Pre-Harvest Dynamic Hedging: An Analysis of Transaction Costs and Contract Lumpiness for Soybean Farmers

by

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Hedging by farmers in the futures markets can occur as a single short position at the futures contracts with no revision during the crop growing period (traditional hedging theory, e.g., Working 1953; Johnson 1960; Telser 1955, 1958; Stein 1961; Ederington 1979; and, Adler and Detemple 1988). On the other hand, changes in the expected total production and changes in the futures prices may require updates in the hedge position sometime during the growing period.

Recent literature has investigated dynamic hedging by grain producers throughout the growing season (Karp 1987, 1988; Martinez and Zering 1992a, 1992b). These papers have used Linear Exponential Gaussian (LEG) control, as an analytical procedure, to derive the optimal hedge solution. One of the short comings in using this procedure is that contract size is a continuous variable. In addition, transaction costs are imposed ex-post. Closed form solution methods used in these papers are not flexible enough to capture contract lumpiness because of the discrete nature in which contracts are bought and sold. Such discrete phenomena render models analytically intractable. Shanker (1993), however, explicitly considered contract lumpiness in a single period model.

In this paper, a stochastic dynamic programming model is used to solve the dynamic hedging problem for a soybean producer. We use a variant of the model presented by Karp and by Martinez and Zering. Our model differs from other dynamic hedging models presented in the literature in that we explicitly incorporate transaction costs and contract lumpiness. In addition, the model captures hedging from a whole farm perspective. Polynomial projection methods are used to solve the underlying stochastic dynamic program which lacks a closed form solution (Judd 1991).

In the first part of the paper, we develop a model of a soybean producer who begins a hedging program at planting time and ends it at harvest time and, in the second part, we use Monte-Carlo counterfactual simulations to evaluate the effects of transaction costs and contract lumpiness on the optimal hedge ratio of a representative soybean farmer. The model shows that transaction costs and contract lumpiness play an important role in hedging behavior.

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Theoretical Model

In our model, a producer with initial wealth ω begins a hedging program at time $t=1$ by selling some futures contracts. The producer may be able to revise his hedge position throughout the growing season from time $t=2$ to $t=T$ by selling or purchasing futures contracts. In the terminal period $T+1$ the producer offsets the remaining hedge positions by purchasing futures contracts and selling the harvested crop on the cash market. To implement the dynamic hedging program the producer chooses an optimal integer number of futures contracts n_t in each period to optimize the expected utility of terminal wealth ω_{T+1}. A power utility function is used which allows for the assumptions of either decreasing absolute risk aversion or constant relative risk aversion.

The producer's decision model and the representative Bellman's equation are given as follows:

$$V_t(\omega_t, P_{t,T+1}, n_{t-1}) = \max_{n_t} \; E_t V_{t+1}(\omega_{t+1}, P_{t+1,T+1}, n_t) \qquad (1)$$

s.t.

$$BS_{t+1} = (1+r)[\omega_t - |n_t - n_{t-1}| \times k \times c]$$

$$MA_{t+1} = -(P_{t+1,T+1} - P_{t,T+1}) \times k \times n_t$$

$$FI_{t+1} = (P_{t+1,T+1} - P_{t,T+1}) \times y_{T+1} \times a \times \delta^{t+1}$$

$$\omega_{t+1} = BS_{t+1} + MA_{t+1} + FI_{t+1}$$

$$\log P_{t+1,T+1} = \log P_{t,T+1} + e_t$$

$$\delta^t = \frac{1}{(1+r)^{(T-t)}}$$

and in the terminal period, the Bellman's equation is explicitly defined as:

$$V_{T+1}(\omega_{T+1}) = u(\omega_{T+1})$$

$$u(\omega_{T+1}) = \frac{(\omega_{T+1})^{(1-\gamma)}}{(1-\gamma)}$$

where E is the expectations operator; u is the von Neumann-Morgenstern utility function; t is time with $t=1,...,T+1$ (e.g., May, ..., November); $T+1$ is harvest time; ω_{T+1} is terminal wealth; γ is the coefficient of constant relative risk aversion; $P_{t,T+1}$ is the futures price at t

for a contract with maturity in $T+1$; BS_t is the balance sheet account and at $t=1$ represents the initial wealth ω, and for the remaining periods it represents the wealth level less the transaction costs of change in futures position; n_t is the decision variable representing the integer number of contracts used to hedge at t which maximizes equation 1 in which n_0 is zero; c is the transaction cost per bushel of the commodity in the futures contract; k is the proportionality factor for contract size (1,000 or 5,000 bu.); MA_t is the margin account which represents the profit or loss from the hedge position; FI_t is the field inventory account which represents the expected present value of the growing crop in each time t prior to harvest; y_{T+1} is the expected per-acre yield at harvest; δ is the per-period discount factor; a is the soybean acreage; e_t is the stochastic term which is normally distributed with zero mean and σ^2 variance; and r is the interest rate for the hedge period.

Simulation Results

Monte-Carlo counterfactual simulations are performed to investigate the influence of transaction costs and contract lumpiness on the expected hedge ratio for 10 periods covering the growing season (May 1 to November 1). Throughout the paper we assume an interest rate r of 5 percent and initial wealth ω of $850,000.

First and foremost, little dynamic behavior occurs in the simulations. Producers basically maintain their hedges throughout the growing season. Typically, the number of contracts purchased to hedge the crop increases from period 1 to 3 and stays constant thereafter, see Figure 1. These results are not surprising given fixed yield, fixed conditional mean and constant futures price variance over time. Although not presented herein, recent simulations indicate that changes in the pattern of price volatility and yield over each period of time generate a more dynamic hedging pattern. In spite of this, our model gives some nice intuitive results.

The first set of simulations investigate the influence of transaction costs on the optimal hedging behavior of a soybean farmer. Figure 2 highlights the actions of our representative producer facing a schedule of round turn transaction costs (TC). The model parameters are shown above the graph. We investigate the hedging behavior over a range of TC's defined in cents per bushel of the commodity in the contract and two contract sizes (1,000 and 5,000 bushels). For example, 1.4 cents round turn transaction cost/bu. of contract represents a $70.00 (5,000 bu.× 1.4 cents/bu. of contract) round turn transaction cost per 5,000 bu. contract and a $14.00 round turn transaction cost per 1,000 bu. contract.

The producer's expected hedge ratio decreases as the transaction cost per bushel increases. The rate of decrease is dependent on the lumpiness of the contract. Figure 2 shows that the decrease in the expected hedge ratio for the 5,000 bushel contract occurs near the transaction cost of 1.20 ($60.00/contract) and 1.55 cents/bu. of contract ($77.50/contract). The expected hedge ratio pattern (decreases as a function of the TC) is smoother for 1,000 bushel contracts. Hedging behavior is driven to zero at 1.60 ($80.00/contract) and 1.7 cents/bu. of contract ($17.00/contract) for the 5,000 and 1,000 bushel contracts, respectively.

Figure 3 illustrates the effect of futures price on the hedging behavior of producers using 1,000 and 5,000 bushel contracts. For both contract sizes, the expected hedge ratio increases as the initial futures price increases. The expected hedge ratio for the 5,000 bushel contract over the schedule of futures prices ($5.50 to $6.70 per bushel) jumps discretely near $5.70 and $6.50/bu. The hedge ratio for the 1,000 bushel contract jumps in more discrete steps at more futures prices. Thus, a producer who uses the larger contract is locked into a hedge position over a wider range of futures prices.

The coefficient of constant relative risk aversion (CRRA) plays a major role in the use of the futures markets. Farmers with a higher risk aversion will seek to use the futures markets to reduce price risk. Figure 4 shows that a producer with a higher risk aversion will initiate hedging activity at a lower futures price. Although not shown here, the differential in hedging activity with respect to changes in the coefficient of CRRA for the smaller contract size is similar.

One of the main impediments to the use of futures contracts is the lack of production to be able to hedge in the futures markets. Even if a particular producer has sufficient production to hedge by using one contract, he may not enter the market because of transaction cost or low risk aversion. Figure 5 demonstrates the influence of total production on the expected hedge ratio. Given the model parameters addressed in Figure 5, hedging behavior does not begin until the total production reaches 20,000 to 22,500 bushels of soybean, depending on the contract size. When the 5,000 bushel contract is used, the expected hedge ratio jumps discretely when total production reaches 22,500 and 27,500 bushels. The expected hedge ratio line, for the 1,000 bushel contract, jumps in smaller discrete steps, reflecting the fact that this contract size is less lumpy.

Summary and Conclusions

In this paper, we develop a dynamic hedging model that explicitly incorporates transaction cost and contract lumpiness. Stochastic dynamic programming techniques were used to solve for the optimal dynamic hedge.

Given a fixed yield, fixed conditional futures price mean, and constant futures price volatility over the periods, the model predicts little dynamic behavior. However, the model shows that transaction costs and contract lumpiness play an important role in hedging behavior. The expected hedge ratio increases in the response to an increasing initial futures price, total production, and risk aversion levels. It decreases in response to an increasing transaction cost level. Most notable in these counterfactual simulations is the discrete nature of futures hedging.

These results are a spring board to further refinement of our model. Future research will focus on incorporating a specification for the futures price behavior that will allow for changing conditional mean and variance of futures prices, and incorporation of an equation for yield updates throughout the growing season. These changes will induce to more interesting dynamic hedging patterns.

References

Adler, M. and J.B. Detemple. "Hedging with Futures in an Intertemporal Portfolio Context." *Journal of Futures Markets* 8(May 1988):249-269.

Ederington, L.H. "The Hedging Performance of the New Futures Markets." *Journal of Finance* 34(March 1979):157-170.

Johnson, L.L. "The Theory of Hedging and Speculation in Commodity Futures." *Review of Economic Studies* 27(June 1960):139-151.

Judd, K.L. *Numerical Methods in Economics*. Manuscript, Hoover Institution, Stanford University, 1991.

Karp, L.S. "Methods for Selecting the Optimal Dynamic Hedge when Production Is Stochastic." *American Journal of Agricultural Economics* 69(August 1987):647-657.

Karp, L.S. "Dynamic Hedging with Uncertain Production." *International Economic Review* 29(November 1988):621-637.

Martinez, S. and K.D. Zering. "Optimal Dynamic Hedging Decisions for Grain Producers." *American Journal of Agricultural Economics* 74(November 1992a):879-888.

Martinez, S. and K.D. Zering. "Effect of Institutional Realities on Dynamic Hedging Performance for a Grain Producer." *Journal of Futures Markets* 12(April 1992b): 237-251.

Shanker, L. "Optimal Hedging under Indivisible Choices." Journal of Futures Markets 13(May 1993):237-259.

Stein, J.L. "The Simultaneous Determination of Spot and Futures Prices." *American Economic Review* 51(December 1961):1012-1025.

Telser, L. "Safety First and Hedging." *Review of Economic Studies* 23(1955):1-16.

Telser, L. "Futures Trading and the Storage of Cotton and Wheat." *Journal of Political Economy* 66(June 1958):233-255.

Working, H. "Futures Trading and Hedging." *American Economic Review* 43(June 1953):314-343.

Figure 1. The effect of time and contract sizes (CS) on the pre-harvest expected hedge ratio of November Soybean futures contract*

(*) Model Parameters: total prod. of soybean - 25,000 bu.; Nov. Soyb. fut. price - $6.00/bu.; round turn trans. cost - 1.4 cents/bu. of contract; CRRA - 0.98; annual fut. price volatility - 0.20

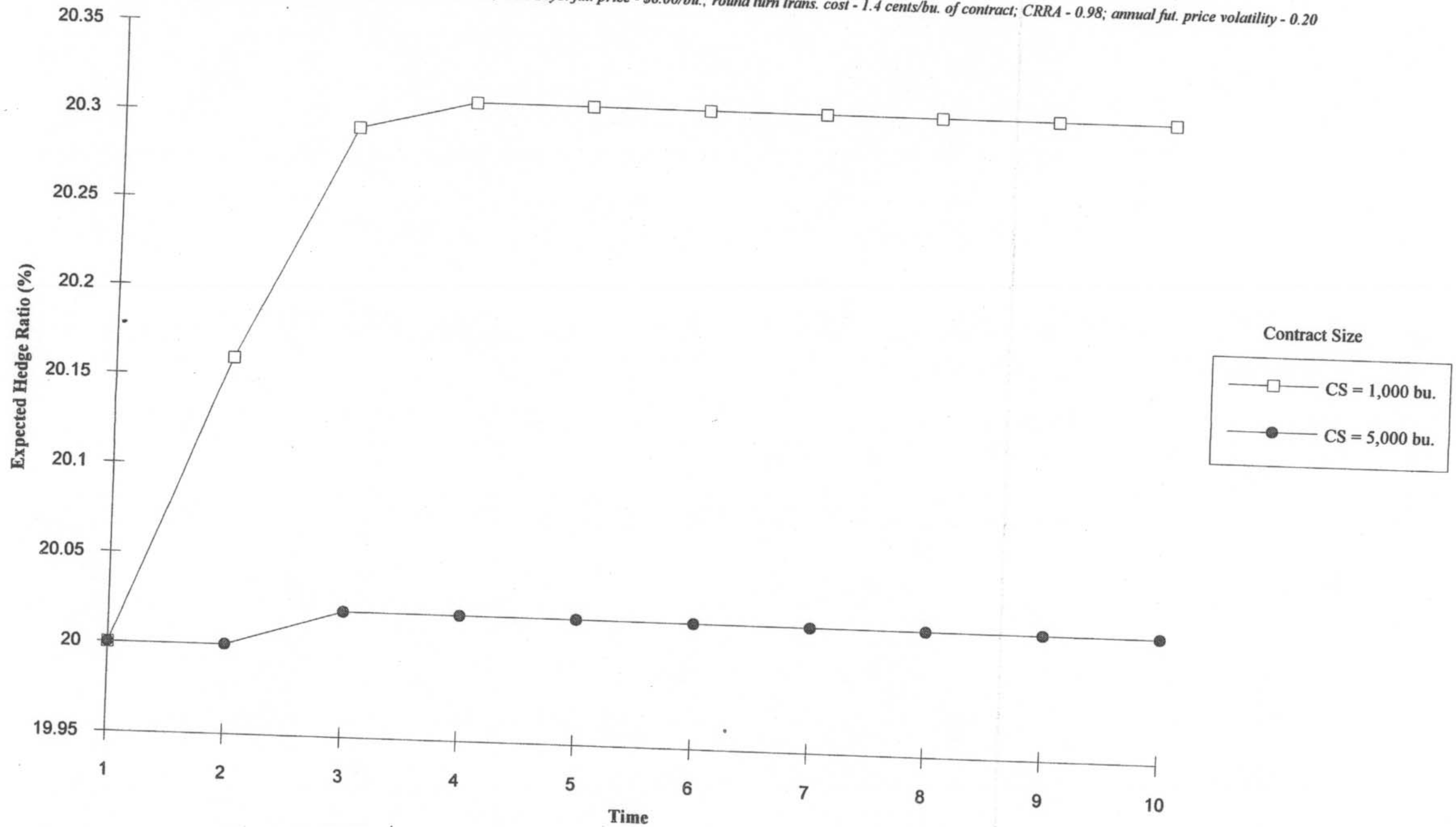

Figure 2. The effect of transaction costs (round turn) and contract sizes (CS) on the pre-harvest expected hedge ratio of November Soybean futures contract (time = 10)*

(*) Model Parameters: total prod. of soybean - 25,000 bu. (area - 500 acres and yield - 50 bu./acre); Nov. Soyb. fut. price - $6.00/bu.; CRRA - 0.98; annual fut. price volatility - 0.20

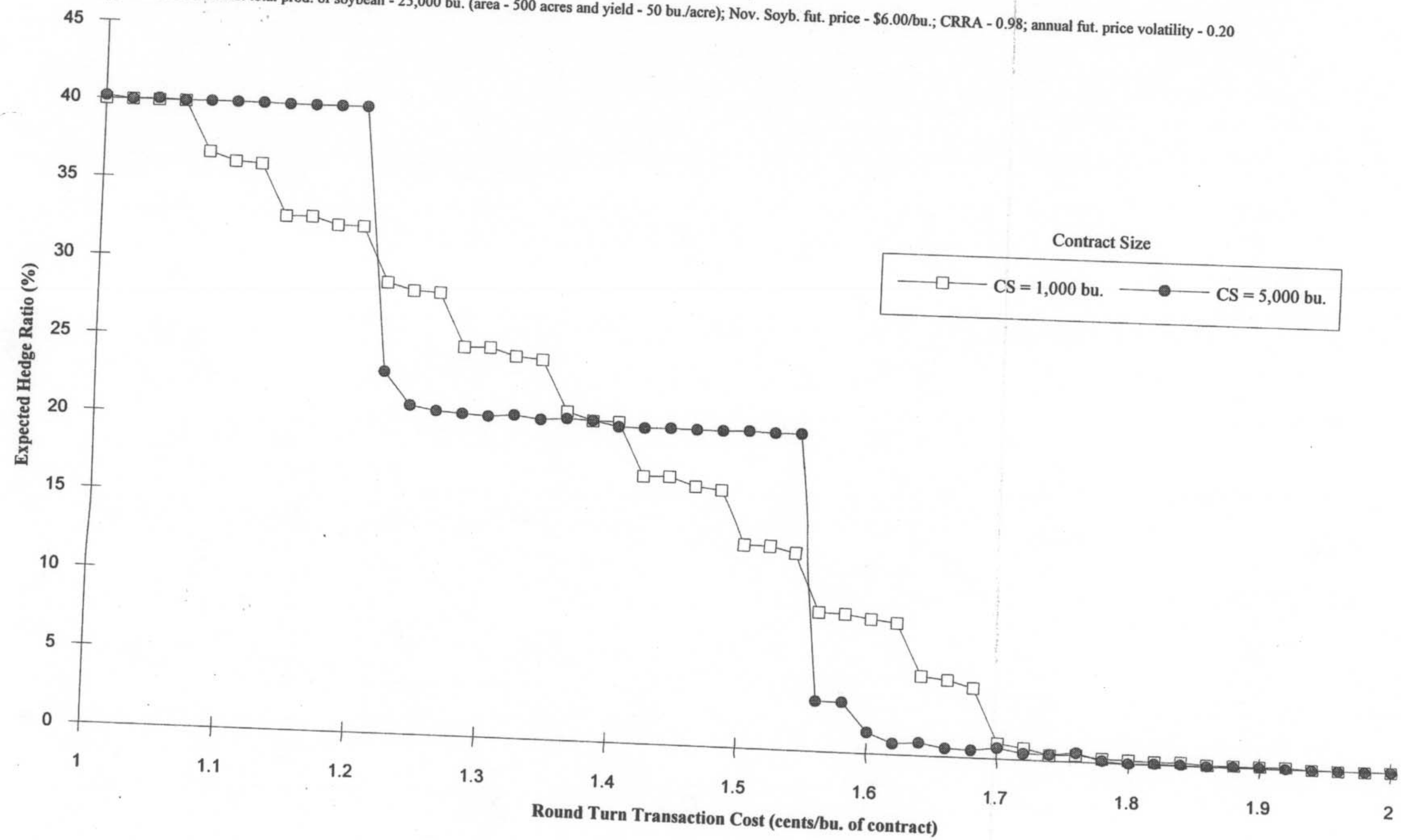

Figure 3. The effect of November Soybean futures prices and contract sizes (CS) on the pre-harvest expected hedge ratio (time = 10)*

(*) Model Par.: total prod. of soyb. - 25,000 bu. (area - 500 acres and yield - 50 bu./acre); round turn trans. cost - 1.4 cents/bu. of contract; CRRA - 0.98; annual fut. price volatility - 0.20

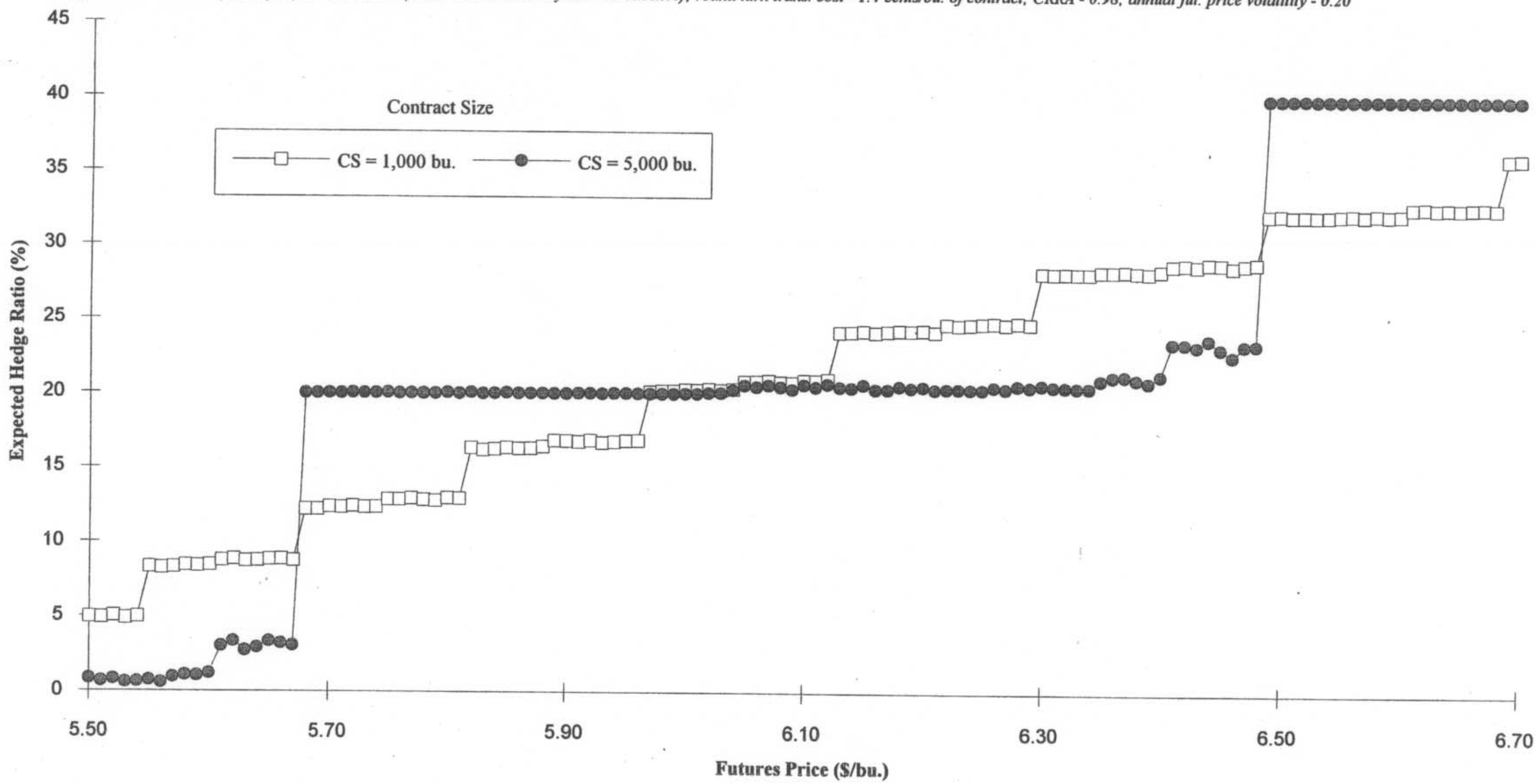

Figure 4. The effect of Nov. Soybean futures prices and coefficient of constant relative risk aversions (CRRA) on the pre-harvest expected hedge ratio (Cont. Size = 5,000 bu. and time = 10)*

(*) Model Par.: total prod. of soybean - 25,000 bu. (area - 500 acres and yield - 50 bu./acre); round turn trans. cost - 1.4 cents/bu. of contract; annual fut. price volatility - 0.20

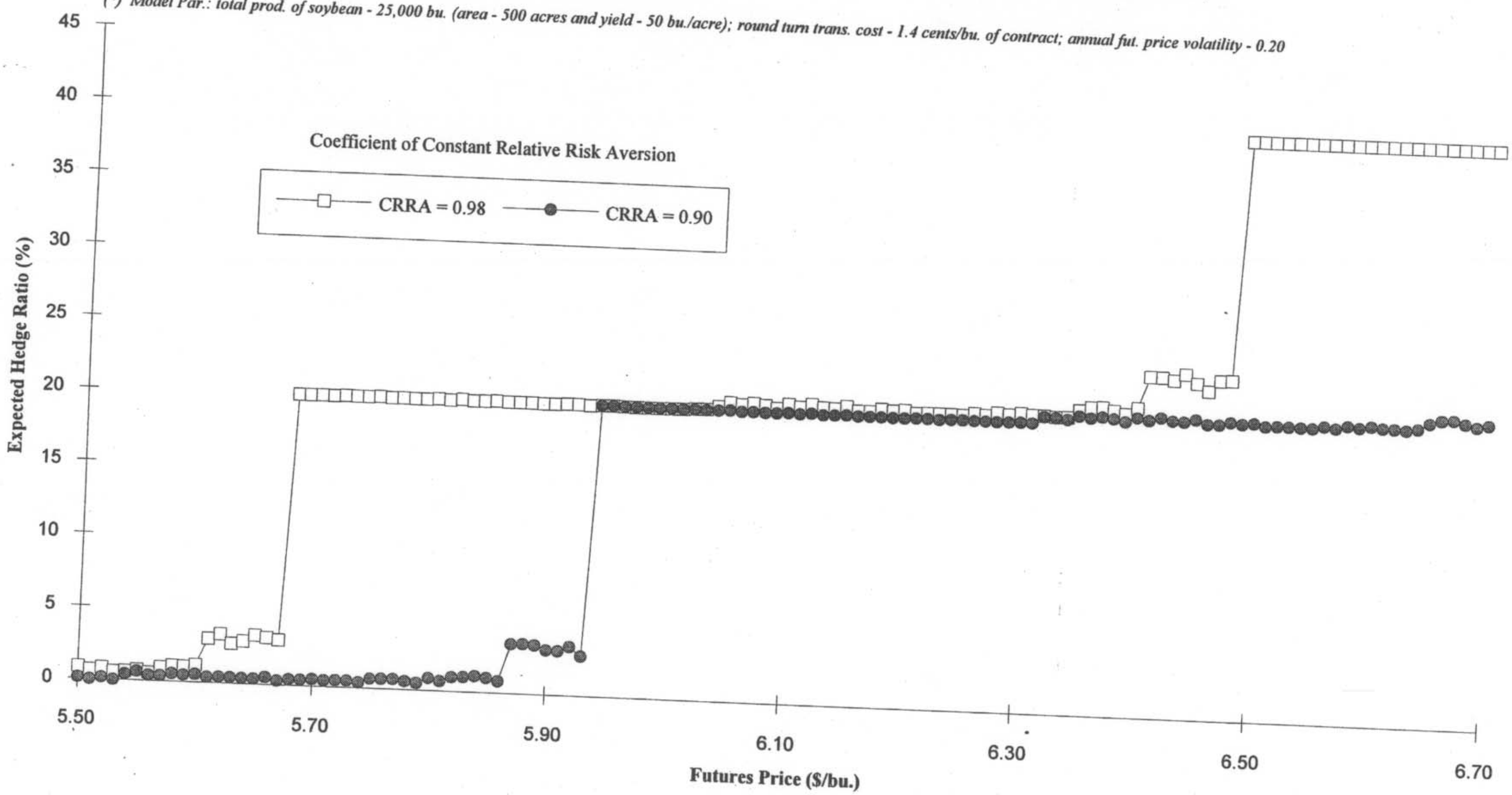

Figure 5. The effect of total production of soybean and contract sizes (CS) on the pre-harvest expected hedge ratio of November Soybean futures contract (time = 10)*

(*) Model Par.: Nov. Soyb. fut. price - $6.00/bu.; round turn trans. cost - 1.4 cents/bu. of contract; CRRA - 0.98; annual fut. price volatility - 0.20