry Situation and Outlook. USDA, ERS, Jul, 1990.
5 Business Statistics, 1961-88. USDC, BEA, Dec, 1989.