Total Response Measures in Systems of Nonlinear Equations: An Application to a Model of the U.S. Dairy Sector

by

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TOTAL RESPONSE MEASURES IN SYSTEMS OF NONLINEAR EQUATIONS: AN APPLICATION TO A MODEL OF THE U.S. DAIRY SECTOR

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I. Introduction

Agricultural economists have long been interested in the proper measurement and interpretation of elasticities and flexibilities between endogenous variables in systems of simultaneous equations (Meinken, Rojko, and King; Buse; Colman and Miah). It is now well known that partial measures commonly used in a single equation context are not valid for obtaining elasticities among endogenous variables in a systems framework. This is because indirect effects are not accounted for by standard partial measures (Buse). To capture indirect effects inherent in any simultaneous system, it is necessary to obtain "total" response measures; measures that show how one endogenous variable changes with respect to another when all remaining endogenous variables adjust accordingly. Only in this manner can the true economic implications of the underlying model be accurately conveyed.

In recent years there has been considerable progress in developing methods for obtaining total response measures in systems of simultaneous equations. For instance, Chavas, Hassan, and Johnson (CHJ) report analytical procedures for obtaining total elasticities and flexibilities in systems of dynamic simultaneous equations. More recently, Holt and Skold (HS) showed the CHJ results, obtained using a model with a two-variable lag structure, could be extended to more general lag specifications. HS also illustrated the potential of the total elasticity approach by obtaining total response measures for a dynamic model of the U.S. pork sector.

While headway has been made, total elasticities and flexibilities have not been used or reported widely. Most analysts continue to summarize a modelling effort using standard partial measures. The problem may be that existing methods for obtaining total response measures, while more comprehensive than in earlier years, are still useful for only a fairly narrow class of models. Importantly, the extensions considered by CHJ and HS assume <u>linear</u> model specifications. To the extent even modest modelling exercises often involve nonlinear equations, it is clear that the above methods will not be especially useful.

The purpose of this paper is to show how the CHJ and HS results can be extended to the most general setting possible. Specifically, we illustrate the potential for obtaining total response measures in a general, nonlinear dynamic system of structural equations. The tradeoff involved is that analytical results for deriving total response measures are no longer available; all results must be obtained numerically. But the clear advantage is that the approach developed here can, in principle, be applied to nearly <u>any</u> system of simultaneous equations. The result is that the economic implications of even the most complicated structural models can be summarized succinctly using total elasticities and flexibilities.

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The potential for obtaining total elasticity measures in nonlinear dynamic models is illustrated with an annual model of the U.S. dairy sector. The dairy market seems well suited for obtaining total response measures in a general setting since previous research has shown that, due to biological lags, supply decisions are characterized by a highly nonlinear dynamic process (Chavas and Klemme). Moreover, interest often focuses on the relationship between prices and/or quantities in markets for Class I and Class II milk (LaFrance and de Gorter; Kaiser, Streeter, and Liu); relationships that can best examined using total elasticity and flexibility concepts.

In the next section, existing methods for obtaining total price and quantity effects in systems of equations are reviewed. These results are then used to motivate the derivation of total response measures in nonlinear models. The third section reports the specification and estimates of a structural model for the U.S. dairy sector. In section four, numerical methods are used in conjunction with the estimated dairy model to obtain partial and total elasticities and flexibilities for selected exogenous and endogenous variables. Conclusions and suggestions for further research are reported in the final section.

II. Partial and Total Elasticities

In this section standard results for obtaining total price and quantity effects are reviewed. The results are then extended to include dynamic and nonlinear models.

General Overview

To motivate subsequent discussion, we follow CHJ and HS and consider a standard three-equation market model consisting of quantity supplied (Y_{1s}), quantity demanded (Y_{1d}), and market clearing price (Y_2). The system can be represented in the following manner:

$$Y_{1s} = f_s(Y_2, \underline{X}) \tag{1a}$$

$$Y_{1d} = f_d(Y_2, \underline{X}) \tag{1b}$$

$$Y_{1d} = Y_{1s} \tag{1c}$$

where $\underline{X}$ is a K-vector of exogenous variables conditioning demand and supply and (1c) denotes market clearing. Relationships between quantities and price are frequently summarized using elasticities. Specifically, letting ϵ_{1s} and ϵ_{1d} denote respectively elasticities of supply and demand with respect to price, we have:

$$\epsilon_{1s} = (\partial Y_{1s}/\partial Y_2)(Y_{1s}/Y_2) \tag{2a}$$

$$\epsilon_{1d} = (\partial Y_{1d}/\partial Y_2)(Y_{1d}/Y_2) \tag{2b}$$

More generally, it is possible that other endogenous variables in addition to Y_{1s}, Y_{1d}, and Y_2 could be determined explicitly within the structure of the model. A generalized version of the market model in (1) would then include equations for all endogenous variables where, in general, each endogenous variable would among other things, be conditioned on the set of remaining endogenous variables. The supply and demand equations in (1a) and (1b) can then be represented as:

$$Y_{1s} = f_s(Y_2, Y_3, \ldots, Y_G, \underline{X}) \tag{3a}$$

$$Y_{1d} = f_d(Y_2, Y_3, \ldots, Y_G, \underline{X}) \tag{3b}$$

where $Y_3, \ldots, Y_G$ denote other endogenous variables in the model. It is clear that the partial elasticities in (2) are not appropriate in the present case since there will be secondary effects resulting from a change in Y_2 on the remaining endogenous variables, $Y_3, \ldots, Y_G$. Of course to capture total effects of a price change on supply and demand, total derivatives must be evaluated. In this context, total elasticities of supply and demand are given by:

$$\epsilon_{1s} = [\partial Y_{1s}/\partial Y_2 + \sum_{j=3}^{G} (\partial Y_{1s}/\partial Y_j)(\partial Y_j/\partial Y_2)](Y_{1s}/Y_2) \tag{4a}$$

$$\epsilon_{1d} = [\partial Y_{1d}/\partial Y_2 + \sum_{j=3}^{G} (\partial Y_{1d}/\partial Y_j)(\partial Y_j/\partial Y_2)](Y_{1d}/Y_2). \tag{4b}$$

Note that the elasticities in (4) differ from those in (2) by the value of the summation terms multiplied by the ratio of the reference values for Y_{1s} (respectively, Y_{1d}) and Y_2. In fact, a necessary condition for the elasticities in (4) to differ from those in (2) is that the sums $\sum_{j=3}^{G} (\partial Y_{1s}/\partial Y_j)$ and $\sum_{j=3}^{G} (\partial Y_{1d}/\partial Y_j)$ not equal zero. In general then, it is not appropriate to use the partial measures in (2) when the model contains more than two endogenous variables.

<u>Static Models</u>

Chavas et al. derive counterparts to the expressions in (4) when all structural equations are linear in the parameters and the variables and when there are no lagged dependent variables. Briefly, their approach is to partition the set of equations into two subsystems. The first subsystem contains the equations for the endogenous variables, say Y_1 and Y_2, for which total elasticities (flexibilities), denoted by ϵ_{12} and/or ϵ_{21}, are sought.[1/] The second subsystem contains equations for the remaining G-2 endogenous variables, denoted by $\underline{Y}.$ The framework developed here is similar to that presented in CHJ, but without the maintained hypothesis of linearity.

Without loss of generality, assume the G-equation system can be written as:[2/]

$$Y_1 = f_1(Y_2, \underline{Y}., \underline{X}) + u_1 \tag{5a}$$

$$Y_2 = f_2(Y_1, \underline{Y}., \underline{X}) + u_2 \tag{5b}$$

$$\underline{Y}. = \underline{f}.(Y_1, Y_2, \underline{Y}., \underline{X}) + \underline{u}. \tag{5c}$$

where $\underline{u} = (u_1, u_2, \underline{u}.)'$ is a G-vector of additive disturbance terms with mean vector zero and variance-covariance matrix Σ. The approach used by CHJ is to obtain the partial reduced form for the subsystem in (5c) where the endogenous variables $\underline{Y}.$ are expressed only as functions of Y_1, Y_2, and $\underline{X}$. That is, an expression of the general form:

$$\underline{Y}. = \underline{g}.(Y_1, Y_2, \underline{X}, \underline{u}.) \tag{6}$$

is obtained by solving for the reduced form of the second subsystem of G-2 equations. Observe that the partial reduced form in (6) shows how the G-2 endogenous variables in the second subsystem will adjust if there is a change in one of the endogenous variables in the second subsystem, Y_1 or Y_2.

The partial reduced form in (6) can be substituted for endogenous variables $\underline{Y}.$ in equation (5a) to obtain the following expression for Y_1:

$$Y_1 = f_1(Y_2, \underline{g}.(Y_1, Y_2, \underline{X}, \underline{u}.), \underline{X}) + u_1 = \tilde{f}_1(Y_1, Y_2, \underline{X}, \underline{u}.) + u_1 \tag{7}$$

which can be expressed in implicit form as:

$$F_1(Y_1, Y_2, \underline{X}, \underline{u}., u_1) = Y_1 - \tilde{f}_1(Y_1, Y_2, \underline{X}, \underline{u}.) - u_1 = 0. \tag{8}$$

Assuming the partial derivatives of F_1 with respect to its arguments are continuous and well defined, and that $\partial F_1/\partial Y_1 = (1 - \partial f_1) \neq 0$, then equation (8) locally defines Y_1 as a function of Y_2, exogenous variables $\underline{X}$, and equation error terms. Incorporated into the implicit equation (8a) are the adjustments that would occur in $\underline{Y}.$ as a result of a change in Y_1 or Y_2. The result is that all essential information implied by the structural system in (12) has been compressed into a single quasi reduced form equation relating Y_1 to Y_2. Expressions (7) and (8) are also the general counterparts to the partial reduced form results obtained by CHJ in the case where the structural model in (5) is linear.

Equation (8) can be used to obtain the total multiplier for Y_1 resulting from a change in Y_2. Specifically,[3/]

$$\frac{dY_1}{dY_2} = -\frac{\partial F_1/\partial Y_2}{\partial F_1/\partial Y_1} = \frac{\partial \tilde{f}_1/\partial Y_2}{(1 - \partial \tilde{f}_1/\partial Y_1)} = \left[\frac{\partial f_1}{\partial Y_2} + \frac{\partial f_1}{\partial \underline{Y}.}\frac{\partial \underline{g}.}{\partial Y_2}\right]\left[1 - \frac{\partial f_1}{\partial \underline{Y}.}\frac{\partial \underline{g}.}{\partial Y_1}\right]^{-1} \tag{9}$$

The multiplier in (9) measures the total effect of a change in Y_2 on Y_1. In other words, (9) measures the change in Y_1 resulting from a change in Y_2 when all remaining endogenous variables are allowed to adjust to the change in Y_2. The corresponding total elasticity (flexibility) is obtained by multiplying (dY_1/dY_2) by the ratio (Y_1^0/Y_2^0) where Y_1^0 and Y_2^0 are reference values for Y_1 and Y_2, respectively. If, for example, Y_1 represents quantity demanded and Y_2 denotes retail price, then the total elasticity $\epsilon_{12} = (dY_1/dY_2)\cdot(Y_1^0/Y_2^0)$ measures the total quantity demand response as retail price is exogenously varied.

An expression analogous to (9) that measures the total response of Y_2 to a change in Y_1 is:

$$\frac{dY_2}{dY_1} = -\frac{\partial F_2/\partial Y_1}{\partial F_2/\partial Y_2} = \frac{\partial \tilde{f}_2/\partial Y_1}{(1 - \partial \tilde{f}_2/\partial Y_2)} = \left[\frac{\partial f_2}{\partial Y_1} + \frac{\partial f_2}{\partial \underline{Y}.}\frac{\partial \underline{g}.}{\partial Y_1}\right]\left[1 - \frac{\partial f_2}{\partial \underline{Y}.}\frac{\partial \underline{g}.}{\partial Y_2}\right]^{-1} \tag{10}$$

and the total flexibility of Y_2 with respect to Y_1 is determined by evaluating $\epsilon_{21} = (dY_2/dY_1)\cdot(Y_2^0/Y_1^0)$.

Several observations regarding the above results are in order. First, the inverse of (9) will not, in general, equal the total flexibility in (10), a result consistent with those obtained earlier by Meinken et al., Houck, Colman and Miah, Chavas et al., and others. In fact, a sufficient condition for the inverse of ϵ_{12} to equal ϵ_{21} is that $\partial\underline{Y}./\partial Y_1 = \partial\underline{Y}./\partial Y_2 = \underline{0}$ and that $\partial f_1/\partial Y_2 = 1/(\partial f_2/\partial Y_1)$. In general, however, the inverse of a total elasticity (flexibility) will not equal the respective flexibility (elasticity).

Second, the total response measure dY_1/dY_2 will not in general equal the corresponding partial response measure, $\partial Y_1/\partial Y_2$. Only if $\partial\underline{Y}./\partial Y_1 = \partial\underline{Y}./\partial Y_2 = \underline{0}$

or if $\partial f_1/\partial \underline{Y}. = 0$ will the total effect equal the partial effect. In the first case, $\partial \underline{Y}./\partial Y_1 = \partial \underline{Y}./\partial Y_2 = \underline{0}$ implies the second subsystem of G-2 equations is not dependent on Y_1 and Y_2. The second condition implies the equation determining Y_1 is not a function of endogenous variables $\underline{Y}$. Lastly, the results in (9) and (10) generalize the total response measures derived by CHJ and HS where linear structural representations were employed. Moreover, the above results illustrate that under suitable conditions, equivalent local results can be derived for nearly any nonlinear model specification.

Dynamic Models

In many situations, lagged dependent variables are an intrinsic part of the model specification. In many agricultural models, for example, lagged dependent variables are included on the basis of the adaptive expectations hypothesis (Askari and Cummings). Sequential or stage-wise production processes, as well as rational expectations models, also give rise to complicated dynamic specifications. Autocorrelation terms also represent an additional source of dynamic response. It is therefore useful to consider how the above results can be extended to a more general dynamic setting.

To begin, we follow HS and consider the case where the model in (5) is extended to include first-order lags on the dependent variables. Such a specification represents the simplest case possible and is useful for elucidating the derivation of total response measures in dynamic nonlinear models. The dynamic representation of G-equation the market model is given by:

$$Y_{1t} = f_1(Y_2t, \underline{Y}._t, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}) + u_{1t} \tag{11a}$$

$$Y_{2t} = f_2(Y_{1t}, \underline{Y}._t, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}) + u_{2t} \tag{11b}$$

$$\underline{Y}._t = \underline{f}.(Y_{1t}, Y_{2t}, \underline{Y}._t, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}) + \underline{u}._t \tag{11c}$$

where time subscripts have been added to all variables and where, as before, the system has been partitioned into two subsystems: one endogenously determining Y_{1t} and Y_{2t} given $\underline{Y}._t$ and the other determining $\underline{Y}._t$ given Y_{1t} and Y_{2t}. Observe that even though only first-order lags are included, the lag structure is general in the sense that lagged values of <u>all</u> endogenous variables are permitted to enter each equation.

The results developed for the static model can be readily applied to the model in (11) to obtain immediate-run total response measures. Specifically, under suitable conditions a local solution of the second subsystem (11c),

$$\underline{Y}._t = \underline{g}.(Y_{1t}, Y_{2t}, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}, \underline{u}._t), \tag{12}$$

can be obtained in a fashion analogous to that in (6). The difference between equation (6) and equation (12) is, however, that (12) includes lagged endogenous variables. As before, substituting (12) for $\underline{Y}._t$ in equation (11a) yields an implicit equation of the form:

$$\begin{aligned} &F_1(Y_{1t}, Y_{2t}, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}, \underline{u}._t, u_{1t}) \\ &= Y_{1t} - \tilde{f}_1(Y_{1t}, Y_{2t}, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}._{t-1}, \underline{u}._t) - u_{1t} = 0 \end{aligned} \tag{13}$$

which, as in the static case, can be used to define locally an explicit function for Y_{1t} of the general form:

$$Y_{1t} = g_1(Y_{2t}, \underline{X}_t, Y_{1t-1}, Y_{2t-1}, \underline{Y}_{\cdot t-1}, \underline{u}_{\cdot t}, u_{1t}). \qquad (14)$$

Equation (13) or, analogously, (14) can be used in conjunction with the results in (9) to obtain a local estimate of the immediate-run total response multiplier, dY_{1t}/dY_{2t}. In dynamic models, however, interest focuses additionally on obtaining estimates of intermediate- and long-run total response multipliers (elasticities). HS derived counterparts to (12) and (14) for the case where the model in (11) is linear. They note, however, in the general case where lags on all endogenous variables are included, analytical expressions for intermediate- and long-run total response multipliers for Y_{1t} with respect to Y_{2t} cannot be obtained.[4]

HS suggest an iterative scheme for obtaining numerical estimates of total response multipliers in the presence of a general lag specification. In the context of the present model, they suggest ordering the equations so Y_{1t}, as determined from (14), is evaluated first. They then suggest "plugging" the resulting estimate of Y_{1t}, conditional on the reference value for Y_2, into (12) to obtain an updated estimate for $\underline{Y}_{\cdot t}$. The model can then be solved iteratively with values for $\underline{Y}_{\cdot t}$, as implied by (12), used in the subsequent iteration to solve for Y_{1t+1}. The entire iterative process can be repeated after perturbing the reference value for Y_2 and numerical estimates of the intermediate-run multipliers for Y_{1t} inferred.

While the procedure suggested by HS is conceptually correct, it is worth noting their method is equivalent to simply solving the subsystem of G-1 equations determining Y_{1t} and $\underline{Y}_{\cdot t}$, holding Y_2 constant. To see this, note that (14), the total response equation for Y_{1t}, was derived using partial reduced form (12) to substitute out $\underline{Y}_{\cdot t}$. Yet, as specified in (12), the variables constituting $\underline{Y}_{\cdot t}$ are themselves dependent on Y_{1t}. Hence, the procedure advocated by HS for obtaining total response measures is akin to solving simultaneously the G-1 system of equations in (12) and (14) or, equivalently, in (11a) and (11c), for alternative reference values of Y_2.[5] This approach is similar to the one advocated by Fair for estimating policy effects in nonlinear models. The distinction is, of course, that in the present case the "policy effects" pertain to shocks in one or more of the endogenous variables.

III. Model Specification and Estimation Results

The aggregate dairy sector model presented here includes both farm and retail components. The model consists of seven behavioral equations and two definitional identities. Equations for heifer and cow numbers and production per cow are specified using the framework suggested by Chavas and Klemme. Retail and manufacturing disappearance of milk, as well as retail, wholesale, and farm-level prices are determined as part of a simultaneous system. The supply block is estimated using maximum likelihood. Demand components, price linkage equations, and production per cow are estimated using two-stage least squares (2SLS). Cow and heifer inventory equations were estimated using annual data from 1950 to 1988. Remaining equations were estimated with annual data from 1960 through 1988. Where necessary, corrections for first-order autocorrelation were made. Estimation results for each behavioral equation are reported in table 1.

Supply Block

Equations for dairy cow (COW_t) and heifer (HEF_t) numbers are specified using information about the biological lags governing inventory response. Since the dynamic relationships included in the cow and heifer inventory equations are instrumental in subsequent simulation results, we briefly review their specifications.

Consider a population of animals $\bar{x}_t$ at time t consisting of n age categories, x_{jt}, $j = 0, \ldots, n$ where, by definition, $\bar{x}_t = \sum_{j=m}^{n} x_{jt}$, m being the earliest age at which reproduction can occur.[6/] Let γ define the (constant) reproduction rate and $k_{j,t}$ denote the survival or retention rate of animals of age j where $0 \leq k_{j,t} \leq 1$. Hence, survival is governed by the first-order difference equation $x_{j+1,t+1} = k_{j,t} \cdot x_{j,t}$ for $0 \leq t \leq n-1$. Noting that with the above difference equation, the adult population of age j at time t, x_{jt}, can be defined as

$$x_{jt} = [\prod_{i=m}^{j} k_{j-i,t-i}] \cdot x_{o,t-j},$$

it follows that the dynamics of the adult population can be characterized as:

$$\bar{x}_t = \sum_{j=m}^{n} x_{jt} = \sum_{j=m}^{n} x_{o,t-j} [\prod_{i=m}^{j} k_{j-i,t-i}]. \tag{15}$$

Equation (15) forms the basis for the specification of the cow inventory equation. Defining $k_{i,t} = [1 + \exp(\underline{x}_t \beta)]^{-1}$ where $\underline{x}_t$ is a vector of exogenous variables, β a parameter vector, and $x_{o,t-j} = HEF_t$, where HEF_t denotes the number of two-year-old heifers, we obtain the specification for the cow inventory equation in table 1. Economic factors, including lagged values of the farm milk price-feed cost ratio (FPM/FC) and the slaughter price-feed cost ratio (SP/FC) (all at time t-j), are allowed to condition retention rates in the cow inventory equation. Other variables included in the retention rates in the cow inventory equation are the age of cows (AGE), price-ratio, age interactions, and the proportion of two-year-old heifers ready to enter the herd (HEF/COW).

Similar logic is used to specify the heifer inventory equation. Assuming $\gamma = 1$ (i.e., one calf per year) and that approximately half of all calves born are female, it follows that the number of two-year-old heifers today is determined by $x_{2t} = .5(k_{1,t-1} \cdot k_{0,t-2}) x_{t-2}$. Again, letting $k_{1,t-1} \cdot k_{0,t-2} = [1 + \exp(\underline{x}_t \beta)]^{-1}$ we obtain the general form for the specification of the heifers equation in table 1. The proportion $(k_{1,t-1} \cdot k_{0,t-2})$ is a function of price ratios (FPM/FC and SP/FC) and, following Chavas and Klemme, is specified to depend on prices at t-1 and t-3.

To complete the specification of the supply block, production per cow (YLD_t) is specified as a linear function of the current milk-feed cost price ratio, FPM_t/FC_t, and a linear time trend. The trend is included to capture technological advancement in production per cow. Since current farm price of milk enters the yield equation, it is estimated using 2SLS.

The results in table 1 indicate all estimated supply equations fit the data reasonably well, with the heifers equation exhibiting the weakest performance. Moreover, all price variables have theoretically acceptable signs and many of the estimated coefficients are statistically significant. An assessment of the economic implications of the estimated supply model is presented in the following section.

Demand Block and Price Determination

The model is completed by adding equations determining average farm price of milk, real class II price of milk, real retail fluid price of milk, per capita fluid demand, and CCC removals. The specification of the demand block and price linkage equations closely parallels that of LaFrance and de Gorter, and Kaiser, Streeter, and Liu. All demand and price linkage equations were estimated using 2SLS. Each equation is discussed briefly.

Fluid demand (FU_t) is estimated in per capita terms and is specified as a function of the retail price of fluid milk ($RPFM_t$), the price of non-alcoholic beverages ($PNAB_t$), personal disposable income ($PINC_t$), a linear time trend (t), and lagged per capita fluid consumption. Following LaFrance and de Gorter, all prices and income are deflated by the CPI index for all items less food ($CPILF_t$). The estimated equation fits the data well and with the exception of the cross-price term, all estimated coefficients are significant at the 10% level and have the expected signs.

Manufacturing demand (MU_t) is estimated in price-dependent form, with the real price of class II milk (PMM_t) as the dependent variable. Other explanatory variables included are the price of fats and oils (PFO_t), a linear trend, and lagged real manufacturing price. All prices are deflated by the CPI. As reported in table 1, all estimated parameters have acceptable signs and all are significant at conventional levels.

The real retail fluid price ($RPFM_t$) is specified as a constant mark-up over the real farm price of fluid milk ($FPFM_t$). Marketing costs ($MCOST_t$) and a linear trend term are included as additional explanatory variables.[7] All economic variables are deflated by the CPI. The estimated retail price equation provides a good fit to the data, as indicated by the high R^2. Moreover, all variables are statistically significant, although the negative sign associate with marketing cost is anomalous.

The nominal farm price of milk (FPM_t) is specified as a time-varying proportion of the weighted averages of the farm price of fluid milk ($FPFM_t$) and the price of manufacturing milk (PMM_t). The weights are derived as the proportion of total milk production going into fluid uses and non-fluid uses, respectively. Since federal milk marketing orders are regional, it is not possible to construct a "true" blend price. Hence, we assume the farm price of milk is a time-varying proportion of the implied national average blend price. As illustrated in table 1, the estimated farm price equation fits the data well.

The final equation is for CCC removals. While CCC removals are in principle exogenous, preliminary estimates of long-run total response measures failed to accurately portray the types of adjustments that would be required in government purchases. The CCC removals equation is specified in double-log form and defines CCC purchases as a function of total milk production ($MPROD_t$), the price support-class II price ratio ($PSUP_t/PMM_t$), and a trend. The signs of the estimated coefficients are acceptable and all are statistically significant.

The model is completed using three identities: one that relates total milk production to cow numbers and production per cow; one that specifies the farm price of fluid milk equals the class II price plus the class I-class II differential; and one that ensures milk production equals all fluid and non-fluid uses. The following section uses the estimated dairy sector model to assess total response measures in a dynamic setting.

IV. Partial and Total Response Measures

The dynamic behavior of the estimated dairy model can be examined using mean path elasticities and flexibilities with respect to both exogenous and endogenous variables. The methods used for obtaining partial and total elasticities and flexibilities are similar to those described by Fair.

Specifically, the estimated model is solved at the means of the sample data using the Gauss-Seidel algorithm. The reference value for the exogenous (endogenous) variable of interest is then altered and the model re-solved.[8/] It is thus possible to obtain numerical estimates of partial and total mean path response measures.

Dynamic Response with Respect to Exogenous Variables

Intermediate run elasticities for selected endogenous variables with respect to feed cost and the price of other fats and oils are reported in table 2. As expected, the initial impacts of a change in feed costs have small initial effects on heifer and cow inventories, as well as total milk production. The production response increases in magnitude, however, for approximately 40 periods.

The response of CCC removals to a feed cost increase is large and negative for the first 60 periods, and the decline in CCC removals more than offsets the decline in production. The net effect of the CCC response is that elasticities of fluid and manufacturing demand for milk turn positive after 10-15 periods. Conversely, impacts on fluid, manufacturing, and farm prices become negative after 15-20 periods, due of course to the increase in fluid and manufacturing demands. Because price impacts turn negative after 15-20 periods, impacts on total milk production begin to decline in magnitude after 40 periods and actually become positive after 70 periods.

The initial response to an increase in the price of other fats and oils is to increase the demand for manufacturing milk, and thus to increase prices. The resulting price increase stimulates production, as well as heifer and cow retention, the result being the production response increases in magnitude for approximately the first 25 periods. The initial response of CCC removals to higher manufacturing demand is negative; however, higher production coupled with a higher price for manufacturing milk causes the impact on CCC stocks to turn positive and rise sharply for the following 25 periods. The net effect of higher manufacturing demand and higher CCC removals is that the fluid demand response is either negative of negligibly small over the entire period. Due apparently to population dynamics, the production cycle peaks after 25 periods, declines, and actually becomes negative after 40 periods.

Dynamic Response with Respect to Endogenous Variables

Additional insights into dynamic properties of the model can be obtained by examining total mean-path elasticities and flexibilities with respect to endogenous variables. Total elasticities and flexibilities for selected endogenous variables with respect to production per cow and retail fluid price are reported in table 3.

The initial production response to an exogenous increase in production per cow is of course positive (figure 1). Higher production levels result in lower prices (figure 2) and, accordingly, higher manufacturing and fluid demands. Lower farm prices result in a negative inventory response, thus providing an offsetting response to the positive effects of production per cow on total milk production. The net result is that after approximately 40 periods, the inventory response completely offsets the yield response, causing the long-run total elasticity of milk production with respect to production per cow to stabilize near zero (figure 1).

Prices respond to an exogenous change in production per cow in a non-monotonic fashion (figure 2). Due to higher production levels, prices fall initially. But at the same time, CCC removals are increasing at a proportionally greater rate than fluid and manufacturing demand. After 10 periods, "squeezeout" fluid and manufacturing uses, resulting in positive price flexibilities. In the long rune, however, all price flexibilities with respect to production per cow also stabilize near zero.

The dynamic response of the sub-system with respect to a change in the retail price of milk is also of interest (table 3). The short-run total elasticity of fluid demand with respect to the retail fluid price is about -.23, while the corresponding long-run measure is near -.65. The short-run production elasticity is small and negative (-.01), while the long-run production response is small but positive (.13).

As indicated in figures 3 and 4, initially an increase in retail fluid price results in offsetting increases in manufacturing use and CCC removals. The result is that manufacturing and farm prices are, in turn, reduced. Lower prices in turn reduce incentives for producers to retain cows and heifers in the dairy herd, thus resulting in a negative production response. Lower production, in turn, results in CCC removals eventually turning negative, with the net effect being that manufacturing demand continues to increase through approximately the first 50 periods, being compensated for ultimately by proportionally higher manufacturing use. Higher manufacturing use is accompanied by lower manufacturing and farm-level prices. This pattern continues until the dynamics associated with population response cause production response to actually turn slightly positive after 50 periods. The long-run implication is that there is a slight positive relationship between production, manufacturing price, farm price, and the retail price of fluid milk.

V. Conclusions

This paper has sought to refine and extend methods for obtaining total elasticities and flexibilities in systems of simultaneous equations. Specifically, we have explored ways of obtaining total dynamic response measures in systems of nonlinear equations. Both the concept and implementation of total response measures in nonlinear systems is quite simple: one need only solve the G-1 subsystem of equations obtained after deleting the equations determining the endogenous variable for which total elasticities are sought.

To illustrate the potential of the total response approach, a model of the U.S. dairy sector was specified and estimated. Due to the inclusion of dynamic terms reflecting population adjustments in the cow and heifer inventory equations, the resulting model is highly dynamic and nonlinear. The model was then used to obtain total elasticities and flexibilities with respect to production per cow and retail fluid price of milk. In both instances, intermediate-run elasticities varied widely in terms of both signs and magnitudes over the simulation period. The implications is that the dynamic interactions occurring in the dairy industry are highly complex—complexities which cannot be uncovered using a partial-equilibrium approach.

While the total elasticity concept is in essence exceedingly simple, it is interesting that the methodology has not been widely used or adopted in previous empirical modeling efforts. In part, this may reflect continuing confusion over the proper interpretation and derivation of elasticities and flexibilities in dynamic systems of equations (Chavas, Hassan, and Johnson). Hopefully this paper will help shed new light on the potential for deriving total response measures in general systems of equations.

Endnotes

1/ In general we may be interested in the relationship between endogenous variables Y_i and Y_j. The notation in the text is perfectly general in that the G-equation system can always be re-ordered so that $i=1$ and $j=2$.

2/ Implicit in the representation in (5) is that the structural model can be normalized so that each endogenous variable is determined uniquely by a single equation. Specifically, the formulation in (5) assumes any implicit equations can be normalized to determine directly a single endogenous variable.

3/ In order for the multiplier in (9) to be defined, it is of course necessary that $\partial f_1/\partial Y_1 \neq 1$, a sufficient condition for the local existence of an explicit function relating Y_1 to Y_2, $\underline{X}$, and model error terms. We assume throughout the conditions for existence of total multipliers are satisfied.

4/ The results obtained by HS contrast with those of CHJ in that the latter assume only lagged values for Y_{1t} and Y_{2t} enter the model. As CHJ suggest, the implication for linear model structures is that the total response equation (14) can always be recast as a system of first-order difference equations by appropriately redefining the lags and increasing the dimensionality of the state space (Chow). It is this manner it is possible to obtain analytical expressions for the intermediate- and long-run total multipliers for Y_{1t} with respect to Y_{2t}. Similar methods could also be applied to nonlinear models, but only if the lag specification is restricted to include Y_{1t} and Y_{2t}. Otherwise, as HS note, the results obtained by CHJ do not apply to the more general case because a change in Y_2 will have a delayed impact on $\underline{Y}$. Moreover, delayed changes in $\underline{Y}$. affect the intermediate-run multipliers for Y_{1t}.

5/ At first glance this result may seem counterintuitive, yet it closely parallels in concept the framework described by Thurman and Wohlgenant for identifying and estimating general equilibrium demand functions.

6/ It is assumed that age j and time t are measured in the same units.

7/ Marketing cost is the average of the wage rate in the food manufacturing sector and a transportation cost index.

8/ The numerical procedure used for obtaining partial and total mean-path multipliers is equivalent to taking one-sided numerical derivatives of the model's implied reduced form.

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Table 1. Estimated Equations of the Dairy Supply-Demand Model.

Replacement Heifers (ML)[a/]

$$HEF_t = (.5\ COW_{t-2})/[\ 1 + \exp(\underset{(.656)}{1.4833} - \underset{(.595)}{.1363}(FPM_{t-1}/FC_{t-1}) - \underset{(.739)}{2.1571}(FPM_{t-3}/FC_{t-3})$$

$$+ \underset{(.081)}{.129}(SP_{t-1}/FC_{t-1}) + \underset{(.083)}{.109}(SP_{t-3}/FC_{t-3})]$$

$R^2 = 0.743$

Inventory of Producing Dairy Cows (ML)

$$COW_t = \sum_{i=1}^{9} (HEF_{t-i})/\{\prod_{j=1}^{i} [\ 1 + \exp(\underset{(2.256)}{9.765} - \underset{(2.210)}{4.434}(FPM_{t-j}/FC_{t-j}) + \underset{(.279)}{.472}(SP_{t-j}/FC_{t-j})$$

$$- \underset{(.588)}{2.821}\cdot AGE + \underset{(.575)}{.994}(FPM_{t-j}/FC_{t-j})\cdot AGE - \underset{(.069)}{.095}(SP_{t-j}/FC_{t-j})\cdot AGE$$

$$+ \underset{(.687)}{2.659}(HEF_{t-j}/COW_{t-j})\cdot(AGE - 3)]\}$$

$R^2 = 0.969$

Production Per Cow (2SLS)

$$YLD_t = \underset{(481.21)}{5832.0} + \underset{(330.27)}{721.11}(FPM_t/FC_t) + \underset{(6.054)}{239.28}t$$

$R^2 = 0.991$

Average Farm Price of Milk (2SLS)

$$FPM_t = (\underset{(.246)}{1.233} - \underset{(.00112)}{.00596}t)[FPFM_t(FU_t/MPROD_t) + PMM_t((MU_t + CCC_t + NCR_t)/MPROD_t)]$$

$R^2 = 0.994$

Real Class II Milk Price (2SLS)

$$(PMM_t/CPI_t) = \underset{(3.124)}{4.011} - \underset{(.00005)}{.000106}\ MU_t + \underset{(.0134)}{.0417}(PFO_t/CPI_t) + \underset{(.0487)}{.0828}\ t$$

$$+ \underset{(.101)}{.734}(PMM_{t-1}/CPI_{t-1})$$

$R^2 = 0.889 \quad \hat{\rho} = -0.240$

Table 1. (Continued)

<u>Real Retail Fluid Price</u> (2SLS)

$$(RPFM_t/CPI_t) = \underset{(5.081)}{82.963} + \underset{(.361)}{4.510}(FPFM_t/CPI_t) \underset{(.067)}{-.341}(MCOST_t/CPI_t) - \underset{(.122)}{.785}\, t$$

$$R^2 = 0.981$$

<u>Per Capita Fluid Demand</u> (2SLS)

$$(FU_t/POP_t) = \underset{(65.740)}{137.44} - \underset{(.161)}{.514}(RPFM_t/CPILF_t) - \underset{(.0632)}{.0692}(PNAB_t/CPILF_t)$$

$$+ \underset{(2.637)}{4.598}(PINC_t/CPILF_t) - \underset{(1.082)}{2.407}\, t + \underset{(.196)}{.673}(FU_{t-1}/POP_{t-1})$$

$$R^2 = 0.988 \quad \hat{\rho} = .449$$

<u>CCC Removals</u> (2SLS)

$$\ln CCC_t = -86.069 + 8.136 \ln(MPROD_t) + 5.625 \ln(PSUP_t/WMPM_t) -.0408 \ln(t)$$

$$R^2 = 0.625$$

Note: Identities used to close the model include:

<u>Total Milk Production</u>:

$$MPROD_t = (COW_t \cdot YLD_t)/1000$$

<u>Market Clearing</u>:

$$MPROD_t = FU_t + MU_t + CCC_t + NCR_t$$

<u>Farm Price of Fluid Milk</u>:

$$FPFM_t = PMM_t + DIF_t.$$

a/ Asymptotic standard errors appear in parentheses. ML denotes estimation by maximum likelihood and 2SLS indicates estimation by two-stage least squares. Also, the variable AGE is defined as AGE = i − j + 3.

Table 2. Mean Elasticities and Flexibilities with respect to Selected Exogenous Variables.

Period	HEIF	COW	MPROD	FU	MU	RPFM	PMM	FPM
				Percent Change in Feed Costs				
0	0.000	0.000	-0.104	-0.006	-0.107	0.027	0.054	0.056
1	0.115	-0.043	-0.142	-0.016	-0.127	0.051	0.102	0.104
2	0.117	-0.053	-0.149	-0.026	-0.113	0.065	0.130	0.130
3	-0.558	-0.064	-0.159	-0.034	-0.109	0.075	0.149	0.147
4	-0.522	-0.074	-0.168	-0.041	-0.102	0.080	0.159	0.157
5	-0.506	-0.062	-0.157	-0.045	-0.076	0.078	0.154	0.152
10	-0.523	-0.141	-0.238	-0.039	-0.037	0.050	0.100	0.106
15	-0.823	-0.471	-0.572	-0.014	0.050	-0.003	-0.006	0.028
20	-1.239	-0.821	-0.938	0.044	0.195	-0.110	-0.219	-0.150
25	-1.786	-1.278	-1.424	0.153	0.485	-0.299	-0.585	-0.469
30	-2.513	-1.812	-2.004	0.346	0.954	-0.597	-1.134	-0.967
40	-3.370	-2.088	-2.327	0.672	1.093	-0.872	-1.611	-1.417
50	-3.211	-1.849	-2.094	0.692	1.014	-0.851	-1.566	-1.391
60	-2.635	-1.284	-1.517	0.579	0.732	-0.678	-1.245	-1.119
70	-0.921	0.180	0.054	0.134	-0.417	0.013	0.024	0.026
80	1.135	0.966	0.942	-0.387	-0.589	0.487	0.893	0.822
				Percent Change in Price of Fats and Oils				
0	0.000	0.000	0.028	-0.030	0.204	0.126	0.254	0.235
1	0.016	0.026	0.066	-0.062	0.348	0.182	0.365	0.334
2	0.022	0.054	0.098	-0.087	0.439	0.200	0.399	0.363
3	0.227	0.084	0.128	-0.103	0.499	0.198	0.395	0.356
4	0.324	0.091	0.134	-0.113	0.516	0.193	0.383	0.344
5	0.370	0.089	0.130	-0.118	0.517	0.188	0.374	0.335
10	0.351	0.163	0.195	-0.110	0.571	0.148	0.297	0.260
15	0.519	0.406	0.441	-0.102	0.488	0.164	0.329	0.280
20	0.823	0.601	0.658	-0.147	0.374	0.253	0.504	0.436
25	1.127	0.739	0.823	-0.221	0.256	0.362	0.708	0.628
30	1.133	0.489	0.561	-0.244	0.520	0.304	0.578	0.523
40	-0.020	0.021	0.035	-0.042	0.646	0.059	0.109	0.102
50	-0.191	-0.078	-0.077	0.010	0.689	0.003	0.006	0.009
60	-0.195	-0.137	-0.142	0.020	0.768	-0.028	-0.051	-0.043
70	-0.138	-0.161	-0.169	0.015	0.841	-0.038	-0.070	-0.061
80	-0.066	-0.145	-0.151	0.004	0.870	-0.028	-0.051	-0.043

Table 3. Total Elasticities and Flexibilities with respect to Selected Endogenous Variables.

Period	HEIF	COW	MPROD	FU	MU	RPFM	PMM	FPM
			Percent Change in Production Per Cow (YLD)					
0	0.000	0.000	1.000	0.038	0.661	-0.162	-0.325	-0.359
1	-0.024	-0.046	0.949	0.078	0.442	-0.226	-0.450	-0.472
2	-0.032	-0.095	0.895	0.109	0.333	-0.246	-0.489	-0.503
3	-0.368	-0.145	0.840	0.130	0.276	-0.248	-0.490	-0.500
4	-0.501	-0.179	0.803	0.141	0.221	-0.236	-0.466	-0.475
5	-0.567	-0.204	0.776	0.144	0.188	-0.219	-0.433	-0.443
10	-0.571	-0.394	0.567	0.074	-0.063	-0.043	-0.086	-0.109
15	-0.527	-0.575	0.368	-0.012	-0.084	0.055	0.110	0.082
20	-0.469	-0.531	0.416	-0.055	-0.152	0.107	0.213	0.177
25	-0.300	-0.446	0.510	-0.103	-0.246	0.179	0.350	0.305
30	-0.195	-0.469	0.484	-0.144	-0.242	0.210	0.401	0.358
40	-0.859	-0.881	0.031	-0.026	-0.029	0.013	0.025	0.022
50	-0.929	-0.943	-0.038	0.005	0.029	-0.019	-0.035	-0.032
60	-0.892	-0.935	-0.028	0.001	0.045	-0.013	-0.024	-0.022
70	-0.871	-0.909	0.000	-0.005	0.025	0.001	0.002	0.002
80	-0.881	-0.891	0.020	-0.006	-0.006	0.010	0.019	0.018
			Percent Change in Retail Price of Fluid Milk (RPFM)					
0	0.000	0.000	-0.010	-0.231	0.152	-	-0.076	-0.085
1	-0.006	-0.010	-0.031	-0.380	0.212	-	-0.160	-0.171
2	-0.011	-0.029	-0.058	-0.474	0.226	-	-0.227	-0.239
3	-0.091	-0.055	-0.090	-0.535	0.218	-	-0.272	-0.283
4	-0.180	-0.074	-0.112	-0.575	0.207	-	-0.300	-0.310
5	-0.258	-0.091	-0.132	-0.602	0.195	-	-0.315	-0.325
10	-0.418	-0.223	-0.258	-0.647	0.124	-	-0.282	-0.289
15	-0.618	-0.493	-0.529	-0.653	0.208	-	-0.327	-0.316
20	-0.964	-0.762	-0.817	-0.654	0.358	-	-0.518	-0.481
25	-1.404	-1.088	-1.176	-0.654	0.638	-	-0.840	-0.773
30	-1.943	-1.379	-1.511	-0.654	0.968	-	-1.235	-1.143
40	-1.360	-0.518	-0.599	-0.654	0.217	-	-0.639	-0.617
50	-0.003	0.068	0.060	-0.654	0.005	-	-0.026	-0.052
60	0.129	0.124	0.125	-0.654	-0.017	-	0.037	0.007
70	0.127	0.130	0.131	-0.654	-0.030	-	0.044	0.013
80	0.117	0.129	0.131	-0.654	-0.037	-	0.043	0.013

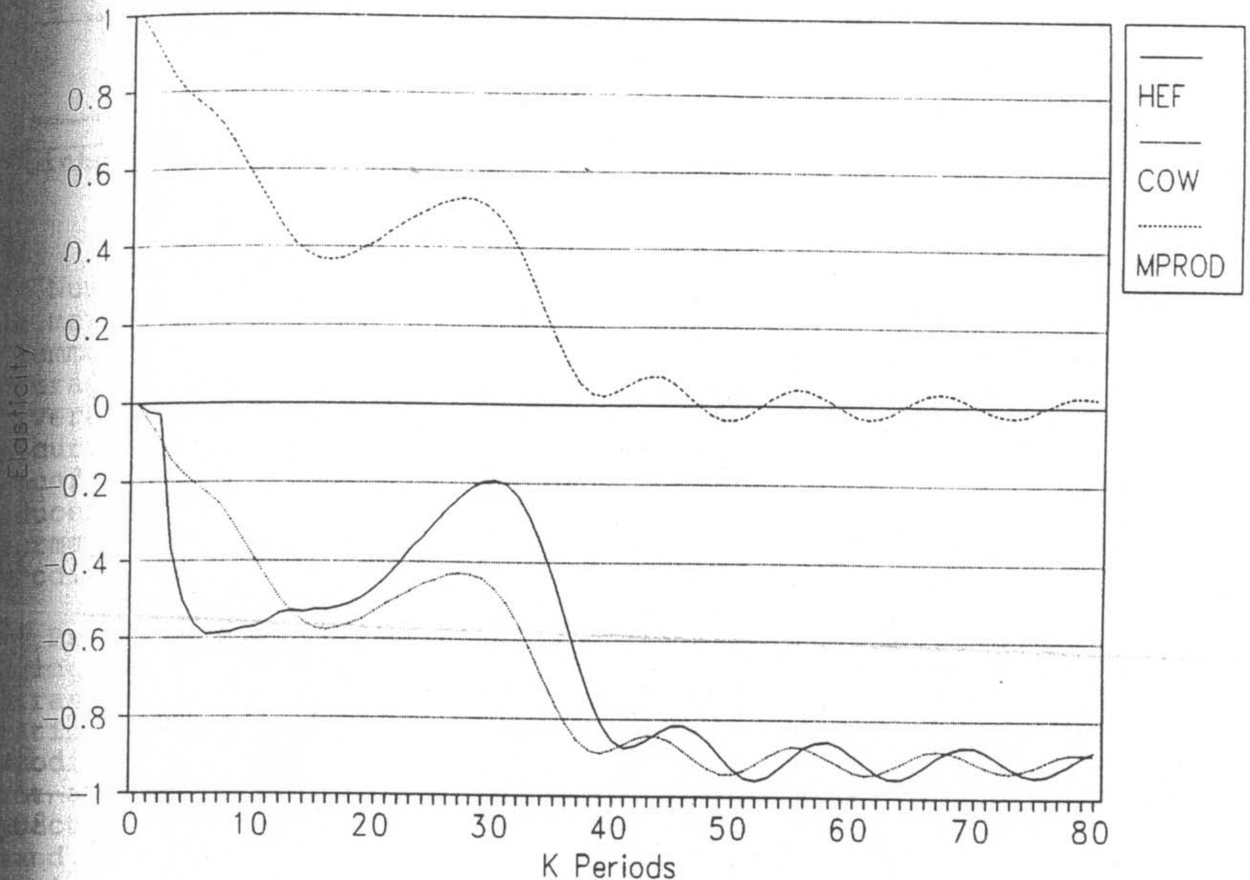

Figure 1. Total supply elasticities with respect to production per cow

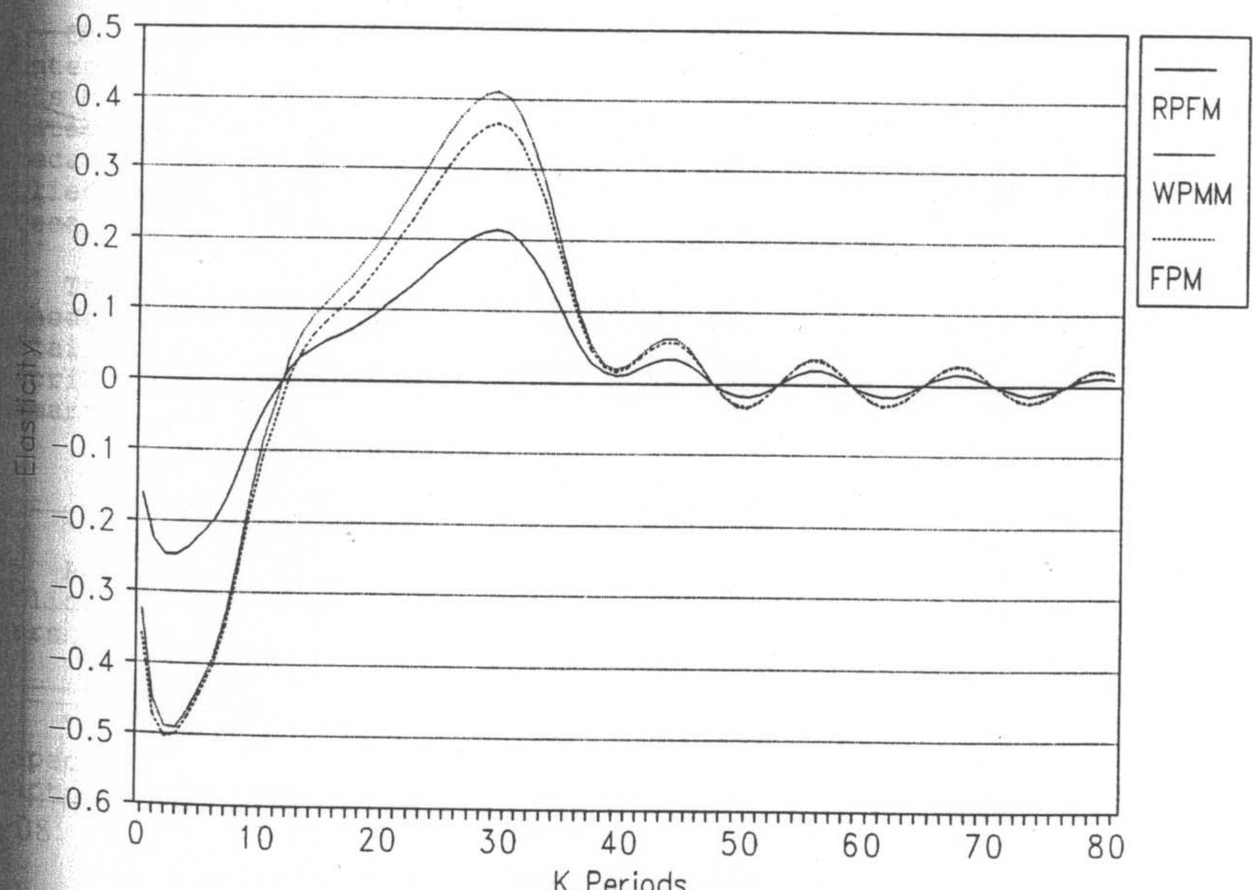

Figure 2. Total price flexibilities with respect to production per cow

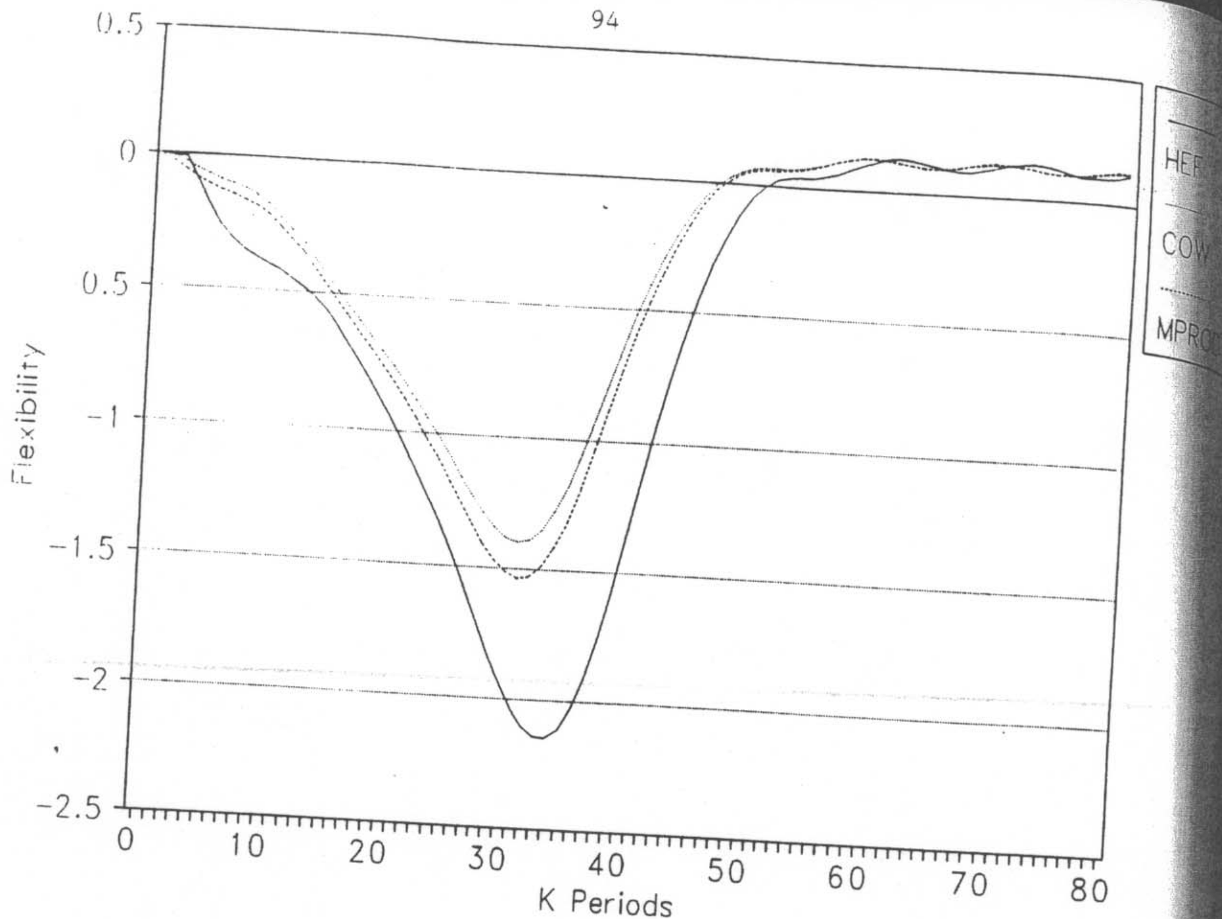

Figure 3. Total supply elasticities with respect to retail price of fluid milk

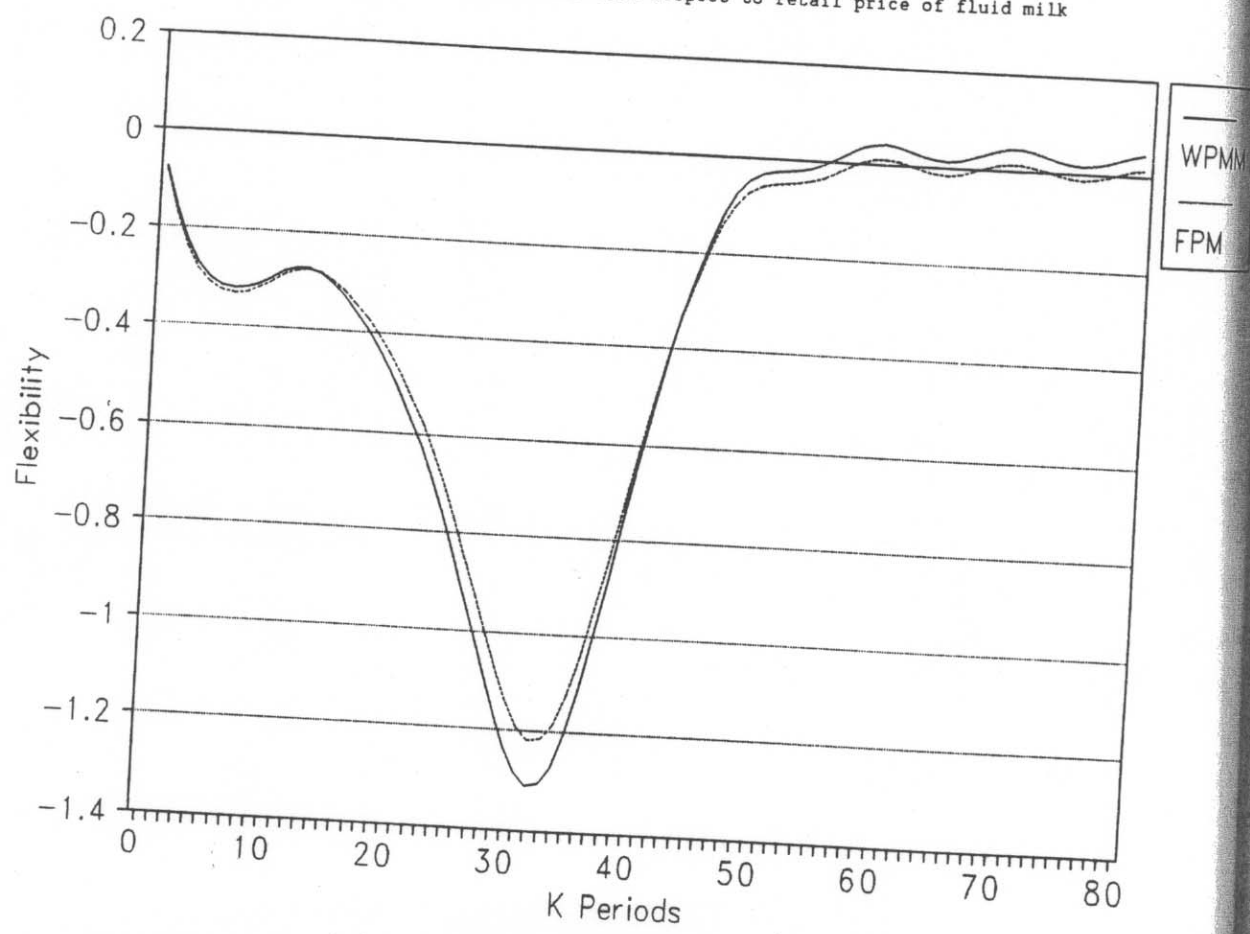

Figure 4. Total price flexibilities with respect to retail price of fluid milk