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Forecasting Volatility for Commodity Option Prices:
Incorporation of Volume and Open Interest

Albert A. Williams and Jack E. Houston*

Volatility is a key factor in the determination of prices of options on futures and in strategies for hedging, speculation, and arbitrage. Large percentage changes in price tend to be followed by large percentage changes, while small percentage changes tend to be followed by relatively small changes. Measures of implied volatility (IV) commonly incorporate data on futures contract returns to compute or estimate a statistic such as the standard deviation and then to make inferences about futures contract volatility. Analysts choose a specification for the returns process, which is then used to estimate the stochastic process of volatility. For example, Black's (1973) option pricing model measured volatility of futures prices by an iterative procedure that changes the standard deviation until the calculated futures option value is equal to the observed market value.

Engle (1982), however, observed traded asset prices to be heteroskedastic and suggested that models should be tested for this property and adjusted, if necessary. Applications of ordinary least squares (OLS), with its constant variance assumption, provide unbiased, but not efficient, estimates in the presence of heteroskedasticity. If lagged endogenous variables are included, OLS estimates may not be appropriate (Judge et al. 1985). Engle (1982) thus developed an Autoregressive Conditional Heteroskedasticity (ARCH) model to provide a structure for the variance. Bollerslev (1986) generalized the ARCH model (GARCH), using a more flexible lag structure and allowing lagged conditional variances to influence forecasts.

Kenyon et al. (1987) hypothesized that the level of production and the level of futures prices relative to the loan rate affect volatility. Najand and Yung (1990) found a relationship between volume and price variability in the return process of Treasury-bond futures contracts, and Turner et al. (1992) described a systematic relationship between prices, volume, and open interest changes for commodity futures contracts. Integrating volume and open interest into prediction of prices and their volatilities enables traders to observe and, in some instances, anticipate market reactions, particularly reversals. Discrepancies between actual options premiums and derived premiums may exist, but Wilson et al. (1988) suggest this does not necessarily imply that the market is inefficient or that the model is inaccurate. Values depend on time-to-maturity, market volatility, and market liquidity.

This paper compares two methods of measuring volatilities for corn, soybeans, and pork bellies futures prices -- OLS and ARCH. Following a review of some theoretical considerations that stimulated this research,

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the third section describes the methodology and procedures for incorporation of changes in prices, volume, and open interest. Empirical results are shown and discussed next, and the final section presents preliminary conclusions and implications.

Theoretical Considerations

Black (1976) developed the most widely used option pricing formula, where the fair market value of an option depends functionally on the interest rate, time-to-maturity of the option, the futures and exercise prices, and the standard deviation of the futures' price return over the remaining life of the option. Of these inputs, only the standard deviation of futures price returns must be estimated. Black's model assumes a constant variance; thus, a measure of past standard deviation can be used. This constant variance assumption has been relaxed to allow for a changing variance (Anderson 1985). Black's model, a derivative of the Black-Scholes (1973) model for stock options, also assumes: (1) the markets for options, futures and pure discount bonds are frictionless; (2) the risk-free rate is constant over the life of the option; (3) the option is a European option and hence cannot be exercised before the expiration date; and (4) the instantaneous percentage price change of the futures contract follows a fixed-variance Wiener process.

To calculate the current price of a futures call option using Black's model:

$$C = B[F N(d_1) - K N(d_2)]$$

where B is the current price of a pure discount bond paying one dollar at the option expiration date, F is the current price of the underlying futures contract, K is the exercise price of the option, and N(.) is the cumulative normal distribution function. Values of d_1 and d_2 are defined:

$$d_1 = [\ln(F/K) + 1/2\sigma^2(T - t)] / [\sigma(T - t)^{1/2}], \text{ and}$$

$$d_2 = d_1 - \sigma(T - t)^{1/2},$$

where σ^2 is the variance of the instantaneous percentage price change distribution, T is the time of the expiration of the option, and t is the current time.

Wolf (1982) suggests four methods to estimate volatility for use in Black's model: (1) the standard deviation from historical data, (2) a weighted average of historical data where more recent observations receive higher weights, (3) a moving average, and (4) implied standard deviation. Wolf used the standard deviation calculated from historical data for gold futures contract to calculate the price of a call option and, using put-call parity, the price of a put. Figlewski and Fitzgerald (1982), testing Black's formula for London options on sugar and cocoa futures, found a significant difference between the actual premiums and those calculated by Black's formula. A likely explanation is that Black's model does not allow for premature exercise of the option.

Kenyon et al. (1987) hypothesized that the level of production (activity) and the level of futures prices relative to the loan rate affect volatility. The variance is expected to increase when the uncertainty of seasonal supply is high and when the production level is low. It is expected to decrease when the futures price-to-loan rate ratio is declining. Kenyon et al. conclude that the futures market price volatility of corn, soybeans, and wheat is not constant over the life of the futures contract. Seasonality effects are significant, and volatility is positively related to the ratio of average futures price to the loan rate. In periods of yield (volume) uncertainty, volatility increases. This implies that option pricing models should allow for changing variance over the life of the contract and that this changing variance might be related to changes in yield/volume. No significant change in volatility for cattle or hogs over time was demonstrated by Kenyon et al., however.

Testing Black's model for soybean futures options, Jordan et al. (1987) used intra-day transaction prices and three volatility estimation methods -- historical, forecast, and implied. The Black model prices accurately when volatility is estimated by implied volatility based on the settlement price of the previous day's nearest at-the-money option for each contract delivery month. The Black model underprices by only \$0.004 cent per bushel, on average, for calls and by only \$0.001 for puts. Call and put options at-the-money are slightly overpriced. However, as the options become in- and out-of-the money, the model underprices. This underpricing results from the use of an at-the-money implied volatility.

Wilson, Fung, and Ricks (1988) examined the differences between actual option premiums versus those derived by Black's model for wheat, soybean, and corn. Discrepancies between actual premiums and derived premiums are expected, as they compare an American option to a European option. Although significant discrepancies may exist, this does not necessarily imply that the market is inefficient or that the model is inaccurate. Rather, such discrepancies empirically support Merton's theorem, "the right to exercise an option prior to the expiration date always has non-negative value" (Merton 1973, p. 144). That exercise right value depends on time-to-maturity, market volatility, and market liquidity. Wilson et al.'s results suggest that as the volatility of the futures market increases, actual premiums decrease relative to Black's premiums. In calculating the predicted premium using the Black model, estimation of the variability of the underlying futures contract is critical to calculated options premiums.

If Black's model is assumed accurate and if the market is efficient, then an implied standard deviation (ISD) can be computed by an iterative technique to reflect all relevant information about the expected standard deviation of the futures returns for the remaining life of the option. Wilson and Fung (1990) tested the informational content of the ISD for corn, soybeans, and wheat options to evaluate whether present option premiums anticipate volatilities. For CBOT corn and soybeans, the correlation between the historical standard deviation (HSD) and the ISD is positive and anticipatory. The corresponding correlation for the wheat market is insignificant and backward-looking. The markets studied processed relevant information in 1985 and 1986 inefficiently, systematically mis-pricing some option premiums. Tests for the joint hypotheses of market efficiency and the validity of the Black model show that if Black's model is assumed

correct, then ISD does not accurately reflect most of the information affecting the variability of futures prices.

Engle (1982) proposed a class of models (ARCH) to address the pricing pattern whereby large percentage changes in price tend to be followed by large percentage changes, while small percentage changes tend to be followed by relatively small changes. This class may be specified:

$$y_t | \psi_{t-1} \sim N(x_t \beta, h_t),$$

$$h_t = h(\varepsilon_{t-1}, \varepsilon_{t-2}, \dots, \varepsilon_{t-p}, \alpha), \text{ and}$$

$$\varepsilon_t = y_t - x_t \beta,$$

where y_t is an endogenous variable and ψ_{t-1} is the information set at $t-1$. The variance function, h_t , can be generalized to include current and lagged exogenous variables, implying that the underlying forecast variance may change over time and can be predicted by past forecast errors.

ARCH stochastic processes have a zero mean and are serially uncorrelated, with nonconstant variances conditional on past information. ARCH models can be used to find time-varying confidence intervals, which are expected to be large for periods of high volatility and vice versa. Engle estimated the variance of inflation in the United Kingdom for 1958 to 1977, using an ARCH model, and showed that the maximum likelihood estimates of the ARCH model are generally smaller than those of OLS. Using confidence intervals from both ARCH and OLS models to examine outliers, Engle found the ARCH model more closely approaches the true random residuals, improves the performance of least squares models, and provides more realistic forecasted variance.

Bollerslev (1986) generalized the ARCH model (GARCH) through a more flexible lag structure. The GARCH(p,q) model is specified:

$$\varepsilon_t = y_t - x_t \beta,$$

$$\varepsilon_t | \Phi_{t-1} \sim N(0, h_t), \text{ and}$$

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}$$

where the variables are similar to those of the ARCH model. The GARCH variance, h_t , now depends additionally on its own lagged values, h_{t-1} . The order of p and q are identified by applying Box and Jenkins (1976) time series techniques to the autocorrelation and partial autocorrelation for the squared process of ε_t . Najand and Yung (1991) examined the relationship between volume and price variability of Treasury-bond futures contracts, finding a GARCH(1,1) specification best described the returns process. Examining only near-month contract data for 1984 to 1989 from the CBOT and using descriptive statistics, they rejected the null hypothesis of normality; specifically, kurtosis was very large. A positive relationship between price variability and volume was found only for the contracts in 1986 and 1988.

The Najand and Yung result may be due to a simultaneity problem, as has been described by Lamoureux and Lastrapes (1990). That is, volume may be an endogenous variable in a larger system of simultaneous equations. Harvey (1989) solved the simultaneity problem by using lagged values of the endogenous variables. When using the lagged values of volume in the GARCH analysis, Najand and Yung found a positive and significant relationship for the overall period and most of the sub periods. Najand and Yung's implications support the saying on Wall Street that "It takes volume to make prices move."

Turvey (1990) evaluated four measures of IV based on options written on a single futures contract. The first measure is a simple average of the different IVs found from different options written on a single futures contract. The second uses the IV of the option nearest to or at-the-money (Becker 1981). A third measure weights the IVs by the partial derivative of the option price with respect to the standard deviation (Latane and Rendleman 1976). The fourth weights the IVs by the elasticity of the option price with respect to the standard deviation. Turvey's results show that IVs weighted by the derivative of the option price with respect to the standard deviation provide better ex ante predictions of put option premia on soybean and live cattle futures, on average. Turvey suggests that this measure of IV may be superior, on average, to the other three measures.

Methodology and Procedures

Our model hypothesizes that changes in volume, open interest, and lagged prices influence changes in current futures prices or their volatilities. That is, a systematic relationship exists between prices, volume, and open interest changes for futures contracts. Integrating volume and open interest into predictions of prices and their volatilities enables traders to observe market reactions to variables other than price (Jiler 1975; Turner et al. 1992). Changes in volume patterns may warn of a reversal in trend before it actually happens (Jiler 1962).

The models used here to compare OLS and ARCH regressions are commodity specific and are specified as follows:
For Corn --

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-1} + \beta_2 \dot{p}_{t-7} + \beta_3 \dot{p}_{t-13} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, h_t)$,

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-7}^2 + \alpha_3 \varepsilon_{t-13}^2 + b_0 \dot{v}_t + b_1 \dot{o}_t$$

and $\alpha_0 > 0, \alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$;

For Soybeans --

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-2} + \beta_2 \dot{p}_{t-7} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, h_t)$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-2}^2 + \alpha_2 \varepsilon_{t-5}^2 + \alpha_3 \varepsilon_{t-7}^2 + b_0 \dot{v}_t + b_1 \dot{o}_t$$

and $\alpha_0 > 0, \alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$; and

For Pork Bellies --

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, h_t)$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + b_0 \dot{v}_t + b_1 \dot{o}_t$$

and $\alpha_0 > 0, \alpha_1 > 0$.

In the above models,

$\dot{p}_t = \ln(P_t/P_{t-1})$ where P_t is the futures price at time t ,

$\dot{v}_t = \ln(V_t/V_{t-1})$ where V_t is the volume at time t ,

$\dot{o}_t = \ln(OI_t/OI_{t-1})$ where OI_t is the open interest at time t , and

$\dot{p}_{t-1} = \ln(P_{t-1}/P_{t-2})$.

An Autoregressive-Integrated-Moving Average (ARIMA) process is employed to assist the identification of the lagged endogenous variables and lagged squared error terms for each model (Bollerslev, 1986).

Three commodity contracts are estimated -- corn, soybeans, and pork bellies. Daily data for the three commodities included observations from 1985 through 1991, and data for each commodity linked contracts nearest to maturity.

The first stage in the ARCH(1) model procedure uses OLS to obtain initial parameter values for a Feasible Generalized Least Squares (FGLS), four-step procedure outlined by Greene (1990). This procedure is asymptotically equivalent to maximum likelihood (Greene 1990). OLS and ARCH results are then compared.

Forecasts of implied volatility are estimated with the ARCH processes:

$$h_{T+i+1} = \alpha_0 + \alpha_1 \varepsilon_{T+i-1}^2 + \alpha_2 \varepsilon_{T+i-7}^2 + \alpha_3 \varepsilon_{T+i-13}^2 + b_0 \dot{v}_{T+i} + b_1 \dot{o}_{T+i},$$

$$h_{T+i+1} = \alpha_0 + \alpha_1 \varepsilon_{T+i-1}^2 + \alpha_2 \varepsilon_{T+i-5}^2 + \alpha_3 \varepsilon_{T+i-7}^2 + b_0 \dot{v}_{T+i} + b_1 \dot{o}_{T+i}, \text{ and}$$

$$h_{T+i+1} = \alpha_0 + \alpha_1 \varepsilon_{T+i-1}^2 + b_0 \dot{v}_{T+i} + b_1 \dot{o}_{T+i}, \text{ where } i = 0, 1, \dots, 19 \text{ for corn}$$

soybeans, and pork bellies, respectively. A forecast of implied volatility is made for $T+i+1$ at time, T . At time, T , the squared residual from the OLS model is estimated for $T+i$. Parameter estimates for both OLS and ARCH models are held fixed at the end of in-sample data, and out-of-sample daily forecasts for 20 days are made for each contract. The IVs from the ARCH model are then compared with the OLS squared residuals.

Empirical Results

The lags of the endogenous variables are chosen by employing an ARIMA process. Hence, the statistical significance of the corresponding parameters are generally consistent with expectations. The ARCH results for corn generally indicate smaller standard errors (Table 1). Changes in the three lagged prices are positively related to price changes at the 99 percent confidence level. For the conditional variance equation, all three lagged squared error terms are significantly positive. Changes in open interest do not appear to be related to the conditional variance. However, there is a positive and significant relationship between the conditional variance and volume changes. This implies that larger changes in volume in the market are associated with increases in price volatility and smaller changes in volume with decreased volatility. The Lagrange Multiplier (LM) statistic, $TR^2 = 11.63$, suggested by Engle (1982), also rejects the hypothesis that disturbance structure is conditionally homoskedastic.

OLS and ARCH results indicate that there is a positive and significant relationship between soybean price changes and price changes lagged seven periods (Table 2). The estimated ARCH parameters have smaller standard errors. The LM statistic is 233.82 and is significant. The conditional variance is positively related to squared errors lagged one, five, and seven periods respectively. Neither open interest nor volume changes are significant in the conditional variance equation for soybean futures prices.

ARCH and OLS results for pork bellies indicate a positive relation between price changes and price changes lagged one period (Table 3). The standard errors of the ARCH process are smaller than those of OLS. The LM statistic, 17.48, is significant. There is a significant and positive relationship between the conditional variance and the changes in open interest. Also, there is a significant and negative relationship between the conditional variance and changes in volume. This finding supports the hypothesis that both volume and open interest changes affect the volatility of pork bellies futures prices.

The OLS and ARCH estimators are used to forecast out-of-sample daily variances for the 20 days following the most current date used in the estimation. Because of the logarithmic transformation of the data, zero values have been set to small positive numbers. This may influence some of the forecasted IVs. Hence, some unusual behavior of the forecasts may be related to the data. These problems will be addressed in follow-up studies.

Forecasted IVs for the commodities futures prices are preliminary. Corn (Figure 1) IV forecasts are reasonably close to the squared errors from the OLS model. The forecasted conditional variance appears to have a one-period adjustment. IV forecasts for soybeans closely track the corresponding OLS squared residuals (Figure 2). The IV forecasts behave like a smoothing process. Forecasts for the implied volatility of pork bellies futures prices are shown in Figure 3.

Conclusions

Forecasting volatility is critical to forecasting prices of options on futures. Engle (1982) demonstrated that the price of traded assets may have distributions that are not homoskedastic and suggested an ARCH model that allows past information to be used in the variance structure. Bollerslev (1986) extended this model to additionally include the past variances. The ARCH model is used here to estimate the parameters of a model testing the relationship of changes in futures prices, volume, open interest, and lagged prices for corn, soybeans, and pork bellies. These parameter estimates and current information are then used to forecast variances, which could be used to assist in hedging and speculating in options futures markets.

ARCH parameter estimates generally differed little in magnitude from those estimated OLS. The standard errors of the ARCH parameters, however, are smaller than those from OLS, and all three commodity models demonstrated disturbances with significant ARCH effects. Open interest changes significantly affected estimation of the conditional variance for pork bellies. Volume changes significantly related to the conditional variances for both corn and pork bellies futures prices. Changes in both open interest and volume added important information to explaining the conditional variance for pork bellies and supported the hypothesis that this information is considered in the market.

The forecasted IVs are preliminary. An extension of the ARCH models to incorporate GARCH procedures will be tested in further research with these and other traded instruments. Other methods of estimating volatility should also be considered, including modified versions of Black's (1973) formula. These could include models where there are one or more stochastic variables other than price.

Table 1. OLS and ARCH Models for Corn Futures (1985 - 1991)

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-1} + \beta_2 \dot{p}_{t-7} + \beta_3 \dot{p}_{t-13} + \varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-7}^2 + \alpha_3 \varepsilon_{t-13}^2 + b_0 \dot{o}_t + b_1 \dot{v}_t$$

Procedure	Dependent Variable	Intercept β_0	Lagged Prices(-1) β_1	Lagged Prices(-7) β_2	Lagged Prices(-13) β_3
OLS	\dot{p}_t	-0.00003 (0.00038) ^a	0.04735* (0.02416)	0.11208** (0.02416)	0.07993** (0.02420)
ARCH	\dot{p}_t	-0.00050** (0.00020)	0.04114** (0.01443)	0.09856** (0.01748)	0.08522** (0.01706)

Procedure	Dependent Variable	Intercept α_0	ε_{t-1}^2 α_1	ε_{t-7}^2 α_2	ε_{t-13}^2 α_3	Changes in Open Interest b_0	Changes in Volume b_1
ARCH	h_t	0.00010** (0.00001)	0.16510** (0.02314)	0.23221** (0.02387)	0.27078** (0.02368)	0.00000 (0.00000)	0.00006** (0.000003)

TR² = 11.63^a Standard errors are in parentheses.

* and ** indicate significance at the 5% and 1% levels, respectively.

1696 Observations

Table 2. OLS and ARCH Models for Soybeans Futures (1985 - 1991)

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-2} + \beta_2 \dot{p}_{t-7} + \varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-2}^2 + \alpha_2 \varepsilon_{t-5}^2 + \alpha_3 \varepsilon_{t-7}^2 + b_0 \dot{o}_t + b_1 \dot{v}_t$$

Procedure	Dependent Variable	Intercept β_0	Lagged Prices(-2) β_1	Lagged Prices(-7) β_2	ε_{t-1}^2 α_1	ε_{t-5}^2 α_2	ε_{t-7}^2 α_3	Changes in Open Interest b_0	Changes in Volume b_1
OLS	\dot{p}_t	0.00001 (0.00038) ^a	-0.03312 (0.02576)	0.05482** (0.02575)					
ARCH	\dot{p}_t	-0.00001 (0.00025)	-0.03303 (0.01911)	0.05589** (0.02054)					
ARCH	h_t	0.00008** (0.00001)	0.17808** (0.03147)	0.30859** (0.04410)	0.13130 (0.03349)	-0.00000 (0.00000)	0.00001 (0.00001)		

TR² = 233.82

^a Standard errors are in parentheses.

* and ** indicate significance at the 5% and 1% levels, respectively.

1514 Observations

Table 3. OLS and ARCH Models for Pork Bellies Futures (1985 - 1991)

$$\dot{p}_t = \beta_0 + \beta_1 \dot{p}_{t-1} + \varepsilon_t$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + b_0 \dot{o}_t + b_1 \dot{v}_t$$

Procedure	Dependent Variable	Intercept β_0	Lagged Prices(-1) β_1
OLS	\dot{p}_t	-0.00007 (0.00064) ^a	0.08447 ^{**} (0.02366)
ARCH	\dot{p}_t	0.00145 ^{**} (0.00055)	0.10063 ^{**} (0.02316)

Procedure	Dependent Variable	Intercept α_0	ε_{t-1}^2 α_1	Changes in Open Interest b_0	Changes in Volume b_1
ARCH	h_t	0.00055 ^{**} (0.00003)	0.01970 ^{**} (0.01130)	0.00020 ^{**} (0.00003)	-0.00025 ^{**} (0.00004)

TR² = 17.48

^a Standard errors are in parentheses.

^{*} and ^{**} indicate significance at the 5% and 1% levels, respectively.

1777 Observations

Figure 1. Forecast of Variance (ARCH)
Corn Futures Price

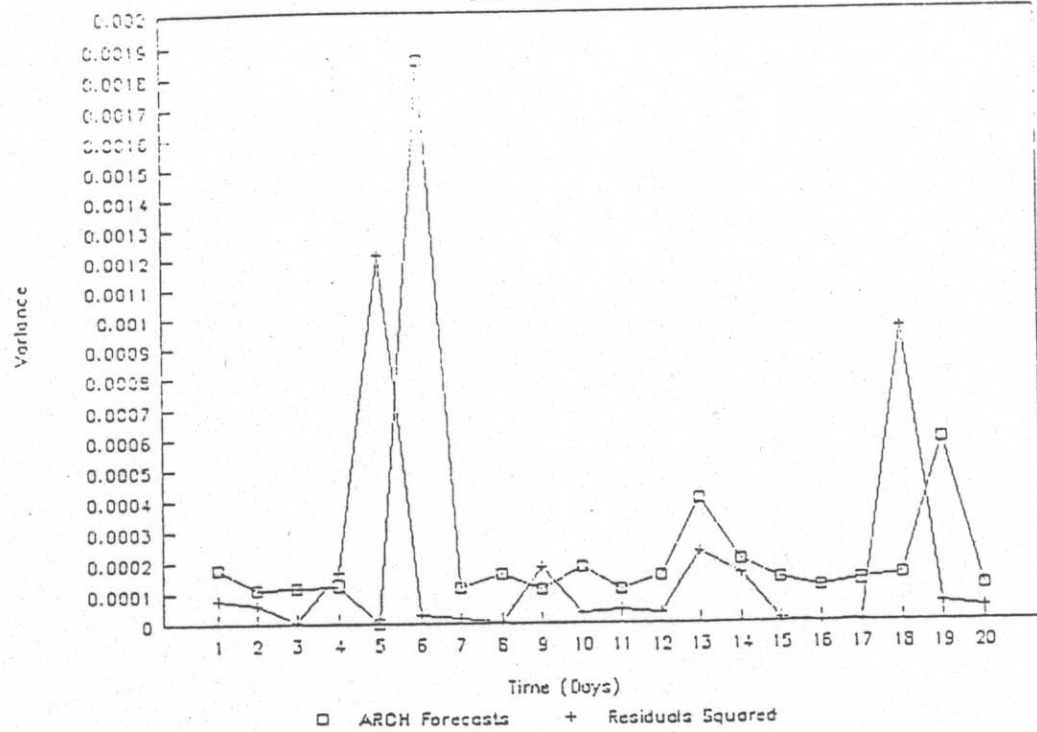


Figure 2. Forecast of Variance (ARCH)
Soybeans

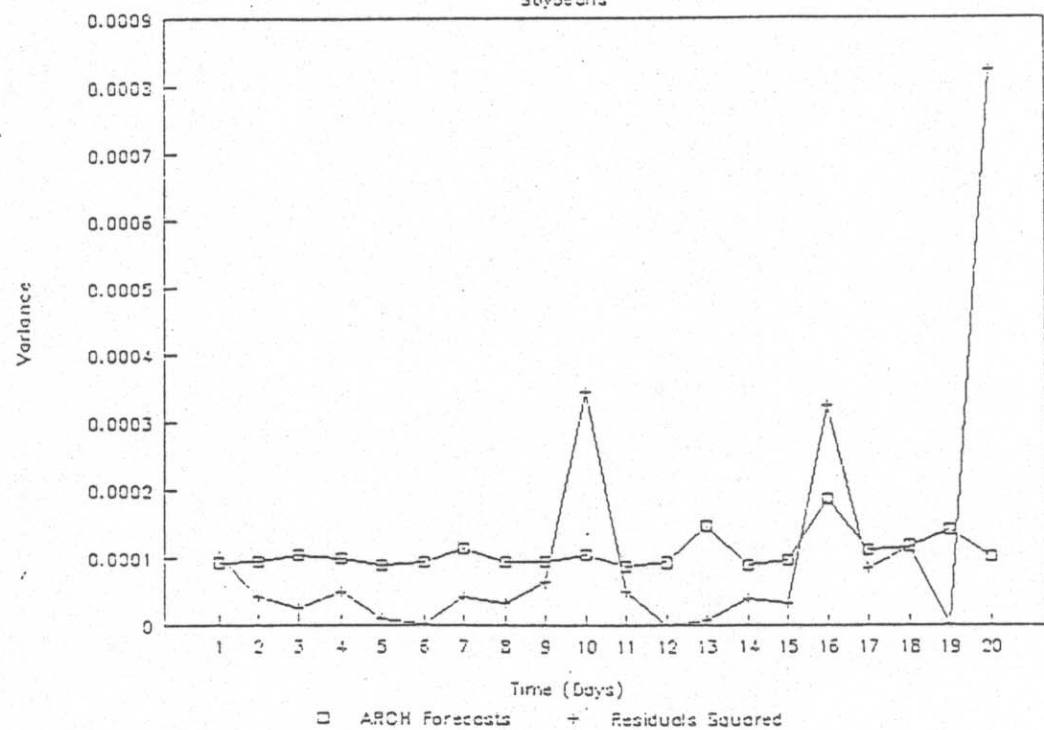
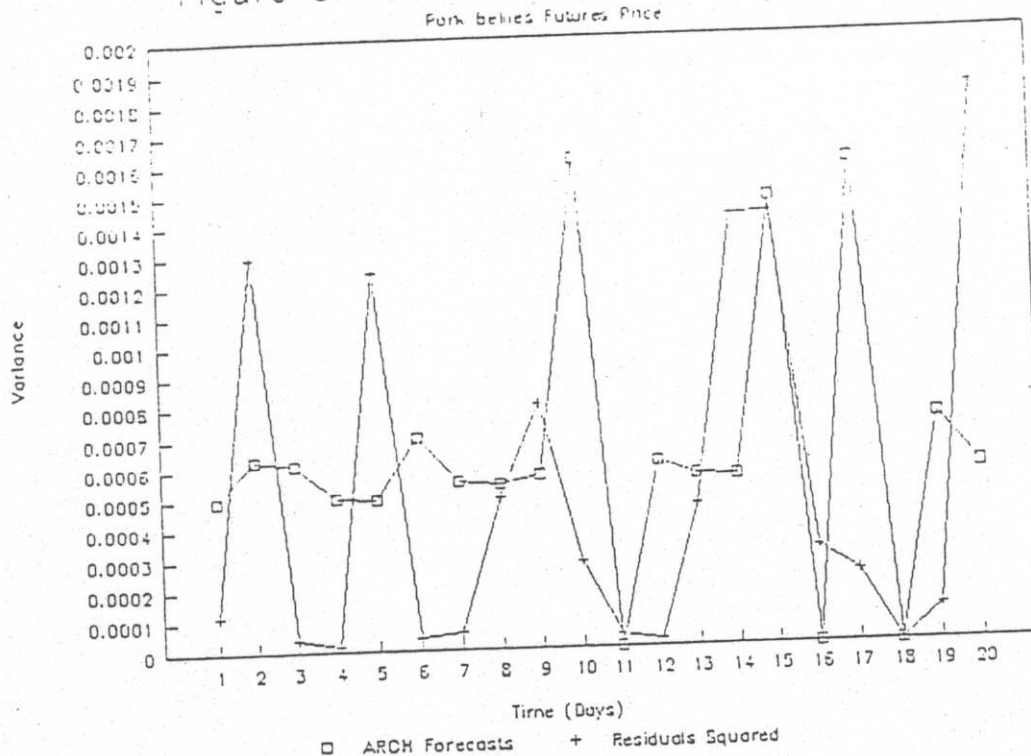


Figure 3. Forecast of Variance (ARCH)



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