The Market Efficiency of Options on Corn Futures

by

Sergio H. Lence

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The variety of financial instruments available for risk management increased with the introduction of institutional options markets in the 1970s, and of institutional markets for options on futures contracts in the 1980s. Options traded in regulated exchanges have been among the most successful financial developments in recent decades.

The popularity of options attracted the attention of the economics profession (see, for example, the selected bibliography listed in Cox and Rubinstein, p. 479-492). Among the issues that have concerned economists the most is whether options markets are efficient. Numerous studies have analyzed market efficiency for options on stocks (Bhattacharya; Frankfurter and Leung; Klemkosky and Resnick 1979; Klemkosky and Resnick 1980; Phillips and Smith; Halpern and Turnbull), options on price indexes (Evnine and Rudd, Chance), options on currencies (Bodurtha and Courtadon), and, more recently, options on futures (Ball and Torous; Followill and Helms; Ogden and Tucker; Wilson and Fung). Although the evidence provided by many of these studies favors the efficient market hypothesis, the results of several are at odds with market efficiency (e.g., Evnine and Rudd; Frankfurter and Leung; Halpern and Turnbull, Followill and Helms; Wilson and Fung).

The only study that addressed the efficiency of futures options for grains concluded that these markets are not efficient (Wilson and Fung). Wilson and Fung analyzed market efficiency for options on corn, soybeans, and wheat futures on the basis of two tests. The first test relied on the implied standard deviations calculated from puts and calls, which is a joint test of efficiency and of the particular option valuation model employed. The second test was based on the number of violations of the boundary restrictions imposed by riskless arbitrage, which is the standard test used in this type of studies. Wilson and Fung concluded that "riskless profits could likely be earned by traders even after transaction costs" (p. 65), and that both tests "suggest substantial evidence to reject the put-call parity condition in these markets" (p. 65). On the positive side, Wilson and Fung noted that the proportion of violations of arbitrage bounds in the second half of their data set was smaller than in the first half, suggesting an increase in efficiency as the markets become better established. They focused on the early stages of the option market (i.e., 1986 and 1987), and used one futures delivery position (the new crop contract). Both characteristics limit the extension of their conclusions to more developed stages of the market and/or to options on other futures delivery months.

The purpose of this study is to reexamine the efficiency of the market for options on corn futures. Corn is among the most important commodities in U.S. agriculture. The efficiency of agricultural futures options is of relevance because of the ongoing interest in substituting traditional price support programs with programs based on options. Over the years, there have been many advocates of this substitution (e.g., Gardner, Kahl). But recently, substitution has become much closer to reality as the government implemented a pilot program that gives producers puts on the December futures contract for corn.

We depart from the previous literature by using a much longer data series that covers all trading days and contracts since corn futures options began trading in February 1985 through to December 1992. The extended data series allows us to use both binary outcome (i.e., logit and probit) and standard regression models to analyze efficiency in terms of frequency and magnitude of violations of arbitrage bounds. We test whether efficiency exhibits patterns more or less unique to crops, such as trading and delivery seasonality. Seasonality in efficiency is of particular interest to producers, whose risk management strategy typically involves buying puts before harvest and buying calls later in the marketing year. Seasonality in efficiency is also of major concern to price support programs, as they would likely be based on particular futures delivery months (i.e., December). We also formally test the hypothesis that efficiency has increased with market

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maturity, as suggested by earlier studies (e.g., Wilson and Fung, Klemkosky and Resnick 1979). In addition, we explore the relationship between efficiency and market liquidity, price volatility, and daily price limits on futures and options. Knowledge of the causes of inefficiency is important in developing and implementing risk management strategies, and in selecting successful price support programs.

Arbitrage Bounds for Options on Futures

The arbitrage bounds for American options on futures contracts are summarized in Table 1. Conditions (1.1) through (1.5) were proven by Ramaswamy and Sundaresan, and condition (1.6) was shown by Ball and Torous. The proof of these bounds is based on the principle of riskless arbitrage. For example, if the lower bound on the value of the American futures call is violated, an individual would be able to make riskless profits by simultaneously buying the underpriced call, exercising it, and selling the underlying futures contract. Similarly, a violation of the lower bound on the value of the American futures put implies the possibility of realizing riskless profits by simultaneously buying the futures contract and the underpriced put, and exercising the latter. The assumptions underlying these conditions are not stringent; conditions (1.1) through (1.5) require investors to be price takers and to prefer more wealth to less, frictionless markets, and markets for pure discount bonds of every maturity; condition (1.6) requires the additional assumption of deterministic interest rates.

These bounds are the basic elements with which we examine the efficiency of the options market for corn futures. Options market efficiency has been traditionally assessed by analyzing the violations of arbitrage bounds such as (1.1) through (1.6) (e.g., Klemkosky and Resnick 1979; Halpern and Turnbull; Chance; Bodurtha and Courtadon; Ball and Torous; Ogden and Tucker; Wilson and Fung)[1] because these bounds require very reasonable assumptions. In particular, bounds (1.1) through (1.6) hold regardless of the option pricing model and the futures price dynamics. Frequent and/or large violations of these bounds provide strong evidence against market efficiency because they imply the possibility of realizing riskless profits by performing the right type of transactions.

Table 1. Arbitrage bounds for American options on futures contracts.

	UPPER BOUND		LOWER BOUND	
CALL	(1.1)	$C(t, T_O) \le F(t, T_F)$	(1.2)	$C(t, T_O) \ge \max[0, F(t, T_F) - K]$
PUT	(1.3)	$P(t, T_O) \le K$	(1.4)	$P(t, T_O) \ge \max[0, K - F(t, T_F)]$
PUT-CALL PARITY	(1.5)	$C(t, T_O) \le P(t, T_O) + F(t, T_F) - b(t, T_O) K$	(1.6)	$C(t, T_O) \ge P(t, T_O) + b(t, T_O) F(t, T_F) - K$

Note: $C(t, T_O)$ = value at date t of an American call option expiring at date T_O on a futures contract with price $F(t, T_F)$, $t < T_O \le T_F$, and with exercise price equal to K.
$F(t, T_F)$ = price at date t of a futures contract maturing at date T_F, $t \le T_F$.
$P(t, T_O)$ = value at date t of an American put option expiring at date T_O on a futures contract with price $F(t, T_F)$, $t < T_O \le T_F$, and with exercise price equal to K.
$b(t, T_O)$ = price at date t of a unit discount bond paying $1 at date T_O.

[1]Recall that conditions (1.1) through (1.6) are specific to futures options. For other types of options, riskless arbitrage imply analogous (but not identical) bounds.

Data

The data employed comprised the period between February 27, 1985 through December 31, 1992. The starting date marks the beginning of trading in options on corn futures at the Chicago Board of Trade (CBOT). Option premia and futures prices were obtained from the CBOT DataBank. The empirical tests on boundary conditions (1.1) through (1.6) were performed on a working closing range built from the CBOT DataBank's closing range, which consists of the highest and lowest quotations at the end of each trading day. Details of the construction of the working closing range are provided in the Appendix.

The working closing range was used to analyze efficiency under two scenarios, denominated *unfavorable* and *midrange* scenarios throughout the paper. The unfavorable scenario assumed that transactions were performed or could have taken place at the end of the working closing range that was most unfavorable to the arbitrageur. The midrange scenario, in contrast, assumed that transactions took place or could have been executed at the average of the upper and lower ends of the working closing range.[2] For example, to profit from a violation of the lower call condition (1.2) it is necessary to buy the underpriced call, exercise it, and sell the underlying futures contract. Hence, under the unfavorable scenario a violation of condition (1.2) occurs when

(2.1) $C_{high}(t, T_O) < \max[0, F_{low}(t, T_F) - K]$

where the subscript high (low) means that the price is that of the high (low) end of the working closing range. Under the midrange scenario, violations of condition (1.2) happen when

(2.2) $C_{midrange}(t, T_O) < \max[0, F_{midrange}(t, T_F) - K]$

where the subscript midrange symbolizes the average of the working closing range quotations. The midrange scenario contained fewer observations than the unfavorable scenario because the midrange was set to missing whenever either end of the working closing range was missing.

Violations of arbitrage bounds under the unfavorable scenario provide stronger evidence against market efficiency than under the midrange scenario. The reason for this assertion is that violations under the unfavorable scenario entail less holding risk and therefore better approximate riskless arbitrage conditions.[3] The purposes of using both scenarios were to verify the robustness of the results and to facilitate the comparison with previous studies, which have used either settlement prices (e.g., Wilson and Fung), transaction prices (e.g., Followill and Helms), or bid-ask quotes (e.g., Chance).

Prices for discount bonds were calculated from T-bill quotations using the formula in Cox and Rubinstein (p. 255). Bid and ask quotations for T-bills maturing immediately after the expiration of the corresponding options were taken from *The Wall Street Journal*.[4] Consistent with the approach employed with options and futures prices, the most unfavorable T-bill quote was used in the unfavorable scenario, and the midrange T-bill quote was used in the midrange scenario.

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[2]Prices are not continuous but discrete variables; according to CBOT rules, the minimum price fluctuation is 0.25 cents/bushel for corn futures and 0.125 cents/bushel for options on corn futures. Restrictions on minimum fluctuations mean that transactions could not have taken place at some midrange quotations. In this instance, the midrange scenario should be interpreted either as an average mispricing or as an expected profit from arbitrage.

[3]Holding risk is the risk of prices moving in the wrong direction during the time interval between the first and last transactions involved in the arbitrage. For example, an arbitrageur wishing to profit from a violation of condition (1.2) must buy the call and sell the underlying futures. If the arbitrageur buys the call first and the futures price falls before he sells the futures contract, he or she will experience a reduction in profits or even suffer a loss if the futures' fall is large enough. In practice, holding risk is important because it precludes the existence of completely riskless arbitrage opportunities, which is the assumption employed to obtain (1.1) through (1.6).

[4]Quotations for T-bills maturing immediately before option expiration were not used because they were generally not reported for days close to expiration (e.g., one week or less before option maturity).

The other data employed besides the quotations for options, futures, and T-bills were the volume of trade and the open interest in the options and futures markets. These data were also furnished by the CBOT DataBank.

The Frequency of Violations of Arbitrage Bounds

For each observation, the working closing range was used to verify whether bounds (1.1) through (1.6) were violated. Table 2 summarizes the results obtained under both the unfavorable and the favorable scenarios. Violations were classified into two groups according to whether they occurred on days in which the settlement price for any of the markets involved was set at the relevant price limit. We proceeded in this way to avoid biased results due to CBOT daily price limits on both options and futures for corn.

If daily price limits were not taken into account, inferences from bound violations alone could be biased towards rejecting the market efficiency hypothesis.[5] The reason for this assertion is that bound violations are not the same as market inefficiency. Bound violations can be interpreted as market inefficiency only if arbitrage can be performed. But in the presence of price limits, it is quite possible that at least one of the transactions involved in the arbitrage operation could not have been executed, thus precluding arbitrageurs from exploiting the mispricing. For example, a violation of the lower call bound (1.2) does not imply profits if futures closed at the lower futures limit and/or calls closed at the upper call limit. To profit from this violation requires selling futures and buying calls, but the price limit(s) could have prevented the arbitrageur from executing at least one of the required transactions.[6]

Table 2 reports the number and percentages of violations of bound conditions that occurred on days in which at least one of the settlement prices in the corresponding markets hit the relevant price limit. In aggregate, 20 (6) percent of the total number of violations took place at price limits in the unfavorable (midrange) scenario. These violations at price limits were eliminated from the data set for the remainder of the analysis. For simplicity, violations not occurring at price limits are just called violations in the rest of the paper.

As reported by other studies on American futures options (Ball and Torous; Ogden and Tucker; Wilson and Fung), we found no violations of the upper call and upper put bounds (1.1) and (1.3). The frequencies of violations of the other four bounds are similar to (albeit slightly smaller than) those reported by previous literature on futures options. Also, and possibly because of the much longer period covered by our data set, the frequencies shown in Table 2 lie in a much narrower interval (within each scenario) than the frequencies estimated by other authors. Specifically, the frequencies of violations range from 0.53 to 0.91 percent under the unfavorable scenario, and from 1.38 to 2.09 percent under the midrange scenario. By comparison, employing settlement prices Ball and Torous found violations ranging from 2.53 to 7.56 percent for gold, from 0.58 to 7.08 percent for Deutsche marks, and from 0.52 to 6.03 percent for sugar. Also using settlement prices, Wilson and Fung calculated between 1 and 6 percent violations for the corn December contract in 1986, and between 1 and 2 percent in 1987. Ogden and Tucker employed transactions data but obtained similar results; they observed violations occurring between 0.9 and 3.1 percent of the time in British pounds, between 0.2 and 1.7 percent in Deutsche marks, and between 0.6 and 2 percent in Swiss francs.

Violations of the lower put-call parity bound (1.6) do not seem more frequent than violations of the upper put-call parity bound (1.5). As stated earlier, condition (1.6) requires the additional assumption of deterministic interest rates, and could therefore be expected to be violated more often than condition (1.5). Nevertheless, the data do not support this hypothesis.

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[5]Inferences are not affected by daily price limits when using transactions data (as in Ogden and Tucker), but they may be affected otherwise (as in Ball and Torous, and Wilson and Fung). However, neither Ball and Torous nor Wilson and Fung mentioned whether they accounted for daily price limits.

[6]Obviously, a violation of bound (1.2) is synonymous of market inefficiency in the opposite situations of (i) futures closing at the upper futures limit and calls not closing at the upper call limit, and/or (ii) calls closing at the lower call limit and futures not closing at the lower futures limit.

Table 2. Number and percentage of violations of arbitrage bounds.

SCENARIO	ARBITRAGE BOUND	NO VIOLATIONS		VIOLATIONS				TOTAL NUMBER OF OBSERVATIONS
				At Price Limit		Not at Price Limit		
		Number	Percent	Number	Percent	Number	Percent	
Unfavorable	Upper Call	53597	100.00	0	0.00	0	0.00	53597
	Lower Call	53984	99.31	86	0.16	290	0.53	54360
	Upper Put	35565	100.00	0	0.00	0	0.00	35565
	Lower Put	33743	99.08	2	0.01	312	0.91	34057
	Upper Put-Call	21884	99.12	62	0.28	133	0.60	22079
	Lower Put-Call	23463	99.18	63	0.27	130	0.55	23656
Midrange	Upper Call	42625	100.00	0	0.00	0	0.00	42625
	Lower Call	42229	98.56	23	0.06	592	1.38	42844
	Upper Put	26189	100.00	0	0.00	0	0.00	26189
	Lower Put	25631	97.90	1	0.01	548	2.09	26180
	Upper Put-Call	14199	97.91	46	0.32	257	1.77	14502
	Lower Put-Call	14189	97.84	34	0.24	279	1.92	14502

In contrast to the methodology used by the aforementioned authors, we did not subtract the transaction costs involved in the execution of arbitrage. This omission means that the frequencies of violations in Table 2 are upwardly biased. For reasons to be discussed in the next section, however, this upward bias is likely to be small.

A Binomial Outcome Model of the Frequency of Violations

The frequency of violations of arbitrage bounds may be related to a number of factors. For example, factors affecting the extent up to which actual market conditions mimic riskless arbitrage conditions are likely to have an impact on the frequency of bound violations, as riskless arbitrage is a key assumption to derive these bounds. Differences in the characteristics of participants in the options and futures markets throughout the year may also be responsible for seasonality in market efficiency. The existence of seasonal patterns in efficiency is of practical interest to market participants. Seasonality in efficiency is also relevant for price support programs.

To investigate the determinants of the frequency of violations, we fitted the following model to the binary outcome violation-no violation:

$$(3.1)\quad g[\text{Prob(No Violation)}] = \alpha_0 + \alpha_1 \text{ Days to Option Expiration} + \alpha_2 \text{ Minimum Open Interest} + \alpha_3 \text{ Minimum Volume} + \alpha_4 \text{ ACHF} + \sum_i \alpha_{5i} \text{ DYi} + \sum_j \alpha_{6j} \text{ DSj} + \sum_k \alpha_{7k} \text{ DDk}$$

where: ACHF = absolute change in the futures settlement price
DYi = 1 if year = i, DYi = 0 otherwise (i = 1985, 1986, 1987, 1988)
DSj = 1 if season = j, DSj = 0 otherwise (j = April-June, July-August, September-November)
DDk = 1 if delivery month = k, DDk = 0 otherwise (k = May, July, September, December)

Prob(No Violation) denotes the probability of no violation of the arbitrage bound. Hence, Prob(Violation) ≡ 1 - Prob(No Violation).

The function g(x) is defined as $g(x) \equiv \log[x/(1-x)]$ for the logit model, and as $g(x) \equiv \Phi^{-1}(x)$ for the probit model, where $\Phi^{-1}(x)$ is the inverse of the cumulative standard normal distribution function. Both logit and probit models were employed because there is no theoretical justification to choose one over the other. Furthermore, these models may yield different results when the probabilities involved are close to zero or one (Greene, p. 666), as it was the case here. Fitting both models allowed us to check the robustness of the results.

The number of days left to expiration is likely to affect the probability of violations in two opposing ways. First, options with fewer days left to expiration are more likely to violate bounds (1.2), (1.4), (1.5), and (1.6). Violations of bounds (1.2) and (1.4) can be expected to occur more often near expiration because the value of options decrease as options approach expiration. Violations of both (1.5) and (1.6) can also be expected to be more frequent with fewer days left to expiration because the price of the unit discount bond $b(t, T_O)$ approaches one as the bond nears maturity. As bond prices tend to unity, boundary conditions (1.5) and (1.6) collapse towards the equality $C(t, T_O) = P(t, T_O) + F(t, T_F) - K$. Second, violations are more likely to be found far from expiration because options with more days left to expiration tend to be less liquid, implying less competitive and less efficient markets.

Minimum open interest (minimum volume) is the smallest open interest (volume) among the options and futures markets involved, measured in contracts/1000. These two variables represent the depth of the market. The inclusion of these two variables allows us to control for the second effect of days to expiration mentioned in the previous paragraph. The probability of violations is expected to be negatively related to both the minimum open interest and the minimum volume because thin markets hamper the successful execution of arbitrage. It is important to note, however, that these two variables reflect the depth of the market for bounds (1.2) and (1.4) better than for bounds (1.5) and (1.6). In the latter two bounds, arbitrage involves in addition the bond market, for which there were no liquidity data available.

The absolute change in the settlement price of the futures contract (ACHF) is a proxy for the expected volatility of futures prices. Option premia increase with the expected volatility of the underlying futures price. Hence, the frequency of violations of the lower bounds (1.2) and (1.4) is likely to be negatively associated to the absolute change in the futures settlement price.[7] The effect of ACHF on violations of lower bounds (1.5) and (1.6), if any, is ambiguous.

The dummy variables accounting for years 1985 through 1988 are included in (3.1) to test the hypothesis that efficiency increased as the market matured. Klemkosky and Resnick (1979) and Wilson and Fung found some indications of violations diminishing over time, but their short data set precluded them from testing the significance of this effect. The only study that formally tested this hypothesis is Halpern and Turnbull. These authors analyzed stock options over a much shorter data period (2 years) than ours. Interestingly, they found that violations increased rather than decreased over time.

The dummy variables DSj and DDk are used to test for patterns in efficiency related to the seasonality of corn supply and demand. In defining DSj, the year was divided into the four seasons with most distinctive market conditions. The dummy variables DDk were set to identify each delivery month. Like most agricultural commodities, corn is characterized by a highly seasonal supply. This affects the behavior of prices and also the composition of participants in both futures and options markets throughout the year. For example, prices are much more volatile during the growing season, which may increase the number of bound violations as arbitrageurs need time to exploit arbitrage opportunities. The presence of such patterns would be uncovered by the dummy variables accounting for seasonality and/or delivery months.

The results obtained from the logit model are reported in Table 3 for the unfavorable scenario, and in Table 4 for the midrange scenario. The probit model had a very similar fit, and is omitted to save space.[8] The results from the unfavorable scenario generally agree with those from the midrange scenario, but there are exceptions (e.g., in the lower put equation, DDMay is negative and not significant in the unfavorable scenario, but positive and significant at the 1 percent level in the midrange scenario). The indices of rank correlation (i.e., Somers' D, Goodman-Kruskal γ, and c) show that the fitted models have good predictive ability, especially for the upper and lower put-call parity bounds (1.5) and (1.6).

The probability of violations is negatively related to the number of days left to expiration. This relationship is significant at the 1 percent level in all bounds and scenarios. If the liquidity effect is not controlled for (i.e., if minimum open interest and minimum volume are excluded from the equations), the explanatory power of days to maturity for bounds (1.2) and (1.4) decreases but not enough to change its sign or to render it nonsignificant. Hence, the first effect of days to expiration discussed earlier, dominates the second effect.

The coefficients corresponding to minimum open interest and minimum volume have the expected signs and are significant at the 1 percent level in the equations for bounds (1.2) and (1.4). Minimum open interest and minimum volume are not significant in the equations for bounds (1.5) and (1.6), with the exception of minimum open interest in the lower put-call bound (1.6). This result is not surprising because, as mentioned before, these two variables do not capture the liquidity conditions of the bond market.

As expected, the probability of violations of bounds (1.2) and (1.4) was significantly negatively affected by the absolute change in futures settlement prices. As hypothesized, this explanatory variable was generally nonsignificant in the equations for the put-call parity bounds (1.5) and (1.6).

The data do not support the hypothesis that the probability of violations decreased with the maturity of the options market. The coefficients of the dummy variables accounting for the early years of this market are very erratic both in significance and sign. In 1988, for example, violations of the lower call bound (1.2) and of the lower put-call parity bound (1.6) were significantly more

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[7]Recall that violations of lower bounds (1.2) and (1.4) occur when the respective option premium is low in relative terms.

[8]These results are available on request.

Table 3. Logit model of no violations of arbitrage bounds under the unfavorable scenario.[a]

EXPLANATORY VARIABLE	ARBITRAGE BOUND			
	Lower Call	Lower Put	Upper Put-Call	Lower Put-Call
Intercept	3.30**	2.62**	3.32**	2.21**
	(223.35)	(128.73)	(62.15)	(30.07)
Days to option expiration	0.00816**	0.01100**	0.0915**	0.110**
	(120.22)	(173.60)	(131.43)	(115.02)
Minimum open interest	0.789**	1.060**	1.12	0.244**
	(87.07)	(118.21)	(2.26)	(7.69)
Minimum volume	4.1**	2.9**	-0.32	-0.58
	(12.48)	(6.98)	(0.12)	(1.37)
ACHF	0.089**	0.059*	-0.082*	0.052
	(12.84)	(4.03)	(4.79)	(2.03)
DY1985	0.63*	0.61**	0.20	0.13
	(5.20)	(10.23)	(0.23)	(0.08)
DY1986	1.55**	0.26	-0.35	-0.45
	(13.68)	(2.83)	(1.08)	(2.02)
DY1987	1.13**	1.77**	-1.33**	-0.07*
	(10.65)	(38.92)	(29.41)	(6.58)
DY1988	-1.29**	1.11**	0.48	-1.13**
	(98.80)	(26.74)	(2.58)	(18.92)
DSApril-June	-0.44**	0.50*	-1.25	-0.8
	(7.19)	(5.47)	(1.59)	(0.58)
DSJuly-August	-0.22	-0.80**	-3.3	2.3
	(1.27)	(18.23)	(1.85)	(3.14)
DSSeptember-November	0.13	-0.61**	-2.6	1
	(0.57)	(12.25)	(1.16)	(0.00)
DDMay	0.04	-0.17	0.6	0.9
	(0.04)	(0.55)	(0.39)	(0.70)
DDJuly	-0.03	-0.70**	0.4	0.5
	(0.02)	(9.67)	(0.14)	(0.16)
DDSeptember	0.35	-0.49*	1.4	-3.6**
	(3.01)	(5.55)	(0.32)	(6.94)
DDDecember	-0.54**	-0.74**	1.3	-0.4
	(8.08)	(15.27)	(0.29)	(0.00)
Log-Likelihood	-1549.605	-1479.0945	-500.19	-469.708
Somers' D	0.683	0.719	0.920	0.683
Goodman-Kruskal γ	0.729	0.743	0.939	0.729
c	0.841	0.860	0.960	0.841

[a]Wald χ^2 statistics are shown within parentheses.
* (**) Significantly different from zero at the 0.05 (0.01) level on the basis of the Wald χ^2 test.

Table 4. Logit model of no violations of arbitrage bounds under the midrange scenario.[a]

EXPLANATORY VARIABLE	ARBITRAGE BOUND Lower Call	Lower Put	Upper Put-Call	Lower Put-Call
Intercept	1.18**	0.72**	1.92**	0.32
	(57.68)	(13.85)	(43.95)	(1.46)
Days to option expiration	0.01710**	0.0225**	0.1031**	0.1414**
	(609.12)	(458.14)	(232.27)	(255.94)
Minimum open interest	0.747**	0.640**	-0.25	0.110*
	(305.64)	(215.81)	(0.63)	(5.48)
Minimum volume	1.63**	3.44**	0.37	-0.43
	(14.63)	(32.51)	(0.73)	(2.08)
ACHF	0.055**	0.074**	-0.031	0.016
	(7.56)	(8.35)	(0.96)	(0.26)
DY1985	1.10**	0.74**	0.56	0.11
	(31.16)	(25.43)	(2.24)	(0.10)
DY1986	1.22**	0.51**	-0.48	-0.32
	(37.70)	(15.18)	(3.69)	(1.38)
DY1987	1.62**	1.62**	-1.22**	-0.56*
	(39.42)	(47.51)	(30.56)	(4.60)
DY1988	-0.56**	2.12**	0.16	-0.91**
	(28.73)	(71.29)	(0.49)	(20.17)
DSApril-June	-0.37**	0.39	-1.9	-0.87
	(9.17)	(3.31)	(3.18)	(1.86)
DSJuly-August	-0.20	-0.98**	-1.6	2.98**
	(1.53)	(23.56)	(1.21)	(10.96)
DSSeptember-November	0.13	-0.69**	-1.2	9.2**
	(0.84)	(13.54)	(0.39)	(28.90)
DDMay	-0.002	1.12**	1.5	0.74
	(0.00)	(17.35)	(2.09)	(1.41)
DDJuly	-0.38*	-0.21	0.9	0.84
	(6.34)	(0.86)	(0.67)	(1.50)
DDSeptember	0.67**	0.39	-0.1	-3.87**
	(15.10)	(3.49)	(0.01)	(17.47)
DDDecember	-0.79**	-1.07**	-2.3	-8.7**
	(25.93)	(41.73)	(1.37)	(26.30)
Log-Likelihood	-2460.9085	-1864.508	-720.0695	-694.068
Somers' D	0.727	0.794	0.919	0.942
Goodman-Kruskal γ	0.748	0.809	0.933	0.945
c	0.863	0.897	0.959	0.971

[a]Wald χ^2 statistics are shown within parentheses.

* (**) Significantly different from zero at the 0.05 (0.01) level on the basis of the Wald χ^2 test.

frequent than in the period 1989-1992, but violations of the lower put bound (1.4) were significantly less frequent than in later years.

Similarly, there does not seem to be any clear seasonality pattern in the probability of violations, with the only possible exception of options on the December futures contract. Options on the December contract exhibit significantly more violations than options on other contracts in seven out of the eight bound-scenario combinations, five of which are significant at the one percent level. This result is important in qualifying the conclusions by Wilson and Fung. In their analysis of corn futures options, these authors only employed data for the December futures contract.

The tendency for options on the December contract to present more frequent violations is somewhat disturbing because the December contract is the most heavily traded. For the typical risk management strategy of corn producers, however, the implications of this finding are not clear. Producers tend to buy puts on the December futures contract to secure a minimum price for their crop. Hence, a straightforward conclusion would be that producers benefited from such inefficiency because premia for puts on December futures were more frequently underpriced than premia for puts on other contracts. But a deeper implication could be that options on December futures were more inefficiently priced in general than options on other contracts. If this is the case, it is quite possible that producers lost because of inefficiencies. Similar conclusions apply to the proposed use of puts on the December futures contract as substitutes for the current price support programs.

The erratic signs and significance of the coefficients corresponding to the dummy variables accounting for early years, seasons, and futures delivery months highlight the need for an extended data set before attempting to draw general inferences regarding the efficiency of these markets. This observation is of particular relevance, given that earlier literature has consistently used data corresponding to relatively short periods and/or to specific futures delivery positions.

The Magnitude of the Violations of Arbitrage Bounds

Cumulative Frequencies

The observations violating bounds (1.1) through (1.6) (and not occurring at price limits) were used to analyze the magnitude of mispricings. The size of the violations provides another way of assessing market efficiency because, other things equal, larger mispricings imply greater potential to profit from arbitrage.

Table 5 contains cumulative sample frequencies at selected mispricing levels, and also the largest violation for each bound and scenario. Mispricings of $ 6.25 per contract (0.125 cents/bushel) and $ 12.5 per contract (0.25 cents/bushel) were selected because the former is the minimum fluctuation possible for corn options and the latter is the minimum fluctuation possible for corn futures. This is the case because the CBOT imposes minimum price fluctuations of 0.125 cents/bushel on corn options and of 0.25 cents/bushel on corn futures.

The large size of the minimum fluctuations relative to transaction costs was the main reason for omitting the latter when calculating violations. Mispricings implied by violations of bounds (1.1) through (1.4) must be at least 0.125 cents/bushel because of the minimum price fluctuation permitted. Hence, transaction costs below 0.125 cents/bushel are irrelevant for computing violations of these four bounds. For comparison, Wilson and Fung used transaction costs of 0.0348 cents/bushel. For bounds (1.5) and (1.6), transaction costs of less than 0.125 cents/bushel may be relevant. However, transaction costs as small as 0.0348 cents/bushel had little effect on the observed violations of these two bounds.

According to Table 5, for example, 2.4 percent of the violations of the lower call bound (1.2) in the unfavorable scenario were of $ 6.25 per contract or less, and 54.5 percent were of $ 12.5 per contract or less. This means that arbitrageurs wishing to profit from violations of the lower call bound (1.2) would have seen their profits vanish 54.5 percent of the time by a single tick move of futures in the wrong direction.

With the exception of violations of the lower call and lower put bounds in the unfavorable scenario, a single tick move of the option price in the wrong direction was enough to eliminate about one third or more of the profitable opportunities. In all instances, approximately one half or more of the profitable opportunities were eliminated by a single tick move of futures in the wrong

Table 5. Cumulative sample frequencies of deviations from arbitrage bounds.

SCENARIO	ARBITRAGE BOUND	CUMULATIVE SAMPLE FREQUENCY (%) Deviations from Arbitrage Bounds ($/contract)				MAXIMUM DEVIATIONS FROM ARBITRAGE BOUNDS ($/contract)
		≤ 6.25	≤ 12.5	≤ 25	≤ 50	
Unfavorable	Lower Call	2.4	54.5	75.9	89.0	625
	Lower Put	1.9	52.2	76.3	88.8	475
	Upper Put-Call	45.1	65.4	82.0	91.7	178
	Lower Put-Call	37.7	46.9	72.3	86.9	396
Midrange	Lower Call	32.1	57.8	79.4	93.8	500
	Lower Put	41.6	58.8	80.7	94.9	181
	Upper Put-Call	33.5	59.1	83.7	95.7	210
	Lower Put-Call	35.8	62.0	78.8	92.1	158

direction. These figures reveal that most of the observed mispricings were very small. In addition, they show that a substantial proportion of violations occurred under conditions not resembling riskless arbitrage conditions, which is the key assumption employed to derive bounds (1.1) through (1.6).

A different perspective is provided by the maximum deviations from arbitrage bounds. These reveal that, albeit very infrequent, there were a few occasions where large profits could have been made from arbitrage. This observation is most apparent in the unfavorable scenario, and in particular for the lower call bound (1.2). In the latter case, the largest violation exceeded the arbitrage bound by \$ 625 per contract, or one hundred times the minimum fluctuation allowed for option premia.

A Regression Model of the Magnitude of Violations

Following the rationale employed to select the explanatory variables in (3.1), it can be concluded that these variables may impact the magnitude of the violations as well. To gain a better understanding of the determinants of the size of mispricings, we fitted (4.1) by means of ordinary least squares:

$$(4.1) \quad \ln(MV) = \beta_0 + \beta_1 \text{ Days to Option Expiration} + \beta_2 \text{ Minimum Open Interest} + \beta_3 \text{ Minimum Volume} + \beta_4 \text{ ACHF} + \sum_i \beta_{5i} \text{ DY}i + \sum_j \beta_{6j} \text{ DS}j + \sum_k \beta_{7k} \text{ DD}k$$

MV is the magnitude of the violations. Logarithms were used in the left-hand side of (4.1) because mispricings are very skewed.

To save space, only the results corresponding to the midrange scenario are reported in Table 6. The explanatory power of the model was very low, as reflected by R^2s ranging from 0.15 to 0.26. The dummy variables accounting for the early years of the option market were generally nonsignificant, implying that the size of mispricings did not change over time. Similarly, the magnitude of violations did not exhibit seasonality and did not seem to be different among alternative futures delivery months.

The variables that were most important in explaining the frequency of violations (i.e., days to expiration, minimum open interest, minimum volume, and ACHF) did a poor job of explaining the magnitude of violations. Furthermore, when significant, the coefficients associated with days left to expiration and with the absolute change in the futures settlement price had the wrong sign.

A careful examination of the data, however, reveals that the nonsignificance of the liquidity variables can be attributed to the inadequacy of the equality model (4.1) to capture the nature of the relationship between mispricing size and liquidity. To help explain this observation, we have attached Figures 1 and 2. Figure 1 depicts mispricings against minimum open interest, and Figure 2 shows violations against minimum volume for the lower put bound (1.4) under the midrange scenario. Other combinations of bounds and scenarios exhibited the same patterns, and are omitted to save space. Figures 1 and 2 show clearly a negative relationship between the magnitude of violations and market liquidity. Large violations occurred only when the market was very illiquid, in terms of both open interest and volume. In contrast, small violations were observed in both liquid and thin market conditions. Model (4.1) is unable to capture the negative relationship between mispricing size and liquidity because the equality sign in (4.1) requires small violations to occur only under thin market conditions. Unfortunately, standard econometric models are unable to reflect well the negative relationship between mispricing size and liquidity that is evident from Figures 1 and 2.

Concluding Remarks

Based on the analysis of violations of boundary conditions implied by riskless arbitrage, the options market for corn futures appeared to be reasonably efficient from 1985 through 1992. The frequency of violations observed was slightly smaller than for other futures options markets. Furthermore, approximately one half or more of the violations computed disappeared with a single tick move of the futures price in the wrong direction. The small magnitude of most violations

Table 6. Log-linear regression model of size of violations of arbitrage bounds under the midrange scenario.[a]

EXPLANATORY VARIABLE	ARBITRAGE BOUND: Lower Call	Lower Put	Upper Put-Call	Lower Put-Call
Intercept	-1.67** (-16.63)	-1.50** (-11.49)	-2.09** (-6.23)	-2.11** (-7.65)
Days to option expiration	0.00197** (4.24)	0.00238** (4.04)	0.0043 (0.89)	-0.0067 (-0.73)
Minimum open interest	0.033 (1.41)	-0.088** (-3.73)	-0.048 (-1.11)	-0.11* (-2.10)
Minimum volume	-0.44* (-2.10)	0.18 (0.66)	-0.19 (-0.62)	0.44 (1.40)
ACHF	0.047** (3.07)	0.040* (1.96)	0.087* (2.44)	0.041 (1.28)
DY1985	0.03 (0.25)	-0.005 (-0.05)	0.22 (0.56)	-0.04 (-0.12)
DY1986	-0.23 (-1.62)	-0.19* (-2.12)	0.23 (0.86)	0.02 (0.07)
DY1987	-0.04 (-0.22)	-0.03 (-0.17)	0.19 (0.86)	0.32 (1.19)
DY1988	0.43** (5.71)	0.28 (1.56)	0.36 (1.45)	0.64** (3.08)
DSApril-June	0.09 (0.99)	-0.04 (-0.24)	-0.64 (-0.84)	0.44 (0.60)
DSJuly-August	0.28* (2.44)	0.02 (0.10)	-1.30 (-1.49)	-0.28 (-0.27)
DSSeptember-November	-0.12 (-1.30)	-0.13 (-1.01)	-0.80 (-0.57)	1.07 (0.52)
DDMay	-0.06 (-0.59)	0.18 (0.93)	0.48 (0.63)	-0.63 (-0.86)
DDJuly	-0.08 (-0.71)	0.30 (1.65)	0.56 (0.70)	-0.31 (-0.39)
DDSeptember	0.21 (1.68)	0.35* (2.42)	1.68 (1.94)	1.00 (0.91)
DDDecember	0.22 (1.94)	0.06 (0.49)	0.89 (0.64)	-1.22 (-0.59)
R^2	0.26	0.15	0.16	0.23

[a] *t*-statistics are shown within parentheses.

* (**) Significantly different from zero at the 0.05 (0.01) level on the basis of the *t*-test.

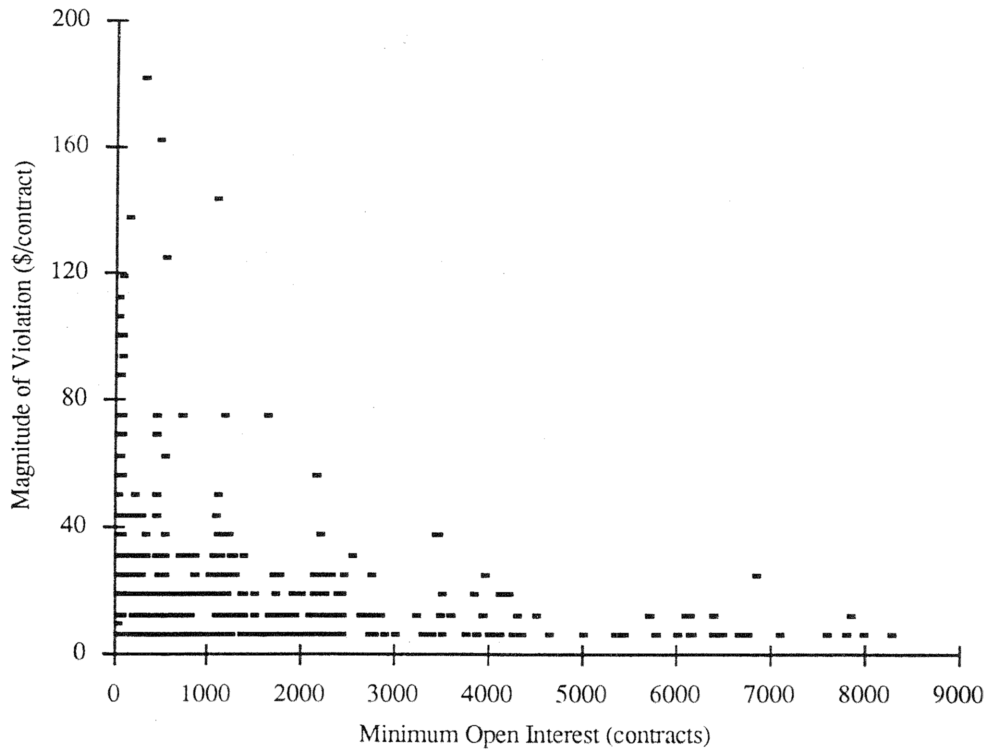

Figure 1. Relationship between magnitude of violations and minimum open interest for the lower put bound (1.4) under the midrange scenario.

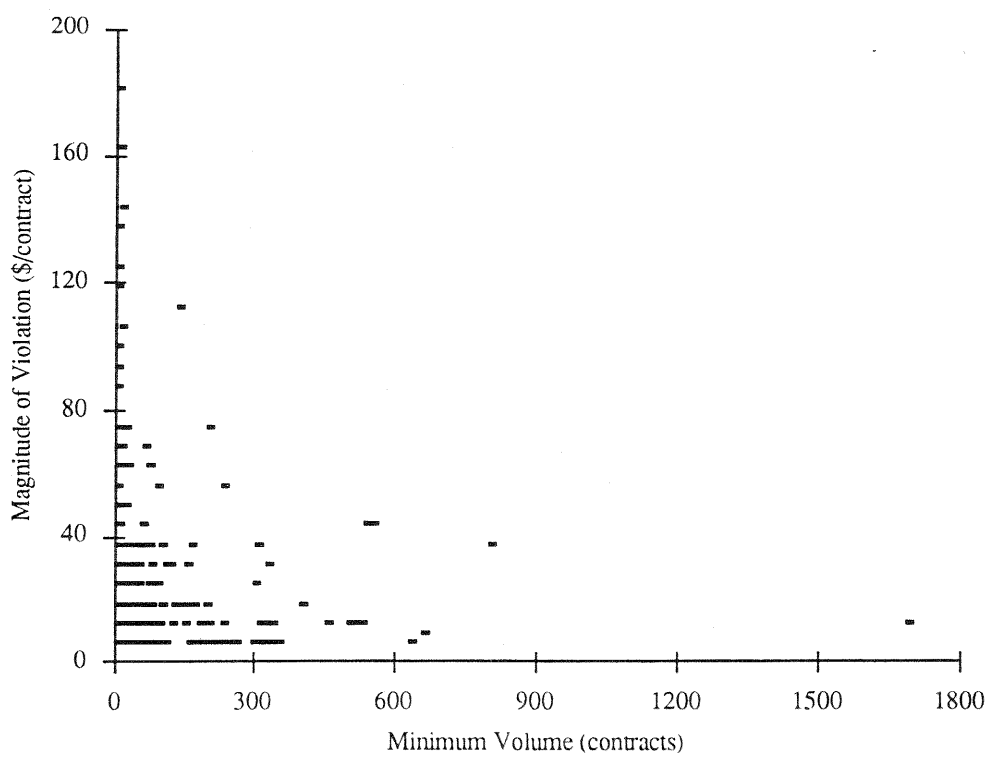

Figure 2. Relationship between magnitude of violations and minimum volume for the lower put bound (1.4) under the midrange scenario.

made them risky for arbitrage purposes, thus inhibiting the execution of arbitrage that could have eliminated them.

The hypothesis that the market was not efficient cannot be discarded because there were a few occasions in which very large profits could have been made from arbitrage. However, strong evidence was found linking the frequency and magnitude of bound violations of the liquidity of the market. More specifically, (i) the probability of violations was significantly higher when markets were thin, and (ii) violations were large only in the presence of illiquid markets. Therefore, bound violations seem more related to the departure of actual market conditions from riskless arbitrage conditions than to the failure of the arbitrage mechanism.

Efficiency did not seem to increase with the maturity of the options market. No support was found for the hypothesis that efficiency varies throughout the year or with different underlying futures delivery months. There is some evidence, however, of bound violations being more frequent for the December futures contract than for other delivery months. These findings are relevant for the risk management strategies of producers, and also for the implementation of government support programs based on options.

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Appendix

Quotations in the CBOT DataBank's closing range are either actual trades, bids, asks, and/or nominal quotes. The working closing range was constructed following the rules exemplified in Table A1. The first two columns of this table show hypothetical closing ranges as reported by the CBOT DataBank, and the last two columns show how they translated into the working closing range. The nature of each hypothetical CBOT DataBank's quotation appears within parentheses. For example, consider the fourth row in Table A1. The first column says that at the end of the trading day there was an offer to sell at 100, and the second column says that there was no buying interest at the closing. This information shows up as a missing value in the lower end and as a quotation of 100 in the upper end of the working closing range.

Table A1. Hypothetical example of working closing range quotations based on quotations from the CBOT DataBank's closing range.

CBOT DATA BANK'S CLOSING RANGE		WORKING CLOSING RANGE	
		Low	High
100 (trade)	0	100	100
100 (trade)	101 (trade)	100	101
100 (bid)	101 (ask)	100	101
100 (ask)	0	missing	100
100 (bid)	0	100	missing
100 (nominal)	0	missing	missing

Option premia on expiration days were also eliminated from the working closing range. We did so because to prevent nonsimultaneous quotations, as trading in the expiring options stops more than one hour earlier than trading in the underlying futures contract.