Chaos Theory and Its Implications for Research on Futures Markets: A Review of the Literature

by

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CHAOS THEORY AND ITS IMPLICATIONS FOR RESEARCH ON FUTURES MARKETS: A REVIEW OF THE LITERATURE

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A desire to discover the price generating function is a major goal of studies concerned with futures markets. Whether spurred by the urge to learn or the desire to get rich, researchers and market analysts have expended great time and effort on the quest. Over time, some have reached the conclusion that price movements are the equivalent of a random walk; others believe future price movements can be predicted by charting or other existing techniques. Recently, chaos theory has emerged as a framework potentially valuable in the search for a price generating function. In this paper, we review some of the recent literature on chaos theory and comment on possible applications useful in the study of futures markets. As part of our review, we hope to increase understanding of chaos theory, both its potential and its limits, for those considering using it in economic studies.

Examples of chaos, unlike other sciences which may seem remote to the average person, can be seen all around us in nature. Chaos theory explains the way a flag flaps in the wind, a column of smoke rises from a fire, even the way a bolt of lightning streaks across the sky. Understanding that these phenomena, which appear rather haphazard and random, are actually completely deterministic, are key to accepting the ideas of chaos. Chaotic systems are, as Gleick (1987) suggested, "...order masquerading as randomness."

The principles of chaos were first discovered and noted by meteorologist Edward Lorenz in his 1963 paper "Deterministic Non-Periodic Flow." Lorenz's title, while possessing neither the catchiness nor the intrigue of the later term 'chaos', is a much more descriptive way to think of this theory of nonlinear dynamics. The primary descriptors identifying a chaotic system are illustrated in the concept map in Figure 1. The left hand side of the Figure shows the three key features of a chaotic system; they are nonlinear, deterministic and non-periodic. Nonlinear systems are those that contain relations beyond the common, simple first-degree terms in linear equations and are computationally more difficult to work with, while a deterministic system is one in which all future changes in value have specific, nonrandom causes. Finally, non-periodic systems continue generating values into infinity without ever converging to a single value or to a repeating pattern. Of these three properties, the combination of deterministic with non-periodic systems is unique since, prior to this, non-periodic behavior has been associated with stochastic, or random, processes. The non-periodic behavior of futures prices, for instance, was responsible for the random walk theory. Economic researchers, according to Weiss (1991) typically assume their models "will ultimately achieve

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either an equilibrium, a cyclic pattern, or some other orderly behavior." Instead, chaos theory suggests that not only are non-periodic models possible but in some cases may be more accurate than alternatives.

Lorenz discovered chaos in 1961 while running a weather simulation model. He accidentally neglected to type in the last decimal places on his variables when performing a duplicate run, and although the numbers only varied after the third decimal place, the results of the two simulations rapidly diverged. This phenomenon, one of the key elements of a chaotic system, came to be called sensitive dependence on initial conditions. In fact, chaotic functions are so sensitive that long run forecasts are deemed impossible (consider trying to get exact values for any parameter without any round-off error!). Thus, as shown in Figure 1, both the random walk theory and the sensitive dependence element of chaos theory imply that long term forecasts are not feasible.

Lorenz published his findings in the Journal of Atmospheric Sciences, which unfortunately did not have a wide audience, delaying somewhat the development of chaos theory. Later papers by others, such as Li and Yorke (1975), and May (1975), rapidly changed this. Li and Yorke showed a method of identifying chaotic functions by proving that any function that converged to a cycle of three points also had cycles of all other lengths. May brought the ideas of chaos to population research.

As can be seen from the above examples, interest in chaos was not confined to any one discipline. Researchers in a number of fields including physics, mathematics, biology and economics soon began to examine how chaos theory related to particular disciplines. To understand the interest shown by economists, one needs only to look at a graph of a price series over time. Examine any typical price series for a futures contract. Notice its up and down movements, its jaggedness, how it seems sometimes to almost have a pattern and yet still seem random.[2] This type of shape can suggest nonlinearity and non-periodicity, two of the key conditions that chaos theory incorporates. In this sense, attempting to use chaos to model futures prices is a natural outcome of this new theory.

Before the discovery of chaotic functions, patterns that seemed to vary without apparent reason were considered to be random and in fact, the dominant theory up to this time about price generation has been the random walk theory. As shown at the bottom of Figure 1, in the random walk theory price changes are independent, and therefore, past prices do not contain any useful information about future prices. Lorenz's weather simulation, however, revealed that sometimes seemingly random patterns could be generated by deterministic functions. In fact it is interesting that the same lack of regular, cyclical or predictable price movements led to the

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[2]Shapes of this sort are said to have fractal, or non-integer dimensions. The term fractal was coined by Benoit Mandelbrot in 1975 to describe shapes more complicated than their integer dimension suggests but less than the next highest integer dimension. For instance, a typical price series line is considered to have a dimension higher than one, but less than two. Fractal analysis (examining a price series by its varying dimension over time) may be another tool technical analysts could use in searching for patterns.

development of both the random walk theory and to the principles of chaos. In this sense, the random walk and chaos advocates may not be as far apart as would at first seem to be the case. For example, Figure 1 shows clearly that proponents of the random walk theory need only to accept the idea that price movements they've thought of as the equivalent of random are in fact the product of a deterministic generating process.

The study of price movements from the perspective of chaos theory may also help settle the ongoing debate between proponents of random walk and the technical analysts. A fictionalized debate between the two groups by Schwager (1984) is both an informative and enjoyable explanation of the key differences. Speaking for the random walk theory, Professor Coin argues that, "...one can no more devise a system to predict market prices than one can devise a system to predict the sequence of colors that will turn up on a roulette wheel." His opponent, Ms. Trend replies "Charts reveal basic behavioral patterns...the past can indeed be used as a guideline for the future." Chaos theory will alleviate most of the differences if the sides are willing to consider it. For instance, technical analysts should be able to adjust easily to chaos theory since it offers a concept that encompasses their views on determinism while explaining the obviously ragged shape of their price charts. However, a more drastic change in viewpoint may be required of the random walk proponents. To understand this it is best to investigate some of the findings on the random walk theory before exploring chaos theory further.

In contrast to chaos theory, the random walk theory possesses an immense literature, beginning with Working's (1958) model which generated unpredictable price changes. The literature is highlighted by the debate over the theory's merits. At various times, economists such as Samuelson (1965) have claimed to prove the theory, others such as Stevenson and Bear (1969) have refuted it. The key decision rule in this process has been whether or not a trading rule could be found that would outperform the buy and hold strategy. In other words, if prices follow a random walk, knowledge of current prices cannot be used to earn an above normal return. However, Working (1958) noted that because of the gradualness of some price changes "...a small degree of very short-term predictability..." should exist. The debate has continued because there have been problems with testing trading rules and a large number of issues must be addressed. A major question relevant to our current discussion: is it possible to have a profitable trading rule and have prices unpredictable? As we shall see, chaos suggests this possibility.

CHAOS THEORY

The chaos literature in economics is increasing steadily and is becoming more broadly based. For example, Baumol and Benhabib (1989) explored the significance and mechanisms of chaos theory for various economic applications while Frank and Stengos (1988) focused on the empirical applicability of chaos theory to macroeconomic issues such as investment and unemployment. In a similar vein, Day (1982) used chaos theory to study irregular growth cycles. The role of chaos theory in understanding rational choice and erratic behavior was explored by Benhabib and Day (1981) and additionally, Weiss (1991) provided a useful guide to chaos terminology.

Many of the papers concerned with the application of chaos theory to economics began with the same example of a chaotic function. The function:

$$x_{t+1}=Ax_t(1-x_t)$$

is considered the simplest that exhibits chaos. Depending on the value of the parameter, A, this function can converge to a cycle of repeating values or to chaos where values never repeat (non-periodic). The discovery of the interesting properties of this function was crucial in showing researchers that complex patterns could be modelled by simple, nonlinear equations.

Van der Ploeg (1986) looked at chaos theory and the insight it provided into the question of rational expectations and risk in financial markets. A primary objective of his study was to present to economists the idea that simple nonlinear models could produce "strange and unexpected dynamic behavior." His paper warned of the dangers of linearizing models, because if a system ruled by nonlinear dynamics is linearized, most if not all interesting aspects of the relations will be either lost or misinterpreted. Van der Ploeg concluded from his study that, due to the complicated aspects of chaotic systems, it may never be possible to accurately predict their behavior.

The next major investigation of chaos and market prices was Robert Savit's "When Random is Not Random: An Introduction to Chaos in Market Prices" (Savit 1988). As the title suggested, the paper served as a good introduction to chaos theory as it explained two important points; how chaotic functions tend to behave and the necessity of considering nonlinear dynamics. He also warned against smoothing data, since "jumpiness" is an important part of chaos. As an introductory paper, it is useful for those interested in chaos, and not just in market prices. Importantly, he noted that nonlinear systems may still contain "noise" and therefore not be completely deterministic.

Whether or not noise exists in chaotic systems is an interesting question and one that is an important area for further research. As Figure 1 shows, chaos theory offers an explanation for what otherwise would be regarded as noise. It may be that chaos can explain movements completely and a correct model of a chaotic system would contain no error term. If a system does contain noise however, many fundamental and important questions arise. For example, how would noise be defined, how much could be allowed before the system can no longer be considered deterministic, what would it represent and what statistics could be used to test for it? Answering any of these questions will require a much greater understanding of chaos than exists currently.

Another important concern in short-term forecasting is the occurrence of feedback loops and their effects, particularly if large numbers of traders began to use chaos theory to analyze market movements. Since feedback is a crucial aspect of chaotic systems, the effects of feedback could prove significant. They are currently beyond our understanding. In conclusion, Savit believes non-linear dynamics and chaos "...will be important for a wide range of practical and theoretical problems in economic theory and market dynamics."

Savit (1989) looked at nonlinear systems again the following year in his paper "Nonlinearities and Chaotic Effects in Options Prices." He proposed in the paper that nonlinear processes are most likely in systems such as the futures market that are self-regulating and have intricate feedback loops. He also included the important reminder that nonlinear does not mean chaotic; non-chaotic nonlinear systems generally would be easier to predict since they would lack the non-periodic element of chaos. While his conclusions were basically the same as in his 1988 paper, the analysis was more sophisticated since it included some recently developed statistical methods for testing the existence of chaos.

While a full investigation of the mathematical methods used to test for the presence of chaos is beyond the scope of this paper, a few references will provide a brief background. Brock (1986) discusses three methods developed for identifying chaos in a time series: examining the size of a series' correlation dimension, checking the sign of the largest Lyapunov exponent and comparing the dimensions and largest Lyapunov exponents of the residual from the estimated series with that of the series. To help support the hypothesis of chaos, results of the tests should yield a low correlation dimension, a positive Lyapunov exponent and equivalent results between the residual and the series. The paper carefully defined these concepts and should be consulted for further explanation. Other papers that rely on these tests also include descriptions of the methods; however, in all cases, a mathematical background is necessary in order to comprehend some of the technical sections. The complicated nature of the current tests is unfortunate since it may discourage some economists from examining their data for chaos. However, if chaos theory were to prove useful in the study of futures markets or other areas in economics, automated test procedures would likely appear and become readily accessible.

Another important paper was Steven C. Blank's "'Chaos' in Futures Markets? A Nonlinear Dynamical Analysis" (Blank 1991). This study was particularly useful because it investigated the statistical techniques available for identifying chaos and was the first to specifically investigate futures markets. Following along the lines of Brock (1986), he derived Grassberger-Procaccia correlation dimensions, performed a residual test for chaos, and estimated the largest Lyapunov exponents. Blank gave as rationale for his study the poor performance of previous attempts to explain price behavior but he also cited the occasional success of previous models as his reason for rejecting the random walk model. As discussed earlier, it is this situation that makes chaos theory so attractive for examining futures markets since it could explain the paradox between apparent trends and the failure of forecasting models. Blank concluded, as did Savit (1988), that there are reasons to believe that nonlinearities are present in futures price movements and that incorporating these into models could yield better short term forecasts. The two main problems he cited were the lack of tests of significance for some of the new analysis methods, and the lack of methods for the construction of an actual, usable forecasting models.

Tvede (1992) examined chaos and financial markets from a perspective different from that of Savit and Blank. Unlike those papers, Tvede's was written with the futures trader as the intended audience. Two important aspects of this article was his explanation for various types of market trends, and his insight into psychological aspects of price movements. His conclusion for traders was that chaos, while perhaps relevant to their profession, doesn't provide them with a "magic formula"; rather the way to make money in the markets was the way they knew already.

The most recent study we reviewed is DeCoster, Labys and Mitchell's (1992) "Evidence of Chaos in Commodity Futures Prices." They also used the correlation dimension technique, and found evidence of nonlinearities, and potentially chaos, in the prices of coffee, silver, copper and sugar. Significantly, they rejected the possibility of the data's structure being either noise, or linear plus noise based. They were also interested in how chaos related to theories of market efficiency.

To summarize, the evidence from previous studies suggests that there are nonlinearities underlying price movements. If the conclusion is further reached that chaos is evident in the futures market and this is combined with the inherent determinism of chaos, it may appear to offer a new and greatly improved prediction method. In this regard, the authors cited above generally concluded that better short-term forecasting would be possible if chaos was present in the price generating function and could be modeled. While long term forecasting would be impossible, any significant improvement in forecasting ability would enable analysts to make large profits at low risk. Already, some groups of investors such as the Prediction Society (Berreby 1993) are trying to use the principles of chaos theory to profit in the markets. However, no model for prediction has yet appeared in the literature, and there are important reasons to doubt such a model is realistic.

OBSTACLES TO APPLYING CHAOS THEORY

The most obvious obstacle people encounter when rejecting randomness for chaos is how to fit a function to the price series. By recalling the sensitive dependence of chaotic functions, one realizes that any error may quickly become significant. This is especially important in economics, where measurement errors may be large. Sensitive dependence makes it impossible to specify exactly a natural chaotic function because even if an exact initial condition could be achieved, errors would quickly collect in successive iterations. This affects both a potential data problem as well as round-off and other problems with the parameters. The data problem also creates the questions: how accurate must parameters be with regard to sensitivity, even in short time intervals, and what time interval will define 'short' term forecasting? These are important questions since the time interval over which a price series is examined is a major factor in the conclusions reached. Are daily closing prices sufficient? Perhaps not since it is possible for trends to exist during the day, therefore a string of daily closes might still appear random. After all, many of the traders earning profits in the markets do so over time intervals of less than a day. Martell and Helms (1979) argue that closing prices are insufficient and that transaction to transaction data must be used in price change studies.

A potentially disastrous problem that seems to have been largely ignored in the literature is bifurcation, which is a shift of the chaotic function caused whenever the values of one or more of the parameters change (Devaney 1989). The importance of bifurcation in chaotic systems can be seen by referring again to Figure 1. Van der Ploeg (1986) discussed bifurcation and saw danger in it, although he remained more concerned about being able to estimate parameters at all than in their later movements. There is a tendency for chaotic functions to shift due to changing conditions. The problem, in other words, is how stable would the parameter values of our chaotic function be? How often does the function change itself? Shifts are apparently inevitable, which rules out the possibility of using chaos theory to make accurate long term forecasts even if sensitive dependence could be overcome. The problem would be to identify how easily and frequently parameters shift in a chaotic function. For instance, a changing condition affecting the futures market may be as large as a major storm, or as small as a single trade; bifurcation could occur on the arrival of any such new piece of information. Imagine the possibility that price series are chaotically generated, but that every transaction alters the function to some degree! It would not be possible to make predictions for any length of time, or even to accurately model the past. In such a situation, the true generator is chaotic, but it would be impossible to show that it was and in fact, it would most likely be called random. The bifurcation issue must be solved in order to determine if short term forecasting is a realistic possibility.

Closely related to the problem of bifurcation is the issue of how large a data set of past prices is necessary to discover a price generating function. Statisticians make much of the importance of large sample sizes, but there is a danger in this as well because trends that may exist in a subset of the period may be canceled out by a later, opposing trend, leading the researcher to conclude the series is random. It may even be that the larger the data series the less likely it will be to show a systematic relationship due to shifts and varying conditions over the period. If points of bifurcation could be identified, they would mark obvious bounds for the time series. Of course this must be counterbalanced with the statistical problems from working with small data sets.

Selection of a lag length is a problem when developing any model. Over the years, many methods and tests have appeared to help in this selection, although it still remains troublesome. Chaos theory, being nonlinear, adds an extra difficulty to this determination. Due to the effects of nonlinearities, any one lag length may not be accurate throughout the sample period. Consider the potential nonlinear relation: a large price change may affect prices for some time to come while a minimal change may have no effect beyond the very next trade. Some types of information may disseminate through the market quicker than others, also altering response times.

While the evidence leaves open the suggestion chaos may be operating in the futures market, no one has proven its existence. Further, it has still not been established that chaos theory can be applied directly to trends of prices in the market. Chaos theory may be a much better fit for natural sciences such as physics and biology since many things under investigation in these fields are constrained by laws of nature, of which chaos theory may be an intricate part. In economics however, and in futures trading in particular, no such 'natural' restraints bound the participants. For instance, a large upward or downward market move can be set off by market psychology as Tvede (1992) pointed out. It is not likely such a phenomenon can ever be modeled or predicted. If chaos theory is part of the laws of nature then, unless it is believed that human decision making is subject to the same forces that determine the spacing of water droplets from a faucet, it may not be appropriate in the field of economics at all.

CHAOS AND NATURAL LIMITS OF PRICE DISCOVERY

It is important to understand the natural limit of price discovery when investigating price series. It is not possible, for most series other than a straight line, to discover a function which will follow precisely all past and future price changes. Perhaps price generating functions do not exist in any form. The best we can achieve probably will be the ability to derive functions that conform at least somewhat to previous prices and, hopefully, may mimic future behavior over a short time span. The hope that long-range forecasts are possible would lead to the uncomfortable realm of predestination, and suggests that every traders' future decisions were somehow already known.

The above discussion is not meant to imply that price changes are random, but simply that they are unpredictable; differentiating between these is an important point when considering chaos. A random event is one that is not based on any previous actions, but rather something that happens without warning. Under this strict definition, nothing that influences commodity prices, can be truly classified as random. This is the conclusion that is reached through chaos theory; chaos is essentially a rejection of randomness and the acceptance of determinism.

Determinism is also different from, and does not necessarily imply, predictability since, in many cases, particularly when dealing with chaotic systems, predictability is outside our range of abilities. For instance, the weather cannot be predicted even though it is deterministic simply because it is beyond our ability to measure and interpret every piece of the atmosphere. Therefore, the key difference between random and unpredictable is that of determinism. Discovering order in seeming randomness is what chaos is all about.

CONCLUSION

What is to be gained from the study of chaos theory applied to price series or other areas in economics? The major accomplishment of the theory should be to get economists to begin thinking more in terms of using nonlinear, instead of traditional linear, models. Comparing patterns generated with chaotic functions and those from a standard futures contract should be enough to convince people of the presence of nonlinearities in price series. Results such as Streeter and Tomek (1992) who looked at models of variability of futures prices demonstrated the potential for nonlinear relationships. Only a few decades ago, researchers were constrained by the intractability of nonlinear problems, but with new methods and powerful computers there is no longer reason for failing to examine data for nonlinear relationships. The study of nonlinear systems should be an important area of future research.

A second important research effort should involve the development of better statistical tests since previous tests, such as serial correlation, analysis of runs and filter techniques, are inadequate when studying chaos theory or other nonlinear systems. Brock's (1986) work is an important effort in this regard, but a considerable number of new techniques are still needed. Development of new statistical tests would have the added benefit of being useful to a wide range of disciplines in addition to the field of economics.

Chaos has become an important concept in many fields, and deserves attention in economics because it offers a way to explain what is now considered "noise". However, there are questions with its applicability to modeling human behavior. So, while chaos is now the 'hot' theory in trading and many are making large sums on products by simply putting the word chaos in their names (Jubak 1993), further effort is needed to determine how appropriate chaos theory truly is for studying market prices and other phenomena in the realm of economics. The next step in this effort should be an attempt to construct an actual forecasting model. As Blank (1991) points out, while chaos analysis provides another set of tests, they "...do not easily lend themselves to direct applications in forecasting model construction." Until this and the previous issues we discussed here are addressed, the usefulness of chaos theory in futures markets research must be questioned.

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Figure 1. Concept Map of Chaos Theory