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Nonlinear Dynamics and Market Anomalies in Daily Futures Prices

Seung-Ryong Yang and B. Wade Brorsen*

Introduction

Research on futures prices (Hudson et al. 1987; Cornew et al. 1984; Gordon 1985; Hall et al. 1988) has found that the distribution is not normal but leptokurtic. That is, the empirical distributions for daily price changes have more observations around the means and in the extreme tails than a normal distribution. Leptokurtosis also appears in stock returns (Fama 1965; Fielitz and Rozelle 1983) and exchange rate changes (Hsieh 1988; Friedman and Vandersteel 1982). Further, nonlinear dependence has been found in futures price changes (Taylor 1985; Blank 1990; Fujihara and Park 1990). Yet, empirical research on market anomalies has either ignored the non-normality and dependence or resorted to nonparametric tests which generally are less powerful than parametric tests.

Past research has suggested distributions such as the stable paretian (Mandelbrot 1963; Fama 1965), and more recently a diffusion-jump process (Akgiray and Booth 1988) as a model of speculative prices. These distributions partly explain leptokurtosis, but since they assume that successive observations are independent, these distributions are inconsistent with empirical work that has found linear and nonlinear dependence. The Generalized Autoregressive Conditional Heteroskedastic (GARCH) model, however, can explain both non-linear dependence and leptokurtosis.

Various time varying variance models have been considered by several authors (Friedman and Vandersteel 1982; McCulloch 1985; Taylor 1986; Akgiray 1989; Fujihara and Park 1990). Results suggest such an approach looks promising. While a GARCH or similar time varying model looks promising, past research on these models has considered few market anomalies and has generally not rigorously tested the adequacy of the models.

Savit (1988, 1989) suggests that asset returns may not follow a stochastic process. Rather, they might be generated by deterministic chaos in which the forecasting error grows exponentially so that the process only looks stochastic. Savit (1989) has shown the effect of chaos on pricing options. Frank and Stengos (1986) found evidence of nonlinear structure for gold and silver markets. Sheinkman and Lebaron (1989) found some support for the stock market following a nonlinear dynamic system. Blank (1990) found results consistent with deterministic chaos in futures prices.

Much research has sought to determine the most appropriate model of the return generating process in stock and exchange markets. However, relatively little has been done for futures price changes. Moreover, no model has successfully explained the non-normality and dependence in speculative price changes. The GARCH process used here will consider market anomalies, such as seasonality, day-of-the-week and maturity effects. Previous studies using simple ARCH or GARCH models (Bollerslev, et al. 1990) could not fully describe the underlying return generating process. However, the possibility of model misspecification can not be ruled out since none considered all possible market anomalies such as seasonality and day-of-the-week effects.

The objective of this research is to test both the GARCH and deterministic chaos processes for a large sample of daily futures price changes and to test for market anomalies. This study extends previous work in several ways.

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This study provides a more appropriate test for market anomalies since the estimates of the GARCH models are heteroskedasticity and autocorrelation-adjusted. Numerous articles have been written which have tested a single market anomaly (e.g. day-of-the-week effects) for only a few commodities. Thus this work goes considerably beyond what has been done in most past research on market anomalies.

The adequacy of the models is rigorously tested. This study does not simply select a model which is more descriptive among alternatives, but tests whether the model can generate the sample data. In addition, the hypothesis that futures returns is generated by a nonlinear deterministic process is tested for the first time using standardized residuals. Since conditional heteroskedasticity is a type of non-linear dynamics, adjusting for conditional heteroskedasticity could lead to very different conclusions about deterministic chaos.

Statistical Models and Estimation Procedures

The GARCH Process

The generalized autoregressive conditional heteroskedastic (GARCH) model (Bollerslev 1986) generates data with fatter tails than the normal distribution. The GARCH process implies dependence in the second moments. The GARCH process can model factors that violate the i.i.d assumption other than conditional heteroskedasticity. Well-documented market anomalies include day-of-the-week effects (Chiang and Tapley 1983; Junkus 1986), seasonality in variance (Anderson 1985; Kenyon et al. 1987)) and maturity effects (Milonas 1986). Ten lagged dependent variables are included to model autocorrelation in the mean (Taylor 1986). Finally, to test for the risk-return relationship, the GARCH model includes the conditional standard deviation in the mean equation.

Bollerslev (1986) suggested that the simplest but often very useful GARCH model is the GARCH(1,1) process. This study follows Bollerslev's proposition. The specific GARCH models to be estimated are

$$(4) \quad y_t = a_0 + \sum_{s=1}^{10} a_s\, y_{t-s} + a_{11}D_M + a_{12}D_T + a_{13}D_W + a_{14}D_H + a_{15}h_t + e_t,$$

$$h_t^2 = b_0 + \alpha_1 e_{t-1} + \beta_1 h_{t-1}^2 + b_1 D_M + b_2 D_T + b_3 D_W + b_4 D_H + b_5 D_{hol}$$
$$+ b_6 M_t + b_7 COS(2\pi I/126) + b_8 COS(2\pi I/252)$$
$$+ b_{10} SIN(2\pi I/252),$$

$$e_t | \psi_{t-1} \sim N(0, h_t^2)$$

Where y_{t-s} denotes the lagged dependent variable, the dummy variable for each day of the week are $D_M = 1$ if Monday and 0 otherwise, and so on (D_T, Tuesday; D_W, Wednesday; D_H, Thursday; D_{hol}, holiday). M_t indicates remaining days to contract maturity. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. I in the sine and cosine functions is the number of trading days after Jan. 1 of the particular year. This avoids any bias from different numbers of trading days in different years. Denominators in the sine and cosine functions are the specified cycle length in days, so 126 indicates a 6 month cycle and 252 a one year cycle.

The GARCH models are estimated by maximum likelihood using the algorithm developed by Berndt, Hall, Hall and Hausman (1974) with a numerical gradient method.

Deterministic Chaos

A predominant assumption about the structure of the economy has been that the real characteristics of the economy such as taste, productivity and technology are not constant, but erratic over time. However, there is no prior reason to rely solely on the stochastic assumption to explain economic fluctuations. Indeed, many early economists tried to identify the internal mechanisms that could explain the observed variations in price movements (Hayek 1933; Schumpeter 1939). Deterministic chaos allows for a nonlinear dynamic model which is deterministic with respect to initial conditions, but errors in estimating parameters and initial conditions exponentially accumulate into forecasting errors (Day, 1982; Brock, 1986). This makes the process look random. Deterministic processes that look stochastic are referred to as 'deterministic chaos'. The deterministic process is distinguished from the stochastic process neither by naked eyes, nor by statistical tests based on spectral densities or autocovariances (Brock and Dechert 1986).

A time series $\{a_t\}$ is deterministic chaotic if there exists a nonlinear dynamic system such that

(5) $a_t = h(x_t)$
$x_t = f(x_{t-1})$, x_0 given as the initial condition at time t=0,

where, $h(\cdot)$ maps R^m to R, and $f(\cdot)$ maps R^m to R^m. The map $h(\cdot)$ is a general function of the unknown state vector x and $f(\cdot)$ the unknown deterministic law of motion that governs the movement of the state variables.

The test for deterministic chaos can be performed by testing whether the calculated largest Lyapunov exponent is positive (Eckmann and Ruelle 1985). However, little is known about the small sample properties of the estimated exponent, so that statistical tests with the estimates are difficult to perform. Therefore, the deterministic chaos test used here is based on the concept of the 'correlation dimension' developed by Grassberger and Procaccia (1983).

Consider a time series, $\{x_t\}$ and a sequence of m-histories, $X_t = (x_t, x_{t+1}, \ldots, x_{t+m-1})$ that is, the m-dimensional vectors obtained by putting m consecutive observations together. The embedding theorem (Takens 1980) says that the behavior of an m-history will mimic that of the underlying unknown dynamic process if m is large enough. To illustrate, a good random number generator for i.i.d. uniform [0,1] should fill the closed interval [0,1]. If the numbers clump together on a few points, then the series is said to be low dimensional. If the 2-dimensional vectors, $[(x_t, x_{t+1})]$ fill out the unit square of $[0,1]^2$, the series $\{x_t\}$ has correlation dimension ≥ 2. Likewise, a good random number generator provides a random series which fills m-cubes for any embedding dimension, m. If a series is generated by a deterministic process, m-dimensional vectors may not fill m-cubes but clump onto a low-dimensional subset or a few points. Consequently, the test is to determine whether m-histories of $\{x_t\}$ fail to fill m-cubes for large embedding dimensions, or, equivalently, whether the estimated dimension is well below the embedding dimension.

The correlation dimension is estimated using the correlation integral $[C_m(\epsilon)]$:

(6) $C_m(\epsilon) = \lim_{T\to\infty} \{\#\text{ of } (i,j) \text{ for which } \|x_i - x_j\| < \epsilon,\ i \neq j\} / T^*$,

where m is an embedding dimension, and ϵ is a sufficiently small number. $T^* = (T^2 - T)/2$, where T is the number of m-histories that can be made out of a series of length N. From the sample size N, T = N-(m+1) m-histories can be made. The norm ($\|\cdot\|$) used in the computer program is the sup norm. The correlation integral measures the asymptotic probability that the distance between any two m-histories are less than ϵ. If the data are generated by a deterministic process, the correlation integral in (6) will be independent of m but increases with ϵ. The correlation dimension (SC_m) is defined as:

$$SC_m = [\ln C_m(\epsilon_i) - \ln C_m(\epsilon_{i-1})] / [\ln \epsilon_i - \ln \epsilon_{i-1}]$$

Eleven values of ϵ are used, 0.91, 0.92,..., 0.911. Four embedding dimensions are used 2, 4, 6 and 8. The median of the ten estimates of SC_m for each m is employed as the estimate of the correlation dimension since the distribution of estimates of SC for each embedding dimension has great dispersion and skewness.

The test of deterministic chaos is the residual test suggested by Brock (1986). He shows that if time series data are chaotic, the estimated dimension of residuals from the best fitting time series model is the same as that of the original data. If the data are stochastic, the dimension of the residuals would increase since they have less structure than the original data. Brock's residual test is easy to apply and especially useful for this study since the GARCH processes provide residuals which are adjusted for possible linear and quadratic dependence.

Model Validation

If the GARCH models are a precise model of the data, the residuals from the models should be i.i.d. The absolute criterion used to validate the GARCH model is to test the null hypothesis that the residuals are i.i.d. normal.

To do this, the hypothesis that the standardized residuals, $\hat{e}_t / \hat{h}_t$ are normal or student is tested with the Kolmogorov-Smirnov D-statistic. The estimated kurtosis and skewness are also calculated and compared to those of the original data.

The test for nonlinear dependence is performed by the BDS test (Brock et al., 1986) which tests the null hypothesis of an independent identical distribution. The BDS test has power against chaotic alternatives, unlike conventional tests. The statistic was developed based on the correlation integral. Under the null hypothesis that $\{x_t\}$ is i.i.d., the BDS statistic is,

$$(8) \quad W_m(\epsilon,T) = T^{1/2}[C_m(\epsilon,T)^m] / \sigma_m(\epsilon).$$

This statistic converges in distribution to a standard normal random variable under the null hypothesis. Based on Monte Carlo results by Hsieh and LeBaron (1988), three epsilons, $\epsilon = 0.5\sigma$, 1.0σ and 1.5σ with embedding dimensions, m = 2,4 and 6 are used, where σ is the standard deviation of the data series. A rejection of the null hypothesis may be due to linear or nonlinear dependence or to chaotic structure.

Sample Data

The data are the first differences of the natural logarithms of the daily closing futures prices. Daily returns are multiplied by 100 to avoid possible scaling problems in estimation and to express them in percentage terms. The data set is composed of 15 contracts actively traded in U.S. futures markets. The list of data and sample periods of each commodity are shown in table 1. The data are for the ten years from Jan. 1979 to Dec. 1988, except for contracts which began trading after Jan. 1979. All available data is used for these commodities. The more than 2,500 observations provides enough degrees of freedom that it seems reasonable to use tests that are only asymptotically valid.

A continuous series of price changes are constructed in the standard manner for future prices. The data consist of changes in the log of daily closing prices of the futures contract closest to delivery until the third Tuesday of the month prior to delivery, after which the log changes in the next nearest delivery month are used. The data are from the Dunn & Hargitt Commodity Data Bank. Volume is measured as hundreds of contracts traded during the day for the contract month included in the data series.

Table 2 provides summary statistics of the data. All means are not statistically different from zero. Estimated variances are very different from commodity to commodity even though the data are unitless due to using log changes. Twelve of 15 contracts are skewed. The skewness is mostly negative, but sometimes positive. All are leptokurtic at conventional levels of significance. Three stock index futures, NYSE, S&P 500 and Value Line have exceptionally large kurtosis. These extremes result from the stock market crash in 1987. Kurtosis can be greatly affected by a single outlier, even in a sample of 2500.

Estimated Results
Test for Deterministic Chaos

Table 3 shows the estimated correlation dimensions for the raw and rescaled data (standardized residuals). If time series data are stochastic, the estimated dimensions should have full dimension, that is, equal or very close to the embedding dimensions. Only silver has an estimated dimension lower than the embedding dimension by more than three. This can be evidence of a deterministic structure. However, silver does not pass Brock's residual test. The estimates for the rescaled data for silver are larger.

Estimated dimensions detect some contracts which seem to pass the residual test. However, they tend to have full dimensions, that is, those estimated dimensions are close to the embedding dimensions implying no evidence of deterministic structure. To confirm deterministic chaos, diagnostics should meet the saturation condition as well as Brock's residual test. That is, beyond some embedding dimension, estimated correlation dimensions for both raw and residual data should be the same and stable with embedding dimension. None of the series that pass the residual test also satisfy the saturation condition. Correlation dimensions are monotonically increasing with embedding dimensions.

Empirical results show no evidence of low-dimension deterministic chaos for the futures data. The assumption that futures price changes are generated by stochastic processes appears reasonable.

Estimated Results of the GARCH Process

Table 4 reports estimates and test results of the GARCH model. First, estimated coefficients for the ARCH term, α and the GARCH term, β are positive and significant at the 5% level for all commodities. Risk is not priced in returns in any equation.

The null hypothesis of no linear dependence is rejected only for silver and platinum. This is less autocorrelation than found in past studies such as Taylor (1986). Mean returns differ from day to day only for coffee. The results, however, show volatility does differ by day of the week. Including the holiday effect, only three series have no significant differences in variances. Thus, different variability of returns for each day of the week seems to be a general phenomenon in futures markets. Specifically, most Mondays and all holidays have significantly higher variances, and most Wednesdays have lower variances.

The maturity effect is significant in six cases, but is positive two of these times contrary to Samuelson's hypothesis. Moreover, the effect is less than 1% of the variance. Consequently, the maturity effect is not a general characteristic of futures markets. This implies that studies which use nearby price series do not need to be concerned about differences in maturity.

Nine cases reveal significant seasonal patterns in price volatility. The results agree with Kenyon et al. (1987) which found price volatility of corn, soybeans and wheat was affected by season of the year.

Model Validation

The Ljung-Box test detects second-order dependence in standardized residuals, $\hat{e}_t / \hat{h}_t$ only for coffee. The test statistic has thirteen degrees of freedom. The results imply that the lag structure of the conditional variance is correctly identified.

While the Ljung-Box test performs the test for serial dependence in a specific moment, the nonparametric BDS test conducts the test of the i.i.d. assumption. The BDS test for the raw data (table 4) rejects the null hypothesis of i.i.d. for all cases, which shows that all futures price changes are not random but are dependent. As other researchers have pointed out (eg. Blank) such a finding is consistent with deterministic chaos. The GARCH model, however, removed serial dependence in the raw data considerably. The BDS test statistics for the rescaled data indicate significant dependence for six contracts. However, in three of these cases, the BDS statistic is negative, which is not consistent with deterministic chaos.[1] Neither the Ljung-Box nor the BDS test for the rescaled data rejected the null hypothesis of independence for eight cases. These eight have similar skewness and leptokurtosis as the other commodities, and thus any remaining dependence is likely not that important in explaining non-normality.

A practical concern is how adequately the GARCH models correct the non-normality in the raw data. Table 6 contains Kolmogorov-Smirnov tests of normality and goodness-of-fit for the rescaled data from the GARCH model. Estimated skewness for the rescaled data from the GARCH model is significant in 10 of 15 cases compared to 12 of 15 for the raw data. The GARCH model's inability to remove skewness is no surprise, because the model captures skewness only through the exogenous factors being skewed, for example, the day-of-the-week effect. In contrast, the GARCH model reduces leptokurtosis considerably. The most dramatic reduction in kurtosis is for the NYSE index futures from 170.8 to 10.1. The other two stock indices, S&P 500 and Value Line show similar reductions. However, only oats passes the Kolmogorov-Smirnov test of fit.

Thus, the GARCH models in the present study could not precisely model the distributions of daily futures price changes under consideration. However, those models did remove most, if not all of the dependence and reduced the observed leptokurtosis considerably. They are still the best currently available models. The hypothesis tests in Table 4 are asymptotically valid via the central limit theorem even though the residuals were still leptokurtic.

Conclusions

The daily price changes considered in the present study were not random but dependent. All of the estimated ARCH and GARCH terms were significant even at the 1% level, which implies that the variances of price changes are not constant but gradually changing over time. The BDS test supported serial dependence in daily price changes. The BDS test rejected the null hypothesis of i.i.d. for all the original data. Moreover, no distribution was normal. They were slightly skewed and had fat tails.

The hypothesis of low-dimension deterministic chaos was not supported by the data. No data pass both Brock's residual test and the saturation condition. Estimated dimensions for rescaled data were either larger than those for the raw data, or estimates for both series were increasing with embedding dimensions. This result favors the stochastic approach when determining the return generating process.

The GARCH(1,1) process is not a perfect model of the data. Kolmogorov-Smirnov test of fit rejected the GARCH(1,1) process for all cases. However, the model provides great improvements. Nonlinear dependence in the raw data was

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[1] Monte Carlo studies by Hsieh suggest the size of the BDS test is too large when used on GARCH residuals in small samples. The results here are consistent with this finding.

removed and leptokurtosis was reduced. Since the GARCH model was able to remove dependence, the hypothesis tests for market anomalies are asymptotically valid provided the variance of the error term is finite.

The most prominent market anomaly was day of the week effects on variance. The variance was larger on Mondays and after holidays. This offers some partial support for the calendar-time hypothesis. Several commodities, especially agricultural commodities, showed seasonality in variance. A few commodities showed significant autocorrelation, day of the week in mean, or maturity effects on variance. In no case was the variance of the futures prices significant in the mean equation.

Results of the present research provide an important addition to knowledge about price behavior in futures markets. The GARCH model, while still not perfect, was shown to be overwhelmingly preferred as a model for daily returns to the random walk and normal distribution assumptions which are the basis for much theoretical and empirical work in finance.

List of References

Akgiray, Vedat, "Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts," Journal of Business 62(1989):55-80.

Anderson, R. W., "Some Determinants of the Volatility of Futures Prices," Journal of Futures Markets 5(1985):332-48.

Berndt, E, K., B. H. Hall, R. E. Hall and J. A. Hausman, "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement 3/4(1974):653-665.

Black, F. and M. Scholes, "The Valuation of Option Contracts and a Test of Market Efficiency," Journal of Finance 27(May 1972):399-417.

Blank, Steven C. "'Chaos' in Futures Markets? A Nonlinear Dynamical Analysis." Center for the Study of Futures Markets Working Paper #204, Columbia Business School, February 1990.

Bollerslev, T., "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics 31(1986):307-327.

Bollerslev, T., "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," The Review of Economics and Statistics 69(August 1987):542-47.

Bollerslev, Tim, Ray Y. Chou, Kenneth F. Kroner. "Arch Modeling in Finance: A Review of the Theory and Empirical Evidence." Department of Finance, Northwestern University, Working Paper No. 97, November 1990.

Brock, W. A., "Distinguishing Random and Deterministic Systems: Abridged Version," Journal of Economic Theory 40(1986):168-195.

Brock, W. and W. D. Dechert, "Theorems on Distinguishing Deterministic from Random Systems," Dynamic Economic Modelling, Cambridge University Press, Cambridge, 1988.

Brock, W., W. D. Dechert and J. Sheinkman, "A Test for Independence Based in the Correlation Dimension," Unpublished Manuscript, University of Wisconsin, and Chicago, 1986.

Chiang, Raymond C. and T. Craig Tapley. "Day of the Week Effects and the Futures Market." Review of Research in Futures Markets 2(1983):356-410.

Chicago Board of Trade, Commodity Trading Manual, Chicago Board of Trade, Chicago, 1985.

Cornew, Ronald W., Donald E. Town, and Lawrence D. Crowson. "Stable Distributions, Futures Prices, and the Measurement of Trading Performance." The Journal of Futures Markets 4(1984):531-557.

Eckmann, J. P. and D. Ruelle. "Ergodic Theory of Chaos and Strange Attractors." Review of Modern Physics 57(July 1985):617-656.

Engle, R. F., "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica 50 (July 1982) :987-1007.

Epps, T. W. and M. L. Epps, "The Stochastic Dependence of Security Changes and Transaction Volumes: Implication for the Mixture-of-Distributions Hypothesis," Econometrica 44(March 1976):305-321.

Fama, E., "The Behavior of Stock Market Prices," Journal of Business 38 (January 1965):34-105.

Frank, M. Z. and T. Stengos, "Measuring the Strangeness of Gold and Silver Rates of Return," Discussion paper No. 1986-13, Dept. of Economics, University of Guelph.

French, K., G. Schwert, and R. Stambaugh. "Expected Stock Returns and Volatility." Journal of Financial Economics 19(1987):3-29.

Friedman, D. and S. Vandersteel, "Short-run Fluctuations in Foreign Exchange Rates," Journal of International Economics 13(August 1982):171-186.

Fujihara, Roger and Keehwan Park. (1990 December): "The Probability Distribution of Futures Prices in the Foreign Exchange Market: A Comparison of Candidate Processes," Journal of Futures Markets, 10:623-641.

Gordon, J. Douglas. "The Distribution of Daily Changes in Commodity Futures Prices." Technical Bulletin No. 1702, ERS, USDA, July 1985.

Grassberger, P. and I. Procaccia, "Measuring the Strangeness of Strange Attractors," Physics Review Letter 50(October 1983):189-208.

Hall, J. A., B. W. Brorsen and S. H. Irwin, "The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypotheses," Journal of Financial and Quantitative Analysis 24(March 1989):105-116.

Hayek, F. A., Monetary Theory of the Trade Cycle, Harcourt, New York, 1933.

Hsieh, D. and B. LeBaron, "Finite Sample Properties of the BDS Statistic," Unpublished Manuscript, Univ. of Chicago, 1988.

Hsieh, D., "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," Journal of International Economics 24(1988):129-145.

__________, "Testing for Nonlinear Dependence in Daily Foreign Exchange Rates," Journal of Business 62(June 1989):339-369.

Hudson, Michael A., Raymond M. Leuthold and Gboroton F. Sarassoro. "Commodity Futures Price Changes: Recent Evidence for Wheat, Soybean and Live Cattle." Journal of Futures Markets 7(June 1987):287-301.

Judge, G. G., W. E. Griffiths, R. Hill, H. Lutkepohl and T.-C. Lee. The Theory and Practice of Econometrics. 2nd edition. New York: John Wiley and sons, 1985.

Junkus, J. C., "Weekend and Day of the Week Effects in Returns on Stock Index Futures," Journal of Futures Markets 3(1986):397-407.

Kenyon D., K. Kling, J. Jordan, W. Seale and N. McCabe, "Factors Affecting Agricultural Futures Price Variance," Journal of Futures Markets 7(1987):73-91.

Mandelbrot, B., "The Variation of Certain Speculative Prices," Journal of Business 36(October 1963):394-419.

McCulloch, J. H., "Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates," Journal of Banking and Finance 9(March 1985):137-156.

Milonas, Nikolaos. "Price Variability and the Maturity Effect in Futures Markets." Journal of Futures Markets 6(1986):443-460.

Perry, P. R. "More Evidence on the Nature of the Distribution of Security Returns," Journal of Financial and Quantitative Analysis 18(June 1983):211-221.

Praetz, P., "The Distribution of Share Price Changes," Journal of Business 45(January 1972):49-55.

Savit, Robert. "Nonlinearities and Chaotic Effects in Options Prices." Journal of Futures Markets 6(December 1989):507-518.

Savit, Robert. "When Random is Not Random: An Introduction to Chaos in Market Prices." Journal of Futures Markets 8(June 1988):271-289.

Schumpeter, J. A., Business Cycle: A Theoretical, Historical and Statistical Analysis of the Capitalist Process, McGraw-Hill, New York, 1939.

Sheinkman, J. A. and B. LeBaron, "Nonlinear Dynamics and Stock Returns," Journal of Business 62(June 1989):311-337.

Takens, F., "Detecting Strange Attractors in Turbulence," in Dynamical Systems and Turbulence. D. Rand and L. Young, eds., Lecture Notes in Mathematics, Berlin: Springer-Verlag, 898(1980):366-382.

Taylor, S., "The Behavior of Futures Prices Over Time," Applied Economics 17(1985):713-734.

__________, Modelling Financial Time Series, John Wiley and Son, New York, 1986.

Table 1. Sample period and contract months for futures commodities

Commodity	Sample Period	Contract Months	Price Limit[a]
Corn	1/79-12/88	Mar May Jul Dec	***[b]
Coffee	1/79-12/88	Mar May Jul Sep Dec	$0.04
Oats	1/79-12/88	Mar May Jul Sep Dec	***
Soybean	1/79-12/88	Jan Mar May Jul Nov	***
Soybean Meal	1/79-12/88	Mar May Jul Dec	***
Wheat (Chicago)	1/79-12/88	Mar Jul Dec	***
Wheat (Kansas City)	2/79-12/88	Mar Jul Dec	$0.25
Copper	1/79-12/88	Mar May Jul Sep Dec	***
Gold (NY)	1/79-12/88	Feb Apr Jun Aug Sep	$25.00/oz
Palladium	1/79-12/88	Mar Jun Sep Dec	$25.00/oz
Platinum	1/79-12/88	Jan Apr Jul Oct	$6.00/oz
Silver	1/79-12/88	Mar May Jul Sep Dec	$0.50/oz
NYSE	1/84-12/88	Mar Jun Sep Dec	none
S & P 500	1/83-12/88	Mar Jun Sep Dec	none
Value Line	1/83-12/88	Mar Jun Sep Dec	none

[a]Source: Commodity Trading Manual. Price limits are not constant over the whole time period.
[b]Subject to change; consult latest official notices from the Exchange.

Table 2. Summary statistics of daily futures returns[a]

Commodity	Sample Size	Mean	Variance	Skewness	Kurtosis
Corn	2524	-0.015	1.433	0.038	1.899*
Coffee	2510	0.018	3.104	-0.279*	3.771*
Oats	2522	-0.002	2.673	0.103*	1.093*
Soybean	2523	-0.024	1.943	-0.131*	1.317*
Soybean Meal	2524	-0.017	2.190	0.005	1.740*
Wheat (Chi)	2524	-0.011	1.874	0.722*	13.289*
Wheat (KC)	2494	-0.000	1.580	0.615*	29.084*
Copper	2519	0.017	3.207	-0.260*	2.994*
Gold (NY)	2521	-0.016	1.350	-0.016	3.469*
Palladium	2496	0.017	4.532	-0.093*	1.742*
Platinum	2508	-0.011	4.528	-0.112*	0.947*
Silver	2514	-0.041	4.893	-0.193*	1.485*
NYSE	1353	0.021	2.286	-6.280*	170.806*
S & P 500	1669	0.038	2.316	-5.720*	156.713*
Value Line	1350	0.003	1.623	-4.819*	72.804*

[a]Asterisks denote rejection of the null hypothesis of a normal distribution.

Table 3. Correlation dimension estimates[a]

Commodity	Original Data m=4	Original Data m=6	Original Data m=8	Rescaled Data m=4	Rescaled Data m=6	Rescaled Data m=8
Corn	3.48	4.93	6.11	3.78	5.52	7.19
Coffee	3.68	5.49	7.13	3.68	5.43	7.39
Oats	3.53	5.09	6.80	3.77	5.72	7.39
Soybean	3.63	5.33	6.90	3.74	5.42	7.12
Soybean Meal	3.57	5.18	6.52	3.64	5.44	6.88
Wheat (Chicago)	3.73	5.45	7.02	3.72	5.44	7.02
Wheat (KC)	3.49	4.94	6.52	3.59	5.31	6.79
Copper	3.63	5.22	5.92	3.71	5.62	7.88
Gold (NY)	3.52	5.35	6.90	3.58	5.44	7.01
Palladium	3.69	5.45	7.02	3.62	5.20	6.91
Platinum	3.78	5.59	7.13	3.73	5.36	6.86
Silver	3.69	4.63	2.19	3.69	5.01	5.84
NYSE	3.63	5.44	7.16	3.58	5.53	7.41
S&P 500	3.60	5.62	7.09	3.66	5.44	6.89
Value Line	3.68	5.44	7.29	3.71	5.52	7.60

[a]For each embedding dimension, ten adjacent values of epsilons, 0.9, n=1,2,...,10 are used to estimate. The median is reported as the estimate of the correlation dimension.

Table 4. Estimated results of the GARCH(1,1) process

	Wheatc	Wheatk	soybean	soyb.m	oats
Mean					
Intercept	-0.025	-0.116	-0.057	-0.055	-0.043
	(-0.26)	(-1.77)	(-0.64)	(-0.58)	(-0.40)
D(Monday)	-0.041	-0.058	-0.064	-0.170*	-0.121
	(-0.54)	(-0.93)	(-0.84)	(-2.04)	(-1.40)
D(Tuesday)	-0.028	0.024	0.037	-0.060	-0.096
	(-0.40)	(0.43)	(0.53)	(-0.75)	(-1.11)
D(Wednesday)	0.118	0.172	0.138*	0.095	-0.004
	(1.71)	(2.82)	(2.00)	(1.28)	(-0.05)
D(Thursday)	-0.120	-0.043	0.012	0.005	-0.184*
	(-1.75)	(-0.73)	(0.18)	(0.07)	(-2.11)
Std. Dev.	0.021	0.102	-0.007	0.035	0.080
	(0.26)	(1.88)	(-0.10	(0.50)	(1.24)
Variance					
Intercept	0.133	0.228*	-0.075	0.003	0.175
	(1.78)	(5.97)	(-0.99)	(0.03)	(1.79)
ALpha	0.084*	0.157*	0.063*	0.067*	0.067*
	(10.26)	(15.46)	(7.48)	(8.46)	(8.17)
Beta	0.881*	0.810*	0.916*	0.912*	0.928*
	(73.28)	(80.97)	(90.91)	(94.33)	(109.7)
D(Monday)	0.270*	-0.074	0.464*	0.435*	-0.205
	(2.18)	(-1.06)	(3.60)	(3.23)	(-1.24)
D(Tuesday)	-0.449*	-0.437*	-0.368*	-0.352*	-0.141
	(-3.72)	(-9.14)	(-3.20)	(-2.95)	(-0.97)
D(Wednesday)	-0.127	-0.117*	0.085	-0.010	-0.162
	(-1.19)	(-2.50)	(0.85)	(-0.08)	(-1.19)
D(Thursday)	-0.121	-0.239*	0.215	-0.040	-0.214
	(-0.91)	(-3.87)	(1.71)	(-0.297)	(-1.42)
D(holiday)	0.594*	0.231*	0.356*	0.399*	0.150
	(5.42)	(4.91)	(3.73)	(4.13)	(1.33)
Maturity	-0.001	0.000	0.008*	0.006*	-0.007
	(-0.39)	(0.23)	(2.85)	(2.61)	(-1.90)
Autocorrelation(10)	1.31	1.27	1.17	0.92	0.99
M(day-of-the-week)	4.56	3.54	1.17	2.66	0.95
V(day-of-the-week)	15.96*	72.26*	54.79*	13.59*	0.81
V(seasonality)	15.72*	78.18*	15.40*	22.32*	3.26
Ljung-Box(15)[b]					
$e(t)/h(t)$	4.80	13.06	7.57	10.01	9.36
$e(t)^2/h(t)^2$	16.25	2.62	15.90	12.60	8.05
BDS tests[c]					
Raw data					
dimension=2	0.92	1.73	1.66	1.65	1.64
dimension=4	4.71*	9.42*	4.77*	8.77*	9.58*
dimension=6	10.45*	23.55*	13.87*	21.42*	30.19*
Standardized residuals					
dimension=2	0.04	0.21	-0.00	0.18	0.04
dimension=4	0.05	0.75	-0.14	1.09	0.54
dimension=6	0.63	1.25	1.12	1.40	0.68

Table 4. Continued

	corn	coffee	copper	gold	silver
Mean					
Intercept	-0.005	0.165	-0.031	-0.055	-0.148
	(-0.08)	(1.95)	(-0.31)	(-0.58)	(-0.89)
D(Monday)	0.019	-0.151	-0.284*	-0.102	-0.392*
	(0.30)	(-1.90)	(-3.19)	(-1.21)	(-3.13)
D(Tuesday)	-0.026	-0.119	-0.096	0.003	-0.135
	(-0.43)	(-1.50)	(-1.01)	(0.04)	(-1.13)
D(Wednesday)	0.090	-0.065	0.114	0.060	0.038
	(1.52)	(-0.87)	(1.29)	(0.73)	(0.32)
D(Thursday)	-0.076	0.006	0.046	-0.043	-0.138
	(-1.31)	(0.08)	(0.08)	(-0.54)	(-1.18)
Std. Dev.	-0.013	-0.048	0.037	0.014	0.142
	(-0.20)	(-0.81)	(0.66)	(0.25)	(1.81)
Variance					
Intercept	0.093*	0.034	0.193*	0.384*	0.254
	(2.22)	(0.45)	(2.32)	(5.13)	(1.12)
ALpha	0.088*	0.119*	0.043*	0.077*	0.101*
	(7.94)	(12.42)	(8.28)	(11.28)	(9.99)
Beta	0.883*	0.864*	0.952*	0.900	0.855*
	(62.00)	(84.86)	(172.6)	(117.0)	(66.67)
D(Monday)	0.186*	0.250	-0.096	-0.141	1.107*
	(2.33)	(1.91)	(-0.76)	(-1.02)	(2.92)
D(Tuesday)	-0.292*	-0.046	-0.168	-0.992*	-1.208*
	(-4.06)	(-0.36)	(-1.11)	(-8.51)	(-3.65)
D(Wednesday)	-0.084	-0.031	-0.572*	-0.202*	-0.106
	(-1.39)	(-0.26)	(-4.89)	(-2.43)	(-0.38)
D(Thursday)	-0.140	-0.116	-0.176	-0.484*	-0.091
	(-1.77)	(-0.80)	(-1.19)	(-4.41)	(0.29)
D(holiday)	0.283*	0.390*	0.556*	0.946*	2.139*
	(4.14)	(3.00)	(5.05)	(9.37)	(6.13)
Maturity	0.001	-0.002	0.001	0.002	-0.018*
	(0.72)	(-0.56)	(0.53)	(0.95)	(-2.11)
Autocorrelation(10)	1.20	1.27	1.81	1.63	3.78
M(day-of-the-week)	2.15	19.00*	3.60	0.82	3.57
V(day-of-the-week)	20.12*	3.08	8.51	26.35*	25.53*
V(seasonality)	16.58*	10.88*	21.91*	4.98	14.49*
Ljung-Box(15)[b]					
$e(t)/h(t)$	6.14	9.31	2.69	15.82	21.13
$e(t)^2/h(t)^2$	9.57	31.68*	11.86	13.39	12.03
BDS tests[c]					
Raw data					
dimension=2	1.94	2.99*	2.09*	2.63*	2.56*
dimension=4	10.55*	14.98*	10.80*	12.45*	11.27*
dimension=6	31.81*	38.54*	28.52*	30.10*	26.72*
Standardized residuals					
dimension=2	0.06	0.42	0.25	0.22	0.55
dimension=4	0.48	0.90	0.72	0.85	1.89
dimension=6	1.92	1.48	1.28	0.68	4.66*

Table 4. Continued

	pall	plat	nyse	sp500	vline
Mean					
Intercept	0.073	-0.105	0.019	-0.085	0.163
	(0.57)	(-0.55)	(0.16)	(-0.87)	(1.81)
D(Monday)	-0.084	-0.146	-0.009	0.007	-0.015
	(-0.75)	(-1.13)	(-0.11)	(0.09)	(-0.17)
D(Tuesday)	0.075	-0.001	0.027	0.018	0.019
	(0.74)	(-0.01)	(0.32)	(0.24)	(0.24)
D(Wednesday)	0.146	0.007	-0.003	-0.003	-0.037
	(1.44)	(0.05)	(-0.04)	(-0.04)	(-0.51)
D(Thursday)	-0.076	-0.125	-0.029	-0.023	0.089
	(-0.73)	(-1.01)	(-0.34)	(-0.30)	(1.09)
Std. Dev.	-0.038	0.067	0.050	0.155	-0.148
	(-0.61)	(0.78)	(0.45)	(1.75)	(-1.52)
Variance					
Intercept	0.093	0.650*	-0.053	-0.084	-0.108
	(0.56)	(3.05)	(-0.70)	(-1.31)	(-1.62)
ALpha	0.141*	0.064*	0.200*	0.148*	0.146*
	(11.15)	(6.65)	(13.89)	(18.31)	(16.61)
Beta	0.821*	0.909*	0.712*	0.822*	0.774*
	(68.47)	(66.61)	(24.16)	(57.90)	(36.67)
D(Monday)	0.786*	-0.044	0.304*	0.266*	0.468*
	(2.58)	(-0.12)	(2.38)	(2.36)	(4.08)
D(Tuesday)	-1.02*	-1.885*	0.222	0.251*	0.183
	(-4.03)	(-5.36)	(1.92)	(2.28)	(1.87)
D(Wednesday)	0.210	-0.385	0.070	-0.001	0.090
	(1.06)	(-1.39)	(0.65)	(-0.09)	(0.87)
D(Thursday)	0.180	-0.405	0.430*	0.277*	0.428*
	(0.77)	(-1.10)	(3.12)	(2.20)	(3.19)
D(holiday)	1.267*	1.200*	0.688*	0.442*	0.764*
	(4.29)	(3.93)	(3.62)	(3.37)	(4.67)
Maturity	0.006	-0.009	-0.011*	-0.007*	-0.013*
	(0.99)	(-1.23)	(-2.21)	(-2.25)	(-4.06)
Autocorrelation(10)	0.44	1.85*	0.40	0.76	0.27
M(day-of-the-week)	0.04	0.36	0.04	0.39	0.70
V(day-of-the-week)	55.97*	43.72*	32.78*	22.83*	45.84*
V(seasonality)	11.10*	6.20	5.52	5.61	10.37
Ljung-Box(15)[b]					
$e(t)/h(t)$	12.95	8.80	11.40	8.89	8.39
$e(t)^2/h(t)^2$	26.44	19.87	8.27	5.59	17.63
BDS tests[c]					
Raw data					
dimension=2	3.38*	2.07*	0.29	0.65	0.34
dimension=4	17.56*	9.35*	0.98	2.53*	1.24
dimension=6	45.31*	21.24*	2.37*	5.71*	2.75*
Standardized residuals					
dimension=2	0.98	0.65	-0.56	-0.49	-0.32
dimension=4	4.07*	1.88	-2.45*	-2.48*	-2.04*
dimension=6	6.93*	3.53*	-3.50*	-3.47*	-3.27*

Table 5. Normality and goodness-of-fit tests with standardized GARCH residuals

	Skewness	Kurtosis	D_{max}
Corn	0.065	0.888*	0.018*
Coffee	-0.120*	1.003*	0.033*
Oats	-0.059	0.473*	0.016
Soybean	-0.017	0.496*	0.021*
Soybean Meal	0.162*	1.038*	0.034*
Wheat (Chicago)	0.141*	1.032*	0.029*
Wheat (Kansas City)	1.838*	27.349*	0.056*
Copper	-0.297*	1.885*	0.030*
Gold (NY)	-0.026	2.093*	0.056*
Palladium	0.026	1.071*	0.037*
Platinum	-0.155*	0.386*	0.024*
Silver	-0.113*	1.352*	0.031*
NYSE	-1.082*	10.132*	0.064*
S&P 500	-1.058*	11.329*	0.057*
Value Line	-0.748*	4.427*	0.057*