Hedging with Commodity Options and Safety-First Rules

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Suggested citation format:

Vercammen, J., and V. Gaspar. 1993. "Hedging with Commodity Options and Safety-First Rules." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

HEDGING WITH COMMODITY OPTIONS AND SAFETY-FIRST RULES

James Vercammen and Victor Gaspar[1]

The problem of constructing the optimal short hedge with commodity futures contracts appears to be well understood. Under standard assumptions (e.g., two-periods, no output uncertainty and mean-variance utility) hedgers locate on the mean-variance efficiency frontier according to their degree of risk aversion (e.g., Johnson 1960; Heifner 1972; Berck 1981; Bond, Thomspon and Lee 1988). If the expected delivery price also coincides with the quoted futures price (i.e., there is no speculative motive for hedgers), the problem reduces to minimizing the variance of the combined return from the spot and futures markets. The optimal hedge in this case entails forward selling a fraction β of the deliverable commodity where β is the covariance between the spot and futures price divided by the variance of the futures price.

With the recent introduction of commodity option contracts one would expect another rash of studies designed to analyze optimal hedging strategies when both futures and option contracts are available. In fact, very little literature on this topic has emerged. A likely reason is that under the standard assumptions, risk-averse hedgers will continue to follow the hedging rule described above and will not include commodity option contracts in their hedging portfolio unless price biases are present (Lapan, Moschini and Hanson [LMH] 1991) or unless the profit function is non-linear in prices (Moschini and Lapan 1992).

The result that hedgers do not use commodity option contracts unless there are price biases is completely contrary to the literature published by the Chicago Board of Trade (CBOT) and other similar institutions. For example, in the publication *The Flexible Choice: Hedging with CBOT Agricultural Options*, ten general hedging strategies are detailed, none of which assume price biases. The following advice is given to hedgers concerning how to choose the most appropriate combination of future and option contracts.

> "Your first step is to determine your market objective. Then, based on your marketing goals, the amount of risk you want to assume, and the profit potential of any particular strategy, determine which ones are most appropriate for you." (CBOT; page 3).

According to the CBOT, one of the main advantages of holding a long put option over a short futures contract is that the option allows hedgers to "...establish a minimum (floor) selling price for a cash commodity prior to delivery and, at the same time, to be able to take advantage of a price rally." (CBOT, page 11). Of interest in this paper is the specification of an objective function that captures the CBOT notion of hedging with option contracts. Clearly we must go beyond the standard assumptions discussed above to specify this function because with such assumptions the hedger prefers to trade away any potential for a price rally in exchange for a relatively higher and more constant price floor. In particular, we require an objective function that allows hedgers to behave quasi

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risk-loving once risk has been reduced to a personally acceptable level.

A safety-first approach to hedging is a natural candidate for the problem at hand because it provides a good representation of the hedging problem as described by the CBOT. As well, it has been used extensively in risk programming applications and, according to a recent survey, reflects the way that many primary commodity producers view risk (Atwood et. al 1988). Interestingly, one of the first formal applications of safety-first rules was in the context of hedging with futures contracts (Telser 1955). Telser's approach assumes that a hedger maximizes the expected net return subject to a sufficiently low probability of the net return falling below some specified level. Telser's specification is not appropriate in the current context, however, because by assumption there is no speculative motive and hence the net return is invariant to the hedging position.

Roy (1952) proposed that agents simply maximize the probability of obtaining above some specified return. We adopt Roy's approach in this paper but assume that agents follow such a strategy only so long as their net return does not fall below some price floor. Because of this additional constraint, the target price that the hedger is trying to obtain may be above the mean return. For example, if the mean price of a commodity is \$5.00/bushel, a hedger may wish to choose the combination of futures and options that maximize the probability of his obtaining \$7.00/bushel or more subject to the constraint that his net selling price does not fall below \$3.50/bushel. The \$3.50/bushel may be the minimum net selling price that the hedger requires to meet fixed financial obligations.

A common criticism of the safety-first approach in general is that it is generally not consistent with the axioms of the expected utility hypothesis (Pyle and Turnovsky 1970). With the specific approach used in this paper, the problem lies with the specification of the upper revenue target. Why would one hedger choose \$7.00 per bushel as an upper revenue target while another hedger would choose \$6.00/bushel or \$8.00/bushel? We show that the actual value of the upper revenue target is not important in of itself. Rather, the upper revenue target serves to parameterize the preferences of a hedger over a family of specific price distributions which emerge from the optimization process.

Specifically, the problem as specified gives rise to family of payoff schedules characterized by a constant net price equal to the floor price for all future price realizations less than the option's strike price and a rising schedule thereafter. The higher the upper revenue target specified by the hedger, the higher the strike price chosen,[2] the longer the flat portion of the payoff schedule and the steeper the rising portion of the payoff schedule. Because a particular distribution is associated with a unique value of the hedger's upper revenue target, a hedger characterized by a specific upper revenue target and floor price is equivalently characterized by his preference for a particular price distribution. In other words, the ranking of price distributions is

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[2] Throughout the analysis we assume that a continuum of strike prices are available to the hedger. In the more realistic case where only a discrete set of strike prices are available, it may not be the case that the hedger will restrict himself to the family of payoff functions described above.

complete.

To keep the analysis focused, we employ a number of simplifying assumptions in addition to the standard ones employed above. Specifically, we assume that the futures price and spot price are identical (i.e., there is no basis risk or transportation cost) and the price follows a uniform price distribution. We begin by showing why we can restrict our attention to the particular class of distributions discussed above. We then characterize the optimal hedging strategy by deriving the optimal number of futures and option contracts and the optimal strike price for select values of the price floor and the upper revenue target. Finally, we illustrate how the optimal hedging combination changes for alternative parameter values.

Model

Basic Assumptions

Consider a simple world where the date 1 futures price, f, follows a uniform distribution on the interval [0,1]. As well, there is no basis risk or transportation charges implying that the date 1 futures price and date 1 spot price are identical. At date 0 an agent has one unit of a commodity in store which is to be sold at date 1. The agent can hedge by contracting for X units of the commodity in the futures market and/or Z units of the commodity in the options market. A positive (negative) value of X indicates a short (long) position in the futures market and a positive (negative) value of Z indicates that the agent has purchased (wrote) put options. For simplicity, there are no margin calls and contracts in both the futures and options market can only be lifted at date 1 which is when the commodity is sold in the spot market.

The date 0 price of the futures contract is assumed to equal 0.5, which is the mean of the price distribution. The price of the option, r, equals $0.5K^2$ which is the expected value of f given that f is less than or equal to the strike price, K. The price of the futures and options contract have both been purposely set equal to their "fair" market values in order to ensure that all results are derived without price biases. Given these assumptions, the net price the agent receives for the commodity at date 1 can be expressed as

$$P = \begin{cases} f + (0.5-f)X + (K-f)Z - rZ & \text{if } f \le K \\ f + (0.5-f)X - rZ & \text{if } f > K \end{cases} \tag{1}$$

Figure 1 is a graph of the net price function for several different hedging scenarios including remaining unhedged. The numbers in parenthesis behind each descriptive label indicates the type and size of the hedging contract. In general the agent has considerable control over the location and shape of the payoff function through his choice of X, Z and K. For the moment we restrict our attention to the case where either futures or options (but not both) are used to hedge.

Figure 1 clearly illustrates the tradeoff between downside protection and upside potential. In the extreme case, the agent can eliminate all downside risk and upside potential by locking in the mean price by choosing X=1. In contrast, using an option with Z=1 protects the agent against "very" low prices yet provides the agent with the potential to receive an above-average price. Of course downside protection is less than in the case of X=1 and upside potential is less than the unhedged case because the agent must purchase the option. Also illustrated in Figure 1 are two intermediate hedging scenarios: X=0.6 and Z=0.6. In both of these cases, the agent's net price is rising over the entire domain of f thus providing relatively less downside protection and relatively more upside potential.

Figure 2 is a graph of the cumulative density functions (cdfs) for the various price distributions displayed in Figure 1. The cdfs are calculated by solving each respective expression in equation (1) for f and substituting the resulting equations into the appropriate probability expression. Specifically,

$$F(\pi) = PR(P<\pi) = \begin{cases} PR\left[f < \dfrac{\pi - 0.5X - (K-r)Z}{1-X-Z}\right] & \text{if } \pi \le K \\ PR\left[f < \dfrac{\pi - 0.5X + rZ}{1-X}\right] & \text{if } \pi > K \end{cases} \tag{2}$$

Because f has a uniform distribution on [0,1], the cdf for P is simply given by the expression on the right-hand side of the top (bottom) inequality in equation (2) when $\pi \le K$ ($\pi > K$).[3]

Notice in Figure 2 that there is a zero probability of obtaining a price either above or below the mean price when X=1. When Z=1, the agent has a price floor of 0.375 (i.e., the strike price less the premium) as indicated by the cdf lying on the horizontal axis until 0.375. For a price just above 0.375, the cdf jumps to the strike price (K=0.5 in this example) because by construction there is a K percent chance that the futures price will not exceed the strike price. For prices above 0.375, the cdf increases at the same rate as the no-hedge cdf because the option has no value for these price states.

For the case of X=0.6, the agent's price floor is only 60 percent of the mean price. The cdf increases from this value until the maximum attainable price has been reached at f=0.7. This contract provides the agent with some upside potential at the expense of relatively lower downside protection. Finally, when Z=0.6, there exists a relatively lower price floor and a region in which the cdf is increasing in f prior to "jumping". Notice that for equal values of X and Z, hedging with a put generally results in a relatively higher probability of receiving below a below-average price and a relatively higher probability of receiving above an above-average price. As is shown below, this property is important because it can be exploited by agents who hedge according to safety-first rules.

Figure 2 can also be used to show why a VNM risk-averse agent who is assumed to maximize a VNM expected utility function will always choose X=1 over any other hedging strategy. Recall (e.g., Laffont 1989) that a risk-averse agent will always choose one distribution over another if it stochastically dominates in a second-order sense. Second-order stochastic dominance requires that the cumulative area under the cdf of

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[3] Assuming, of course, that X, Z and K are chosen such that equation (2) evaluates between 0 and 1.

the preferred distribution be never greater than the cumulative area under a competing distribution for all values of f. In Figure 2 it is easy to see that the X=1 contract satisfies this property with respect to all other contracts. Moreover, the X=0.6 contract dominates the Z=0.6 contract and the unhedged position. In general, when there are no price biases, a distribution involving an option always has a greater variance and the same mean as compared with a distribution involving a comparable amount of short futures. Hence, as shown formally by LMH, under the standard assumptions a risk-averse hedger who maximizes a VNM expected utility function will never choose to hold option contracts in their hedging portfolio unless their is a speculative motive for doing so.

The remainder of this section is used to show that hedgers who use safety-first rules rather than maximize VNM expected utility functions will never choose X=1 as an optimal hedging strategy and will generally always demand a combination of options and futures.

CBOT Recommendations and Safety-First Rules

As discussed in the Introduction, we wish to determine the set of hedging instruments which maximize the probability of obtaining a net price equal to or above π_U subject to a price floor of π_L. If $\pi_U \leq 0.5$ then the solution to the problem is trivial and generally non-unique. This is because when $\pi_U < 0.5$ there are a multitude of hedging combinations (always including {X=1, Z=0}) that results in a zero probability of falling below either π_L or π_U. Thus, we restrict our analysis to the case where $\pi_U > 0.5$.

The first major result of this paper is that when $\pi_U > 0.5$ and an agent's behaviour is consistent with the safety-first approach described above, then it is optimal for the price floor to be binding for all f≤K. This result immediately implies from equation (1) that X+Z=1. Hence, agents choose from a continuum of net payoff functions that differ only with respect to the location of the kink point on the price floor and the steepness of the schedule after the kink point. In Figure 3 several such functions are illustrated. Notice that for payoff functions characterized by a relatively high strike price, there are more states that the hedger will receive the floor price but during the other states the potential payoff is relatively higher. Hedgers who prefer "large" payoffs that occur less frequently to "small" payoffs that occur more frequently will choose a relatively steep payoff function in Figure 3.

Figure 4 uses three arbitrary cdfs to illustrates why it is optimal for the hedger to ensure that the price floor is binding for f<K. Each of the three distributions can be achieved through an appropriate choice of X, Z and K.[4] Two of the cdfs are consistent with a binding price floor since they jump from 0 to K at P=0.3 which is the price floor assumed for this particular example. The third cdf does not "jump" at P=0.3 but rather rises at some positive rate before reaching its kink point at F(P)=K. For this cdf, X+Z<1 implying that the floor price is binding only at f=0.

Figure 4 shows that for any arbitrarily chosen upper revenue target above 0.5 (e.g., π_U^1 or π_U^2), a hedger whose price floor is not perfectly binding when f<K can

[4] The areas under each cdf up to the price where the right-most cdf function takes on a value of 1 are equal. Hence, the underlying distributions have the same means.

increase the probability of obtaining a price above that upper revenue target. This is accomplished by adjusting the hedge to the extent that the price floor is completely binding and by adjusting the strike price of the option. For example, notice in Figure 4 that moving from the cdf with a non-binding price floor to the cdf with a binding price floor and a comparatively low strike price, increases the probability of obtaining a price in excess of π_U^1. Similarly, moving from a non-binding to a binding price floor and increasing the strike price increases the probability of obtaining a price in excess of π_U^2.

Optimal Hedge

Because it is optimal for the price floor to be binding, optimal values for both X and Z are completely specified by the safety-first constraint for a given value of K. Hence, K is effectively the only choice variable for the hedger. The solution values of X and Z for a particular value of K are derived as follows. At f=0 it follows from equation (1) that P=0.5X+(K-r)Z. Similarly, at f=K it follows that P=K+(0.5-K)X-rZ. Setting these two expressions equal to the price floor, π_L, and solving for X and Z results in:

$$(3) \qquad X^* = \frac{2[\pi_L - (K-r)]}{1 - 2(K-r)} \quad and \quad Z^* = \frac{1 - 2\pi_L}{1 - 2(K-r)}.$$

Equation 3 shows that for $\pi_L<0.5$ and $\pi_U>0.5$, the optimal amount of futures to hold, X^*, is a decreasing function of K while the optimal amount of options to hold, Z^*, is an increasing function of K. For moderate to high values of K, $X^*<0$ implying that the optimal futures position is a long rather than a short position. Notice in Figure 4 that higher values of K extend the height of the jump of the cdf above P=0.3 and also reduces the slope of the cdf for P>0.3.

The value of K that maximizes the probability of achieving above some upper revenue target, π_U, can be derived by substituting the expressions for X^* and Z^* from equation (3) into equation (2) and choosing K to minimize the height of the cdf at $P=\pi_U$. Figure 5 provides a graphical illustration of that optimization process under the assumption that π_U^* is the upper revenue target. The cdf corresponding to the optimal hedging combination is given by the dark shaded line. This is because choosing either a higher or lower value for K (i.e., either increasing or decreasing the height of the "jump") does not decrease the height of the cdf at π_U^* an in general will increase it.

Sensitivity Analysis

Table 1 details the optimal hedging combination for alternative pairs of values for π_L and π_U. Notice that for a given value of π_L, a higher value of π_U results in more options being used, a less positive or more negative position in the futures market and a higher strike price for the options that are used. This result makes sense because increasing Z and K and decreasing X extends the price floor which in turn increases the probability of obtaining a price that is relatively high above the mean (see Figure 6). For example, with a floor price of 0.25, maximizing the probability of obtaining a price above 0.6 is best achieve by choosing X=0.01, Z=0.99 and K=0.29. If 0.8 was the upper revenue target, however, it would be necessary for the hedger to set X=-2.46, Z=3.46 and to choose a strike price of K=0.62.

The effect of alternative values of π_L on the optimal hedging combination for a given value of π_U is more ambiguous. For relatively low values of π_U (e.g., 0.6 or less), higher values of π_L result in relatively fewer options being used, a more positive futures position and a higher strike price. However, the opposite is true for relatively high values of π_U (e.g., 0.8 or above). For intermediate values of π_U (e.g., 0.7), the use of options initially decreases but eventually increases as π_L is increased in value. Further research is needed to understand the reasons for these latter effects.

Table 1: Optimal Hedging Combinations

π_L	π_u	X	Z	K	$1\text{-}F(\pi_u)$
0.1	0.55	0	1	0.11	0.44
	0.60	-0.25	1.25	0.20	0.40
	0.70	-0.78	1.78	0.33	0.33
	0.80	-1.46	2.46	0.43	0.28
	0.90	-2.20	3.20	0.50	0.25
0.25	0.55	0.27	0.73	0.17	0.42
	0.60	0.01	0.99	0.29	0.36
	0.70	-0.65	1.65	0.45	0.28
	0.80	-1.47	2.47	0.55	0.23
	0.90	-2.46	3.46	0.62	0.19
0.4	0.55	0.55	0.45	0.33	0.33
	0.60	0.20	0.80	0.50	0.25
	0.70	-0.84	1.84	0.67	0.17
	0.80	-2.20	3.20	0.75	0.12
	0.90	-4.00	5.00	0.80	0.10

Conclusions

This paper used a modified safety-first approach to examine a hedger's demand for option contracts. If hedgers are assumed to maximize the probability of obtaining above a prespecified price subject to a specified price floor, the problem reduces to selecting from a series of net payoff schedules, each characterized by a binding floor price for the futures price less than the strike price and rising thereafter. The payoff schedules differ only with respect to where the kink point on the floor price occurs and the steepness of the schedule after the kink point. Agents who prefer infrequent high payoffs subject to a price floor over frequent small payoffs subject to the same price floor will use more options, choose a higher strike price and take either less of a short or more of long position in the futures market.

This analysis demonstrates that options are likely to be highly useful for hedgers who take a safety-first approach to decision making, an approach that appears to be consistent with the recommendations of the CBOT. This finding is in stark contrast to conventional results which conclude that a hedger has no use for options unless price biases or non-linearities are present. This analysis is only a preliminary step toward understanding a hedger's demand for options when using a safety-first approach. Alternative specifications of the objective function and the constraint are certainly possible. As well, the effects of basis risk and a more general price distributions should also be considered. These topics will hopefully be the focus of future research.

References

Atwood, J., M. Watts, G. Helmers and L. Held. "Incorporating Safety-First Constraints Into Linear Programming Production Models." *Western Journal of Agricultural Economics* 13 (1988): 29-36.

Berck, P. "Portfolio Theory and the Demand for Futures: The Case of California Cotton." *American Journal of Agricultural Economics* 63 (1981): 466-74.

Bond, G, S. Thomspon and B. Lee. "Application of a Simplified hedging Rule." *Journal of Futures Markets* 7 (1987): 65-72.

Chicago Board of Trade. *The Flexible Choice: Hedging with CBOT Agricultural Options* (1990).

Heifner, R.G. "Optimal Hedging Levels and Hedging Effectiveness in Cattle Feeding." *Agricultural Economics Research* 24 (1972): 25-36.

Johnson, L. "The Theory of Hedging and Speculation in Commodity Futures Markets." *Review of Economic Studies* 27 (1960): 139-51.

Laffont, J. *The Economics of Uncertainty and Information* MIT Press, Cambridge, Massachusetts (1990).

Lapan, H., G. Moschini and S. Hanson. "Production, Hedging and Speculative Decisions with Option and Future Contracts." *American Journal of Agricultural Economics* 73 (1991): 66-74.

Meyer, J. and J. Robison. "Hedging Under Output Price Randomness." *American Journal of Agricultural Economics* 70 (1988): 268-72.

Moschini, G. and H. Lapan. "Hedging Price Risk with Options and Futures for the Competitive Firm with Production Flexibility." *International Economics Review* 33 (1992): 607-618.

Pyle, D., and S. Turnovsky. "Safety-First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis." *Review of Economics and Statistics* 52 (1970): 75-81.

Roy, A. "Safety-First and Holding of Assets." *Econometrica* 20 (1952):431-49.

Telser, L. "Safety-First and Hedging." *Review of Economic Studies* 23 (1955):1-16.

Figure 1

Payoffs for Various Hedging Alternatives

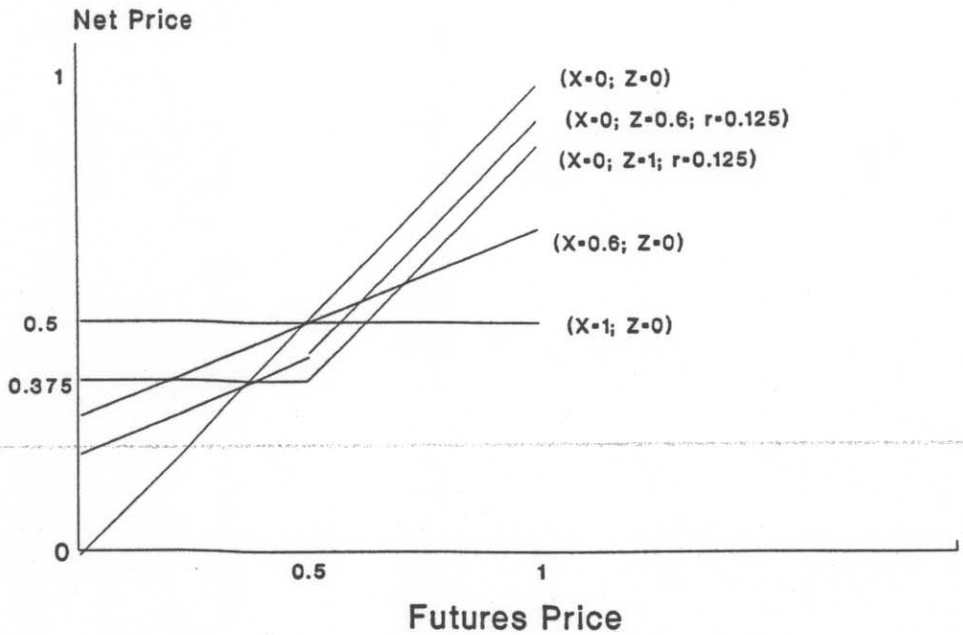

Figure 2

CDF's for Alternative Hedging Strategies

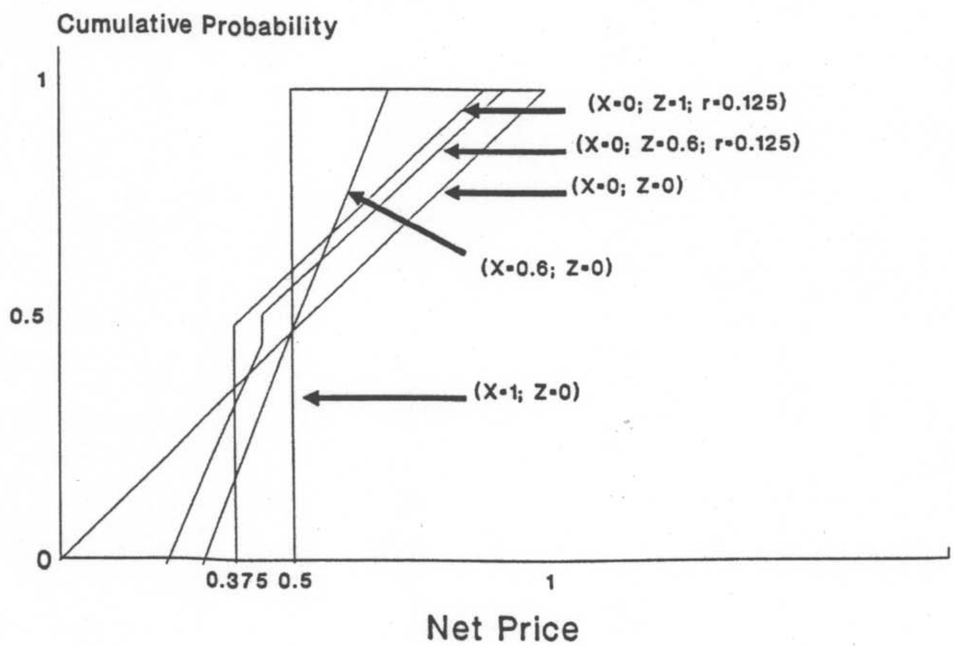

Figure 3
Optimal Payoffs for Alternative K

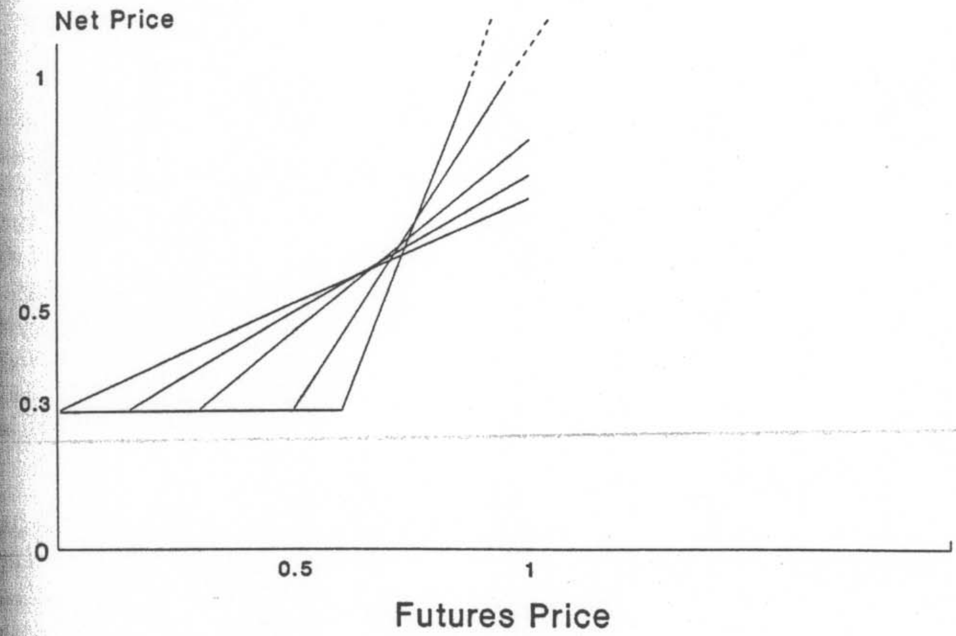

Figure 4
Binding versus Non-Binding Floor Price

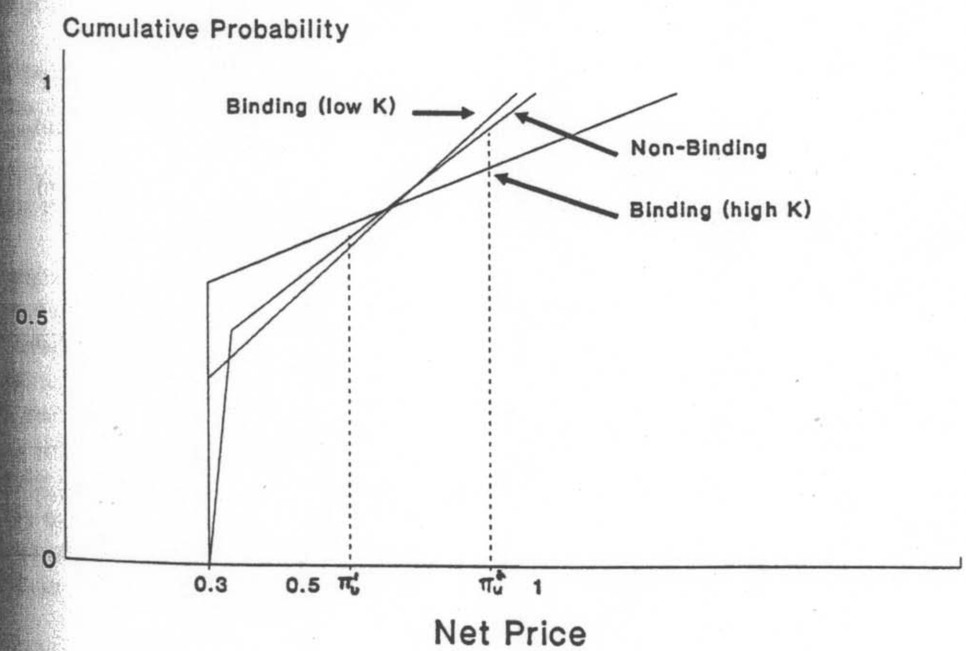

Figure 5
Marginal Conditions for Optimal Hedge

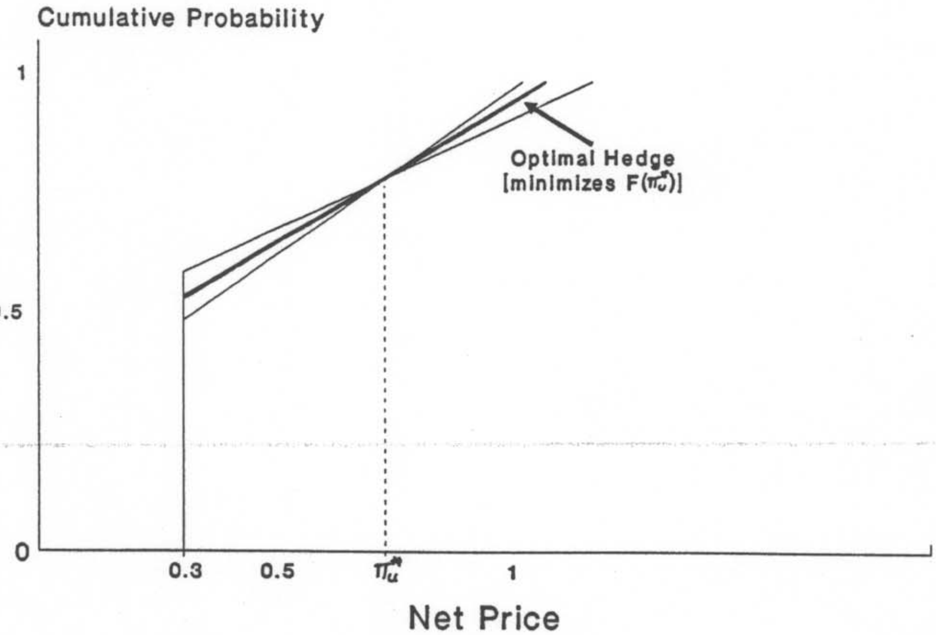

Figure 6
Sensitivity of Optimal Strategy to π_u