GARCH Option Pricing with Asymmetry

by

Taehoon Kang and B. Wade Brorsen

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The distribution of commodity futures prices is not normal but leptokurtic (Hall et al.; Hudson et al.). That is, there are more observations around the mean and more extreme values relative to a normal distribution. Skewness and serial dependence of successive price changes are also well documented (Taylor; Yang and Brorsen). Ignoring the observed non-normality and stochastic volatility is likely to lead to biased estimates of option premia. Therefore, a correct option pricing model should model not only the stochastic variance but also the conditional non-normality. It is well documented that the Black-Scholes option pricing model (OPM) based on constant volatility yields systematically biased estimates of deep in-the-money and deep out-of-the-money options (Johnson and Shanno; Hull and White). These inaccuracies may be due to the inappropriate assumption about the futures price distribution. However, departures from normality have not been considered extensively in commodity option pricing because there is no clearly superior specification (e.g. Eales and Hauser).

Time-varying variance models can explain nonlinear dependence and leptokurtosis. The generalized autoregressive conditional heteroskedasticity (GARCH) process of Bollerslev (1986) was developed as an effective way of modeling the nonlinear dynamics in volatility. Using a more general distributional assumption than normality, this model can be extended to capture the observed non-normality. Bollerslev (1987) suggested the GARCH(1,1)-t model, with one lag on the conditional variance and one on the squared residuals and with a conditional t distribution as a simple and very useful model. This GARCH(1,1)-t process fits the empirical data better than the GARCH(1,1)-normal or a mixed diffusion-jump process (Yang). Myers and Hanson provided evidence that the GARCH OPM with a student distribution performs better than Black's OPM in predicting soybean option premia.

However, one limitation of the GARCH process is that it does not model the observed skewness. An exponential GARCH (EGARCH) model that captures skewness was suggested by Nelson. The EGARCH model considers skewness by allowing the ARCH process to be asymmetric. Asymmetry in the dynamics of the mean return has not been considered in past research. This study introduces an asymmetric GARCH model that captures skewness in the mean equation and determines the most descriptive model of daily Kansas City wheat futures price distributions among several models that consider non-normality and conditional heteroskedasticity. The study also seeks to determine whether time-varying volatility and conditional non-normality can explain the biases in Black's option pricing model.

One of the primary contributions of the paper is its new approach to modeling skewness. The GARCH-t process fails to consider skewness and the EGARCH process models skewness only in the variance equation. The asymmetric GARCH-t process enables the GARCH process to model skewness and the asymmetric EGARCH-t process models skewness both in the mean and the variance equations. The GARCH-t, the asymmetric GARCH-t, and the asymmetric EGARCH-t processes are estimated and the most likely model is selected.

This study improves upon Yang's methods in several ways. Yang assumed the length of lags in the mean equation was ten and that the t-distribution had ten degrees of freedom. He also assumed a GARCH(1,1) process. Such restrictive assumptions are avoided in this paper. Further, this study provides more accurate statistical tests of the i.i.d. assumption and goodness-of-fit.

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[1] Graduate Research Assistant and Professor, respectively, Oklahoma State University, Stillwater OK. 74078-0505. Authors wish to thank Robert Myer for helpful comments.

Also, Yang did not consider effects of distributional assumptions on option pricing.

Previous option pricing models allowing stochastic volatility (e.g., Hull and White) other than Myers and Hanson have not explicitly considered a conditional non-normal distribution. This paper considers non-normality as well as stochastic volatility and analyzes their effects on option pricing. This study extends Myers and Hanson in several ways. Significance tests are conducted on the performances of Black's OPM and GARCH OPM, and the simulated biases between the two OPM's are analyzed in terms of time to maturity effects and futures-exercise price ratio effects. Most important is that this paper uses models which can capture skewness while Myers and Hanson did not.

Procedures

The GARCH and EGARCH processes with and without asymmetry are estimated using maximum likelihood. The models are selected using likelihood ratio tests or the Schwarz model selection criterion. Then the effects on the option pricing of the model allowing time-varying volatility and conditional non-normality are analyzed for wheat options at the Kansas City Board of Trade.

Statistical Models and Skewness

Skewness of the rate of return has received increasing attention in the past decade, as portfolio theory has been extended to include skewness along with mean return and variance to explain security preferences (Conine and Tamarkin; Beedles and Simkowitz; Junkus, 1991; Kang et al.). Skewness in daily futures returns is well documented. Twenty two of the thirty six commodities considered by Yang showed significant skewness. However, the observed skewness has not been well modeled with most models of asset price distributions.

Many competing statistical distributions have been proposed to model the departures from normality: a symmetric stable Paretian distribution (Mandelbrot; Fama), student t-distribution (Blattberg and Gonedes), a mixture of normal distributions (Kon), and a mixed diffusion-jump process (Akgiray and Booth). However, since these models assume the independence of successive asset returns, they are inconsistent with empirical data that is known to be linearly or nonlinearly dependent. Further, these models are focused on capturing leptokurtosis. Jorion found that combining a jump process with a simple ARCH process provides a significantly better model of weekly exchange rates than either process alone. The mixed jump-diffusion process also models skewness. Combining the GARCH(1,1)-t process, however, with a jump process is not significantly better than the GARCH-t process alone (Brorsen and Yang). Thus, a GARCH(1,1)-t process is used as the benchmark model, and other alternative models will be compared with it.

While the GARCH model elegantly captures the volatility clustering in asset returns, it ignores the possible asymmetric response of variance to positive and negative residuals and restricts the parameters in the variance equation to be non-negative. Nelson suggested an Exponential GARCH (EGARCH) model that meets these objections. LeBaron reported that the EGARCH model explains skewness better in the distribution of weekly and monthly stock indices than the GARCH model. However, Nelson's EGARCH model considers skewness only in the variance equation.

In this paper, the GARCH under a student t distribution and the EGARCH under student distributions are considered. Each model will be estimated with and without asymmetry in the mean equation. Specifying the dynamics in the mean equation as asymmetric together with the GARCH or the EGARCH process might capture skewness more effectively. The GARCH process can model well-documented market anomalies such as the day-of-the-week effect (Chiang and Tapley; Junkus,1986) and seasonality (Anderson; Kenyon et al.) both in means and variances and maturity effect (Milonas) in the variance equation. In the GARCH process, the futures price changes, R_t, can be expressed as a stochastic process:

(1) $R_t = f(I_{t-1};\theta) + \varepsilon_t$

and $f(I_{t-1};\theta)$ denotes a function of I_{t-1} (the information set at time t-1) and the parameter vector θ. ε_t has a discrete time stochastic process, (ε_t), of the form;

$$(2)\quad \varepsilon_t = \begin{cases} z_t h_t & \text{in the GARCH-normal model and} \\ ((v-2)/v)^{1/2}\omega_t h_t & \text{in the GARCH-t model.} \end{cases}$$

where z_t is i.i.d. normal with $E(z_t)=0$ and variance $Var(z_t)=1$ and ω_t is i.i.d. student with degrees of freedom v, $E(\omega_t)=0$ and $Var(\omega_t)=v/(v-2)$. Therefore, h^2_t is the time varying variance of ε_t. The GARCH(p,q) model expresses h^2_t as a linear function of past variance and past squared values of the process,

$$(3)\quad h_t^2 = \alpha_0 + \alpha_i\sum_{i=1}^{q}\varepsilon_{t-i}^2 + \beta_j\sum_{j=1}^{p}h_{t-j}^2 .$$

Equation (1) is the mean equation and equation (3) is the variance equation. If h^2_t is to be the conditional variance of ε_t, it must be nonnegative. The GARCH model ensures this by making h^2_t a linear combination of positive random variables, which may unduly restrict the dynamics of the conditional variance process. The EGARCH model ensures that h^2_t remains nonnegative by replacing h^2_t by $\ln(h^2_t)$ in equation (3). For example, the conditional variance of EGARCH(1,0) is:

$$(4)\quad \ln(h^2_t) = \alpha_0 + \beta h^2_{t-1} + \eta(\varepsilon_{t-1}/h_{t-1}) + \varphi(\,|\,\varepsilon_{t-1}/h_{t-1}\,|\, - (2/\pi)^{1/2}).$$

In the GARCH process, the mean and the variance equations to be estimated are, respectively,

$$(5)\quad R_t = a_0 + \sum_{i=1}^{m}\lambda_i R_{t-i} + a_1 D_{MON} + a_2 D_{TUE} + a_3 D_{WED} + a_4 D_{THU} + a_5 SIN(2\pi K/252) + a_6 COS(2\pi K/252) + a_7 SIN(2\pi K/126) + a_8 COS(2\pi K/126) + \varepsilon_t,$$

$$(6)\quad h^2_t = \alpha_0 + \alpha_i\sum_{i=1}^{q}\varepsilon_{t-i}^2 + \beta_j\sum_{j=1}^{p}h_{t-j}^2 + b_1 D_{MON} + b_2 D_{TUE} + b_3 D_{WED} + b_4 D_{THU} + b^5 SIN(2\pi K/252) + b_6 COS(2\pi K/252) + b_7 SIN(2\pi K/126) + b_8 COS(2\pi K/126) + b_9 TTM,$$

where R_t is the logarithmic difference of daily returns at time t, the λ_i's are the coefficients of lagged price changes, and m is the length of lags. The length of lags in the mean equation is identified with the Schwarz criterion[2]. D denotes dummy variables for each day of the week; D_{MON} = 1 if Monday and 0 otherwise, and so on. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. K in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, so 252 indicates a one year cycle and 126 a half year cycle. TTM is the time to maturity measured in the number of trading days prior to maturity.

The asymmetric pricing model has been extensively applied to the structure of farm-retail price transmission (e.g., Kinnucan and Forker; Boyd and Brorsen). The asymmetric model can be used as an appropriate way to capture the asymmetric responses of current variables to the change of past

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[2] Schwarz's SC criterion is obtained by $SC(m) = \ln(SSE_m) + Q_m * \ln(T)/T$, where m is the length of lags, SSE_m is the squared sum of residuals, and Q_m and T are the number of parameters and observations, respectively. The value of m that minimizes SC is selected as the length of lags in the model.

shock variables. Restrictions on short selling, asymmetries in information, preferences of investors, and market psychology might cause the differing responses of price to the past price rising or falling.

The asymmetric model in the mean equation is obtained by segmenting the lagged price changes into one set for rising changes and another set for falling changes. The logarithmic changes in returns, R_t, are segmented as,

$$RP_t = \begin{cases} R_t, & R_t \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$RN_t = \begin{cases} R_t, & R_t < 0 \\ 0, & \text{otherwise,} \end{cases}$$

and the asymmetric model is specified as,

$$(7)\ R_t = a_0 + \sum_{i=1}^{m} \delta_i RP_{t-i} + \sum_{i=1}^{m} \omega_i RN_{t-i} + a_1 D_{MON} + a_2 D_{TUE} + a_3 D_{WED} + a_4 D_{THU} + a_5 SIN(2\pi K/252) + a_6 COS(2\pi K/252) + a_7 SIN(2\pi K/126) + a_8 COS(2\pi K/126) + \varepsilon_t.$$

where δ_i and ω_i represent the net effect of the i^{th} positive or negative changes of R_t, respectively.

The variance equations to be estimated in the GARCH and EGARCH models are obtained by adding the day-of-the-week effect dummies, seasonality, and time to maturity variables into equations (3) and (4), respectively.

The maximum likelihood estimates are obtained for alternative models using the statistical software package GAUSS. For the GARCH model, since the GARCH terms (α and β in the variance equation) are restricted to be nonnegative, inequality restrictions are imposed using the exponential of the parameters. The starting algorithm is Polak-Ribiere-type Conjugate Gradient method and a step size of one which performs well initially when the starting point is poor. After a few iterations, the algorithm is switched to the Davidon-Fletcher-Powell method and Brent method is used. The final estimates are obtained with the Newton method so that the Hessian is used to estimate the information matrix. All derivatives are calculated numerically.

The asymmetry hypothesis is tested in two ways. One is that the total impact of past price increases is the same as that of past price decreases:

(8) $H_0: \sum_{i=1}^{m} \delta_i = \sum_{i=1}^{m} \omega_i$

H_A: H_0 is not true.

The other is that the speed of adjustment to price increases and to price decreases are the same (Boyd and Brorsen).

(9) $H_0: \delta_i = \omega_i$, for all i

H_A: H_0 is not true.

For the hypothesis tests of equations (8) and (9), Wald-F statistics are given. The Wald test is not invariant in nonlinear models, but it is still asymptotically valid (Dagenais and Dufour).

Estimated t-ratios of the parameters on the skewness term (η) in equation (4) are given for tests of asymmetry in the variance equations of the EGARCH models.

Model Selection and Validation

Selecting between the GARCH-t and the asymmetric GARCH-t models is performed by the likelihood ratio test, since the models are nested. Model selection between the asymmetric GARCH-t and the asymmetric EGARCH-t models are conducted by the difference of the Schwarz criteria of the two models. The Schwarz criterion penalizes the model with more parameters and thus is useful for selecting among nonnested models with different numbers of parameters.

If the GARCH models are well-specified and fit the sample data, the standardized residual generated by the GARCH models should be i.i.d. normal or student. The Ljung-Box and McLeod-Li test statistics are provided to determine if the serial dependence is revealed in the rescaled residuals of the selected model. The Brock-Dechert-Scheinkman (BDS) test (Brock et al.) is used to test the null hypothesis that $\{R_t\}$ is i.i.d.. The BDS statistic is based on the correlation integral. The statistic is asymptotically distributed as a standard normal random variable under the null hypothesis. The tables in Brock et al.(p.279) can be used to do the test. Brock et al. have shown that the distribution of the BDS statistic is not standard normal when the data are GARCH residuals. Therefore, the test in this study is more accurate than the test by Yang which assumed the BDS statistic with GARCH residuals had a standard normal distribution.

The Kolmogorov-Smirnov goodness-of-fit test is used to determine if the residuals really have a t-distribution. If the largest absolute deviation between the cumulative distributions of the rescaled residuals and the theoretical student distribution is bigger than the critical value, the null hypothesis of no significant difference between the two distributions is rejected. The rescaled residuals are multiplied by $(v/(v-2))^{1/2}$, where v is the estimated degrees of freedom of the model. This adjustment is needed because the variance of a t-distribution is $v/(v-2)$. Yang's test is biased, since he used the rescaled residuals without multiplying by $(v/(v-2))^{1/2}$.

Implications for Option Pricing

Using implied volatility, McBeth and Merville argued that the Black-Scholes formula overprices out-of-the-money options and underprices in-the-money options. However, Rubinstein argued that their results were not always true. Johnson and Shanno obtained numerical results for general cases in which the instantaneous variance obeys some stochastic processes. Under the situation where the volatility is a separate stochastic variable from the stock price, Hull and White have shown that the Black-Scholes formula overprices options that are at- or close-to-the-money and underprices options that are deep in- and deep out-of-the-money. This paper examines how well Black's OPM estimates option premia under non-normality as well as conditional heteroskedasticity.

Under the assumption of risk-neutrality, commodity option prices are a function of five underlying parameters: the current futures price (F_t), the exercise price of the option (X), the time to maturity of the option (T-t), the risk-free rate of interest (r), and the variance of futures prices (σ^2).

$$(10) \qquad B = \begin{cases} X^*N(-d_2) - F_t^*N(-d_1) & \text{for put,} \\ F_t^*N(d_1) - X^*N(d_2) & \text{for call,} \end{cases}$$

where $d_1 = [\ln(F_t/X) + (\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$ and $d_2 = [\ln(F_t/X) - (\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$.

Monte Carlo integration is used to obtain the expected option prices at maturity. The GARCH or EGARCH models are estimated using data available at time t. Two sets of T-t random numbers are generated: one from a t-distribution with v degrees of freedom and another from a standard normal distribution. Time is measured in number of trading days. The time-varying conditional variances are generated for T-t periods using estimates from the selected model. Then, with the conditional variances, the futures prices F_t are simulated for T-t periods to get the futures price at maturity. Denoting this price at maturity $\{F_T\}_i$, the simulated option prices are

$$(11) \quad G = \begin{cases} e^{-r(T-t)}(1/n)\sum_{i=1}^{n} \max[X - \{F_T\}_i, 0] \text{ for call,} \\ e^{-r(T-t)}(1/n)\sum_{i=1}^{n} \max[\{F_T\}_i - X, 0] \text{ for put,} \end{cases}$$

where n=10000 is the number of replications of this procedure.

One efficient way to improve the accuracy of this calculation is the control variate technique (Hammersley and Handscomb; Boyle). This technique replaces the problem under consideration by a similar but simpler problem which has an analytical solution. The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem. For this purpose, the GARCH option price, G in equation (11), is replaced by Black's option price, B in equation (10). The biases in B are reduced by an error correction term which is the difference between the GARCH option price G_1 and the control variate G_2 that mimics the behavior of GARCH option price and can be easily evaluated (see Boyle for details). The GARCH option price under control variate technique is, then,

$$(12) \quad G^* = B + (G_1 - G_2).$$

In the simulation, B is obtained analytically using the Black's OPM, and G_1 and G_2 are obtained from Monte Carlo methods as given in equation (12). Conditional variances, generated using the estimates of the most likely model and random numbers from a student distribution, are used for the simulation of G_1, but a constant variance and a standard normal random numbers are used for G_2.

Simulated Differences

The issue in this portion of the paper is to determine the differences between Black's OPM and the GARCH OPM. The difference between the Black's OPM and the GARCH OPM may be caused by not considering the observed conditional heteroskedasticity and non-normality in Black's OPM. The extent of the difference can be measured by the absolute difference $(B - G^*)$ or by the percentage difference $\xi = (B - G^*)/G^*$. The differences for short-lived options differ from those for long-lived options. The differences are also different by how much the option is in the money or out of the money. Since the time to maturity effects and the futures-exercise price ratio effects (or moneyness effects) are not distinguishable in most cases, the futures prices are scaled by the exercise price. For the case of a put option, the out-of-the-money option is examined at the levels of 1.10 and 1.05 in the futures-exercise price ratio (F_t/X), the at-the-money option at the at the level of 1.0, and the in-the-money option at the level of 0.95 and 0.90. For the case of a call option, the out-of-the-money option is examined at the levels of 0.90 and 0.95 in the futures-exercise price ratio, the at-the-money option at the level of 1.0, and the in-the-money option at the level of 1.05 and 1.10. In the Monte Carlo simulation, exercise price is set equal to \$1.00. The differences are measured from six months prior to through one half month to maturity.

The asymptotic t-statistics for the simulated differences are provided. These could be used to determine whether the reported pricing biases are significantly different from zero. They are computed by the ratios of the simulated differences to the standard deviations of the differences.

Actual Differences

To determine the ability of the GARCH OPM and Black's OPM to predict actual premia, option prices are estimated for July 1991 Kansas City wheat futures for two month periods prior to maturity (from April 24, 1991 through June 21, 1991). Results are shown for put options. Historical 20-day volatilities are used for Black's option prices. The risk-free interest rate is assumed constant during the simulation period at $r=5.5\%$[3]. Option prices are simulated for exercise prices of $2.70 as out-of-the-money case, and $2.90 and $3.00 as near at-the-money cases. During the simulation period, the actual futures prices were mostly distributed in the range of $2.85 and $2.95. The mean errors and root mean squared errors (RMSE) of Black's OPM and GARCH OPM are computed. Further, the Ashley-Granger-Schmalensee test is used to determine if the mean squared error of Black's OPM is equal to that of the GARCH OPM. A regression model in the test was estimated by OLS using heteroskedastic-consistent covariance matrix to correct heteroskedasticity in the model.

Sample Data

To estimate the alternative statistical models, the first differences of the natural logarithms of the daily futures closing prices of wheat at Kansas City Board of Trade are used. The data are for the period from Jan. 1982 to Sep. 1990. The data were created using Continuous Contractor from Technical Tools. The data consist of the log changes of daily closing prices of the underlying futures contract. Kansas City wheat futures contracts are traded based on five maturities: March, May, July, September, and December. The price series used is a continuous combination of the five contracts. The rollover date is the 21st day of the month prior to delivery. Log differences are taken before splicing so that no outlier is created at the rollover date.

Table 1 shows summary statistics for daily logarithmic changes in the closing prices of July wheat futures contracts at Kansas City Board of Trade. The departures from normality are apparent from the high kurtosis and skewness. The daily put option premia over the simulation period are collected from the Kansas City Grain Market Review.

Empirical Results

Model Selection and Validation

Table 2 shows the estimated log-likelihoods and the test statistics associated with the null hypotheses of no asymmetry. In the asymmetric GARCH model, responses to rising prices are not significantly different from those to falling prices by the total effect. However, significant asymmetries are detected in the mean returns by the speed of adjustment. The lagged adjustment for falling prices is faster than that for rising prices. The absolute values of coefficients for the lagged falling prices are greater than those for lagged rising prices in Table 4. Significant asymmetries are detected both in the mean and in the variance equation in the asymmetric EGARCH model. The skewness term in EGARCH model is positive and significantly different from zero.

Table 3 contains the test statistics of model selection. Likelihood ratio and differences in Schwarz criteria are provided for selecting between nested models and between nonnested models, respectively. Asymmetric EGARCH(1,0)-t is not selected as better than asymmetric GARCH(1,1)-t. GARCH(1,1)-t is rejected in favor of asymmetric GARCH(1,1)-t which is rejected in favor of

[3] During the simulation period, the rate of return on Treasury bills were ranged (0.052, 0.058).

asymmetric GARCH(2,1)-t. Asymmetric GARCH(2,1)-t is rejected neither in favor of asymmetric GARCH(2,2)-t nor of asymmetric GARCH(3,1)-t. Thus, the asymmetric GARCH(2,1)-t process is the most likely and so its estimates are used for simulating option prices.

Table 4 reports estimates and test results of the asymmetric GARCH(2,1)-t model. The estimated GARCH terms are all positive and significant. Mean returns differ by day of the week, but variances do not. Significant seasonal patterns are revealed both in the mean and in volatility. In Table 4, the Ljung-Box and McLeod-Li tests for the standardized data do not detect any linear or second moment dependence, which implies the GARCH-t process removed all the linear or higher order dependencies. The BDS statistics show that the null hypothesis of i.i.d. is rejected in all dimensions for the raw data (Table 4), implying that Kansas City wheat futures price changes are not random but are dependent. For the rescaled residual, the BDS statistics, however, do not identify nonlinear dependence at all. Dependence is removed by the model. The Kolmogorov-Smirnov test shows that the null hypothesis that the GARCH rescaled residuals follow a student distribution is not rejected at the 5% significance level. Yang found a GARCH-t model without asymmetry could be rejected and thus the adjustment for asymmetry is shown to be especially important.

Differences between Black's Option Pricing and GARCH Option Pricing

Table 5 presents absolute and percentage differences between put option premiums by Black's OPM and GARCH OPM. The Black's OPM significantly underprices relative to the GARCH OPM for deep in- and deep out-of-the-money put options. The put option value depends on the left tail of the terminal distribution. Therefore, Black's OPM based on the normal distribution tends to yield lower option premiums than the GARCH OPM. Absolute differences increase as time to maturity increases in deep in- and deep out-of-the-money put options (Table 5, Panel A). Percentage differences for deep in-the-money option also increase as time to maturity increases, but those for deep-out-of-the money option decreases as time to maturity increases (Table 5, Panel B). As time to maturity decreases, the time-value of deep out-of-the-money option decreases very fast and eventually becomes zero. Therefore, deep out-of-the-money options close to maturity show extremely high percentage differences.

In-the-money put option is underestimated by Black's OPM relatively to GARCH OPM, but not always. Percentage differences for in-the-money put option are significantly different from zero. Black's OPM underestimates short-lived out-of-the-money put option and overestimates long-lived out-of-the-money put option, but not always significantly. No strong time to maturity effects are found for in- and out-of-the money put options. Black's OPM overestimates at-the-money option significantly, especially in terms of absolute differences. Percentage differences for at-the-money put option decreases as time to maturity increases.

Table 6 presents absolute and percentage differences between call option premiums by Black's OPM and GARCH OPM. The Black's OPM significantly underprices relative to the GARCH OPM for deep out-of-the-money call options. Since the call option value depends on the right tail of the terminal distribution, the Black's OPM based on the normal distribution tends to yield lower option premiums than the GARCH OPM. Absolute differences increase as time to maturity increases in deep out-of-the-money call options (Table 6, Panel A). However, percentage differences decreases as time to maturity increases (Table 6, Panel B), because as time to maturity decreases, the time-value of deep out-of-the-money option decreases very fast and eventually becomes zero.

Unlike deep in-the-money put option, deep in-the-money call option is not always underestimated by Black's OPM. Although it is not significant, deep in-the-money call option one half month prior to maturity is overestimated. Time to maturity does not affect deep in-the-money call option as much as deep in-the-money put option. Black's OPM underestimates short-lived out-of-the money call option premia, but overestimates premia of long-lived ones. No evidence is found that in-the-money call option is underestimated or overestimated by Black's OPM. At-the money call option is overestimated by Black's OPM, especially in terms of absolute differences.

The simulation results confirm Hull and White's findings that the Black-Scholes model underprices in- and out-of-the-money options under stochastic volatility when successive observations are independent and normally distributed. Their argument that the Black-Scholes model overprices close-to-the-money options is also confirmed. Note that the absolute differences are very small, which agrees with Hull and White, and that absolute differences between the two OPM's are bigger for at-the-money option than for in- or out-of-the-money options.

As the asymptotic t-statistics show (Tables 5 and 6), there is some evidence that the reported option pricing errors are not due to sampling errors. Especially, differences between Black's option price and GARCH option price for out-of-the-money options are significantly different from zero.

Performance of Black's and GARCH option Pricing

Table 7 shows the result for an out of sample simulation. Performance of each model at various exercise prices for the July put option is shown. The mean errors and root mean squared errors of the GARCH OPM are smaller than those of Black's OPM at each level of exercise prices. This is evidence that the option premia of Kansas City wheat is significantly underpriced by Black's OPM. During the simulation period, futures prices are distributed between $2.85 and $2.95. Therefore, the option of $2.70 exercise price refers to a deep out-of-the-money option, and options of $2.90 and $3.00 exercise prices refer to close-money options. Results show that Black's OPM performs worse than GARCH OPM for the three levels of exercise price, especially for deep-out-of-the money option. The differences of mean errors and root mean squared errors are bigger for the option of $2.70 exercise price than those of $2.90 or $3.00. Ashley-Granger-Schmalensee's test supports the finding that mean squared error of Black's OPM is significantly larger than that of GARCH OPM for the three levels of exercise prices.

Conclusions

This paper introduces an asymmetric GARCH model that captures asymmetries in the mean equation, and determines the most likely distribution among alternative autoregressive conditional heteroskedasticity models. The asymmetric GARCH(2,1)-t, which considers asymmetry in the mean equation, was selected as the most likely among the alternative models. The alternative models considered were the GARCH-t, the EGARCH-t, and the asymmetric EGARCH-t models.

The Monte Carlo integration using the estimated asymmetric GARCH-t parameters shows that Black's model underprices the simulated GARCH option prices for deep in- and deep out-of-the-money put options and deep out-of-the-money call option. However, Black's option pricing model overestimates at-the-money put and call option premiums. Differences between Black's model and GARCH option model increase as time to maturity increases, except for percentage changes of deep out-of-the-money case, which confirms Hull and White's findings. The GARCH option pricing model predicts actual option premiums more accurately than Black's model.

Table 1. Summary Statistics of Daily Kansas City Wheat Futures Prices over January 1982 through August 1990.[a]

	Statistics
Sample Size(n)	2191
Mean (μ)	-0.0109
Standard Deviation (σ)	0.9774
Skewness[b]	0.6471*d
Kurtosis[c]	11.1585*

[a] Units are percentages. $R_t = [\ln(P_t) - \ln(P_{t-1})]*100$.
[b] Skewness is computed by $\sum^n_{t=1}(R_t - \mu)^3/(n-1)\sigma^3$.
[c] Excess kurtosis is computed by $\sum^n_{t=1}(R_t - \mu)^4/(n-1)\sigma^4 - 3$.
[d] Asterisks denote the null hypothesis of normality (i.e., zero skewness and zero kurtosis) are rejected at a 5% level based on the critical values by Snedecor and Cochran.

Table 2. Estimated Log-likelihoods and Statistics for Asymmetries in Alternative Models of Daily Futures Prices of Kansas City Wheat

Model	Maximized Log-Likelihood	Statistics for Asymmetries		
		Mean[a]		Variance[b]
		Total	Speed	
Asymmetric GARCH(1,1)-t	-2521.3	3.22	5.93*c	na[d]
Asymmetric GARCH(2,1)-t	-2515.4	3.33	5.86*	na
GARCH(2,1)-t	-2520.6	na	na	na
Asymmetric GARCH(2,2)-t	-2515.0	3.50	6.05*	na
Asymmetric GARCH(3,1)-t	-2514.7	0.87	3.75*	na
Asymmetric EGARCH(1,0)	-2523.5	79.66*	168.82*	2.81*

[a] Statistics for asymmetries in the mean equations are distributed as F(1,2191) for total impact and F(3,2191) for the speed of adjustment under the null hypothesis that there is no asymmetries.
[b] Statistics for asymmetries in the variance equations are the t-statistics of the parameter representing skewness (η in equation (5)).
[c] Asterisks denote rejection of the null hypothesis of no asymmetry at the 5% significance level
[d] Not applicable.

Table 3. Test Statistics for Model Selection of Alternative Models[a]

Hypotheses		
Null	Alternative	Statistics
AGARCH(1,1)-t	AEGARCH(1,0)-t[c]	-3.29
GARCH-(1,1)-t	AGARCH(2,1)-t[b]	11.8*[c]
GARCH(2,1)-t	AGARCH(2,1)-t[b]	10.4*
AGARCH(2,1)-t	AGARCH(2,2)-t[b]	0.42
AGARCH(2,1)-t	AGARCH(3,1)-t[b]	1.4

[a] AGARCH denotes the asymmetric GARCH model and AEGARCH denotes the asymmetric EGARCH model.

[b] Likelihood ratio test statistic is obtained by $2T*(L_1 - L_0)$, where T is the number of observations, L_1 is the loglikelihood values under alternative hypothesis, and L_0 under null hypothesis.

[c] Statistic reported is the difference in Schwarz criteria which is obtained by $2T*(L_1 - L_0)-(K_1 - K_0)*\ln(T)$, where K_1 and K_0 are the number of parameters under alternative and null hypothesis, respectively.

[d] Asterisk denotes rejection of the null hypothesis in favor of the alternative hypothesis at the 5% significance level.

Table 4. Statistics and Test Results for the Estimated Asymmetric GARCH(2,1)-t Process

	Estimated Coefficients	(t-ratio)
Mean		
Intercept	-0.125*[a]	(-3.05)
Lag 1 positive	0.027	(0.64)
Lag 1 negative	0.151*	(3.69)
Lag 2 positive	0.010	(0.26)
Lag 2 negative	-0.134*	(-3.51)
Lag 3 positive	0.050	(1.23)
Lag 3 negative	-0.071	(-1.81)
D_{MON}	0.041	(0.88)
D_{TUE}	0.077	(1.68)
D_{WED}	0.152*	(3.55)
D_{THU}	0.036	(0.82)
SIN252	-0.018	(-0.81)
COS252	0.061*	(2.79)
SIN126	0.025	(1.19)
COS126	-0.043*	(-2.09)
Variance		
Intercept	0.061	(1.74)
Alpha	0.160*	(11.17)
Beta1	0.190*	(2.27)
Beta2	0.592*	(7.25)
D_{MON}	0.048	(0.81)
D_{TUE}	0.014	(0.22)
D_{WED}	-0.096	(-1.63)
D_{THU}	-0.039	(-0.70)
SIN252	0.006	(1.03)
COS252	-0.018*	(-2.47)
SIN126	0.007	(1.21)
COS126	-0.003	(-0.53)

(Table 4 Continued)

Maturity	-0.001	(-0.31)
Degrees of Freedom		
ν^b	7.31*	(5.99)
Wald F statistics		
Day of Week in Mean	3.66*	
Seasonality in Mean	3.12*	
Day of Week in Variance	1.26	
Seasonality in Variance	2.92*	
Ljung-Box and Mcleod-Li[c]		
$\varepsilon_t/h_t(12)$	13.66	
$\varepsilon^2_t/h^2_t(18)$	20.41	
BDS tests $(\varepsilon=\sigma)$[d]		
Raw Data		
Dimension = 3	13.04*	
Dimension = 6	17.58*	
Dimension = 9	24.53*	
Rescaled Data		
Dimension = 3	0.47	
Dimension = 6	-0.57	
Dimension = 9	-0.07	
Goodness-of-fit[e]		
D_{max}	0.013	

[a] Asterisks denote the rejection of the null hypothesis at the 5% significance level. Values in parentheses are the t-statistics.

[b] Inequality constraints were imposed on these parameters using exponential transformation.

[c] Both null hypotheses that ε_t/h_t are not autocorrelated and that ε^2_t/h^2_t are not autocorrelated are tested with twelve degrees of freedom.

[d] The null hypothesis is that the standardized residuals are i.i.d. The hypothesis test is based on Table F.4 in Brock et al. (p.279).

[e] The critical value of this test is $D_c = 1.36/T^{1/2} = 0.0299$ where T is the sample size.

Table 5. Simulated Biases of Black's Option Pricing for Put Option

	Time to maturity (months)					
	0.5	1	1.5	3	4.5	6
Panel A: Absolute differences[a]						
Deep In-the-money	-0.002	-0.008	-0.018	-0.030	-0.047	-0.059
($F_t/X=0.90$)	(-0.20)	(-0.66)	(-1.09)	(-1.30)	(-1.71)	(-1.91)
In-the-money	-0.001	-0.007	-0.005	-0.019	0.059	-0.010
($F_t/X=0.95$)	(-0.15)	(-0.59)	(-5.36)	(-0.88)	(2.39)	(-0.39)
At-the-money	0.051	0.052	0.078	0.070	0.069	0.040
($F_t/X=1.0$)	(7.89)	(5.26)	(6.37)	(3.78)	(3.16)	(1.54)
Out-of-the-money	-0.021	-0.015	-0.017	0.008	0.061	0.024
($F_t/X=1.05$)	(-5.19)	(-2.45)	(-1.81)	(0.55)	(3.46)	(1.14)
Deep Out-of-money	-0.009	-0.019	-0.036	-0.025	-0.028	-0.036
($F_t/X=1.10$)	(-3.67)	(-5.35)	(-5.92)	(-2.58)	(-2.00)	(-1.99)
Panel B: Percentage differences (%)[b]						
Deep In-the-money	-0.02	-0.08	-0.18	-0.30	-0.46	-0.57
($F_t/X=0.90$)	(-8.11)	(-9.85)	(-6.09)	(-4.08)	(-9.87)	(-0.16)
In-the-money	-0.03	-0.14	-0.10	-0.32	0.95	-0.17
($F_t/X=0.95$)	(-14.88)	(-9.20)	(-8.83)	(-3.09)	(3.48)	(-0.83)
At-the-money	4.89	3.47	4.28	2.67	2.17	0.01
($F_t/X=1.0$)	(0.40)	(0.58)	(0.41)	(0.91)	(1.01)	(0.96)
Out-of-the-money	-31.67	-6.73	-4.06	0.86	4.42	1.31
($F_t/X=1.05$)	(-11.35)	(-0.43)	(-2.69)	(1.50)	(0.91)	(1.27)
Deep Out-of-money	-97.67	-64.92	-44.54	-9.05	-5.04	-4.20
($F_t/X=1.10$)	(-5.58)	(-12.46)	(-12.95)	(-4.48)	(-0.63)	(-3.66)

a Black's option price minus GARCH option price when exercise price is set equal to $1.00.
b [(Black's price - GARCH price)/GARCH price]*100.
Asymptotic t-statistics are in parentheses.

Table 6. Simulated Biases of Black's Option Pricing for Call Option

	Time to maturity (months)					
	0.5	1	1.5	3	4.5	6
Panel A: Absolute differences (cents per bushel)[a]						
Deep Out-of-money	-0.003	-0.016	-0.037	-0.046	-0.036	-0.061
(F_t/X=.90)	(-2.63)	(-4.96)	(-5.62)	(-4.39)	(-2.41)	(-2.75)
Out-of-the-money	-0.026	-0.028	-0.016	0.027	0.002	0.017
(F_t/X=.95)	(-7.42)	(-4.24)	(-1.24)	(1.69)	(0.07)	(0.66)
At-the-money	0.050	0.063	0.074	0.093	0.061	-0.037
(F_t/X=1.0)	(7.32)	(5.77)	(5.05)	(4.65)	(2.03)	(-1.03)
In-the-money	0.0003	-0.016	0.013	-0.0004	-0.022	0.003
(F_t/X=1.05)	(0.03)	(-1.09)	(0.80)	(-0.01)	(-0.64)	(0.07)
Deep In-the-money	0.003	-0.006	-0.066	-0.011	-0.001	-0.043
(F_t/X=1.10)	(0.24)	(-0.37)	(-3.09)	(-0.36)	(-0.03)	(-1.01)
Panel B: Percentage differences (%)[b]						
Deep Out-of-money	-98.37	-79.15	-61.96	-21.92	-8.68	-9.02
(F_t/X=.90)	(-3.32)	(-9.53)	(-14.97)	(-3.61)	(-2.38)	(-2.11)
Out-of-the-money	-43.25	-14.32	-4.41	3.24	0.12	1.03
(F_t/X=.95)	(-1.28)	(-8.23)	(-4.03)	(5.28)	(0.81)	(1.35)
At-the-money	4.79	4.19	4.03	3.61	1.91	-0.99
(F_t/X=1.0)	(0.92)	(0.95)	(-0.97)	(0.95)	(0.07)	(-1.12)
In-the-money	0.01	-0.30	0.26	-0.01	-0.35	0.04
(F_t/X=1.05)	(13.00)	(-4.61)	(4.89)	(-2.35)	(-0.47)	(1.57)
Deep In-the-money	0.03	-0.06	-0.66	-0.11	-0.01	-0.41
(F_t/X=1.10)	(-1.00)	(-13.95)	(-14.33)	(-6.96)	(-4.04)	(-0.39)

a Black's option price minus GARCH option price when exercise price is set equal to 100.
b [(Black's price - GARCH price)/GARCH price]*100.
Asymptotic t-statistics are in parentheses.

Table 7. Performance of Black and GARCH Option Pricing for Kansas City Wheat over April 1, 1991 through June 21, 1991

	Black			GARCH		
Exercise Prices	270	290	300	270	290	300
Mean Error[a]	-0.86	-2.36	-2.44	-0.79	-2.35	-2.37
Root Mean Squared Error[a]	1.240	2.672	3.134	1.145	2.638	3.071
AGS statistics[d]	80.52*	18.50*	30.74*			

[a] Mean errors and root mean squared errors are in cents per bushel.
[b] Ashley-Granger-Schmalensee test statistics. Distributed as F(2,37) under the null hypothesis.
* Asterisks denote the rejection of the null hypothesis at the 5% significance level.

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