Modeling Futures Prices with the Student's t Autoregressive Model

by

Rob Murphy and Anya McGuirk

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Introduction

Traditional linear time-series models are generally rejected as models of futures price changes. The inability of these models to capture the probabilistic features of the data is often traced to the inappropriateness of the probability assumptions underlying these models. While linear models typically assume normality, changes in futures prices have been observed to follow decidedly non-normal distributions. Specifically, these data are characterized by symmetric, but leptokurtic, distributions (Gordon; Hudson, et. al.).

Several authors have indicated that this distributional property of futures price changes appears to result, at least in part, from non-constant variance in the time series (Brorsen and Yang; Hall, et. al; Venkateswaran, et. al.). Previous research efforts have therefore focused on non-linear models that allow for a time dependent variance structure. Most popular among these models are the autoregressive conditional heteroskedasticity (ARCH) model proposed by Engel and the generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev. Typically, these model formulations specify that the unobserved model error term is normally distributed with a conditional variance which depends on past values of the squared errors.

Recently Spanos (1993) suggested an alternative modeling approach for data which exhibit leptokurtosis and non-linear dependence. He proposes the Student's t Autoregressive model with dynamic heteroskedasticity (STAR) as an alternative to the ARCH-type models. This paper discusses the modeling approach of Spanos and compares the performance of his STAR model with the more usual ARCH and GARCH specifications for modeling changes in futures prices. Motivating this work is recent evidence that the STAR model performs well in modeling weekly exchange rates on the spot market (McGuirk, et. al., 1993b). In that study, the STAR model provided evidence against the efficient market hypothesis that was not uncovered by alternative (and misspecified) models. The STAR model may be a useful tool in similar applications with futures prices.

The remainder of the paper proceeds as follows. In the next section, the conceptual underpinnings of the STAR model are explained and the model is developed. The data used in this study are then described and their distributional properties examined. Next, results of the STAR estimations are compared with ARCH and GARCH model formulations and all models are checked for misspecification. This evidence is then used to draw conclusions regarding the usefulness of the STAR model for describing the conditional variance structure of futures prices.

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STAR Model Conceptualization

The joint distribution of the statistical model observables, the dependent (y_t) and exogenous random variables (X_t), can be represented by $D(Z_1,Z_2,...,Z_T;\phi)$ where $Z_t=\{y_t\ X_t\}$, t is a time subscript and ϕ is a matrix of parameters. Naturally, this joint density function of all of the variables in all time periods is not operational for modeling purposes since it is over-parameterized. However, by making assumptions about the distribution of Z_t, it is possible to reduce $D(Z_1,Z_2,...,Z_T;\phi)$ such that an operational model is obtained. This approach to specifying econometric models has been termed the Probabilistic Reduction approach (Spanos, forthcoming).

In the case of the classical linear regression model, $D(Z_1,Z_2,...,Z_T;\ \phi)$ is assumed to be multivariate normal with each Z_t being independently and identically distributed. The reduction then takes the form:

$$D(Z_1,\ Z_2,\ ...,\ Z_T;\ \phi) = \prod_{t=1}^{T} D(Z_t;\psi_t) = \prod_{t=1}^{T} D(Z_t;\psi) = \prod_{t=1}^{T} D(y_t \mid X_t;\ \psi_1)\, D(X_t;\ \psi_2)$$

The first equality follows from the independence assumption; the joint distribution can be re-written as a product of the marginal distributions. Assuming Z_t is identically distributed allows the t subscript to be dropped from the parameter vector, ψ, in the second equality. Finally, utilizing Bayes' Theorem, $D(Z_t;\psi)$ can be expressed as the product of the distribution of y_t conditional on X_t and the marginal distribution of X_t. The normality assumption ensures that X_t is weakly exogenous with respect to ψ_1 (Engle, 1983), thus only $D(y_t| X_t;\psi_1)$ is important in defining the classical linear regression model. The model can be represented by:

$$y_t = E[y_t \mid X_t = x_t] + u_t$$

where the conditional mean embodies all of the systematic information in the data and u_t is the remaining un-systematic information. The joint normality assumption implies that the conditional mean is linear in the conditioning variables and that the conditional variance is homoskedastic, leading directly to the familiar regression model:

$$y_t = \beta_0 + \beta' x_t + u_t \quad \text{with} \quad VAR[y_t \mid X_t = x_t] = \sigma^2 \ .$$

The key to model formulation via the reduction approach lies in selection of appropriate reduction assumptions (NIID in the case of the normal linear regression model). By changing these assumptions a large number of alternative statistical models can be derived.

To derive the STAR model, the normality assumption is replaced by the assumption that the joint distribution of the observables is multivariate Student's t. In addition, y_t is assumed to be a stationary, asymptotically independent process with a stable autoregressive representation. With these reduction assumptions, the joint distribution $D(y_1,y_2,..., y_T;\phi)$ can be decomposed as follows:

$$D(y_1, y_2, \ldots, y_T; \phi) = D(\mathbf{y}_p; \psi_0) \prod_{t=p+1}^{T} D(y_t | Y^0_{t-1}; \psi)$$

where $\mathbf{y}_p=(y_1,y_2,...,y_p)$ is the initial p observations on y and Y^0_{t-1} is the conditioning information set generated by the past history of y_t. Ignoring the p initial conditions, $D(y_t|Y^0_{t-1};\psi)$ is the relevant distribution for formulating the statistical model which consists of the first two moments of this conditional Student's t distribution (see Spanos, 1993, for details).

Thus, the STAR(m,p,υ) model is given by:

$$y_t = \beta_0 + \sum_{i=1}^{m} \beta_i y_{t-i} + u_t, \quad m \geq 1$$

$$\omega_t^2 = \left[\frac{\nu}{(\nu + t - 3)}\right]\sigma^2\left[1 + \sum_{i=1}^{t-1}\sum_{j=-p}^{p} q_{|j|}(y_{t-i} - \mu)(y_{t-|j|-i} - \mu)\right], \quad q_j = 0 \ \forall \ |j| > p$$

where $u_t=y_t - E[y_t|Y^0_{t-1}]$ is distributed $St(0,\omega_t^2,\nu)$. The expected value of $y_t=\mu$ and $\upsilon>2$ is the degrees of freedom parameter. The conditional mean of the STAR model is linear in the lagged endogenous variables and the conditional variance is a quadratic function of all the past history of the time series. This quadratic variance structure makes use of only p+1 unknown parameters, q_j, and closely resembles a smoothed form of the unconditional variance.

The log likelihood function for the STAR model is:

$$\ln L \propto -\frac{T}{2}\ln(\pi) + \ln\left(\Gamma\left(\frac{1}{2}(\nu + T)\right)\right) - \ln\left(\Gamma\left(\frac{\nu}{2}\right)\right)$$

$$-\frac{T}{2}\ln(\nu\sigma^2) - \frac{1}{2}\sum_{i=p+1}^{T} \ln(c_t^2) - \frac{1}{2}\sum_{i=p+1}^{T} (\nu + t)\ln(\gamma_t^2)$$

where

$\gamma_t^2 = \left(1 + \frac{u_t^2}{\nu\sigma^2 c_t^2}\right)$, $\quad c_t^2 = \left(1 + (Y^0_{t-1} - \mu)' Q_{t-1} (Y^0_{t-1} - \mu)\right)$, and Q_{t-1} is the inverse of the temporal covariance matrix of Y^0_{t-1}. This matrix is banded (number of bands=p) and persymmetric.

Coefficients in the conditional mean and variance of the STAR model are related and must be estimated jointly. There is no maximum likelihood estimator of υ (Zellner), therefore υ must be pre-specified. Choice of υ is guided by examining bivariate density plots of the data and by the sample kurtosis coefficient, $\alpha_4 = 3 + 6/(\upsilon - 4)$.

Normal ARCH and GARCH Models

The normal ARCH model of order p is:

$$y_t = \beta_0 + \sum_{i=1}^{m} \beta_i y_{t-i} + u_t$$

$$u_t \sim N(0, h_t^2)$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2$$

where $m \geq 1$, $\alpha_i \geq 0$, and $\sum_{i=1}^{p} \alpha_i < 1$. This model assumes the errors attached to the conditional mean are normally, but not identically, distributed.

The normal GARCH(p,q) parameterization is similar except that the conditional variance is specified as:

$$h_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \gamma_i h_{t-j}^2 .$$

The GARCH specification approximates a higher-order ARCH process where the α_i, i > 1, decay exponentially.

In contrast to the STAR model, these models are not derived from the joint distribution of the sample by making reduction assumptions. Rather, they are formulated by 'modeling' the conditional variance of an unobservable error term.

Sample Data

Daily closing futures prices for live cattle, live hogs, corn and soybeans are used in this analysis. These price series include all futures contracts for these commodities that traded on the Chicago Mercantile Exchange (live cattle and hogs) and Chicago Board of Trade (corn and soybeans) between January 1983 and December 1992. First differences of the natural logarithms of these prices are used. A continuous series on the nearby contract for each commodity was constructed utilizing the futures contract nearest to delivery until 15 trading days into the month prior to delivery at which time data for the next nearest delivery month were used.[1]

Table 1 gives the descriptive statistics for each data series. All of the series are leptokurtic. The null hypothesis of normally distributed data was rejected for all of the series using the D'Agostino fourth sample moment test (Anscombe and Glynn). Nevertheless, the two non-storable commodities (cattle and hogs) display much less excess

[1]The log differences were taken prior to combining the data for different contracts.

Table 1. Descriptive Statistics, Log Difference of Futures Price, 1983-1992.

	Mean	Std. Dev.	Min.	Max.	Skewness	Kurtosis	N
Cattle	0.0004	0.0094	-0.0281	0.0290	-0.1280	3.6434	2248
Hogs	0.0003	0.0128	-0.0398	0.0369	-0.0669	3.2473	2248
Corn	-0.0001	0.0122	-0.0542	0.0531	-0.0422	5.5063	2247
Soybeans	-0.0002	0.0138	-0.0640	0.0562	-0.1680	4.8210	2224

kurtosis than the storables (corn and soybeans).

Each of the data series is plotted against time in Figures 1 through 4. Leptokurtocity is also indicated in these plots; a large number of observations are very near the mean and a greater number of data points are in the tails of the distribution than would be anticipated from normally-distributed data. Clearly, the livestock data are much less leptokurtic than the grain data. Evidence of non-linear dependence in the data comes in the form of large (small) changes being followed by large (small) changes. Thus, large and small changes appear to be clustered together over time. This is more apparent in the two grain series than in the livestock data.

This graphical evidence and the descriptive statistics indicate the STAR model may be appropriate. To further assess the appropriateness of the STAR reduction assumptions, non-parametric bivariate densities were estimated for each series (see Figures 5 though 8). In these plots, the current value of the variable (CAT=cattle, HOG=hogs, COR=corn, SOY=soybeans) is plotted against the variable lagged one period. For reference purposes, a bivariate normal distribution is plotted in Figure 9 and a Student's t (6 degrees of freedom) is plotted in Figure 10. The density estimates for the livestock series closely resemble the bivariate normal while the density estimates for the grains are more leptokurtoic—suggesting these data may be more adequately described by a Student's t distribution. As normality was rejected for the livestock series, it is possible that they too are distributed Student's t, but with a larger degree of freedom parameter than suggested by the bivariate plots for the grains.

Estimation Results

The previous data plots, the estimated sample moments and the model's ability to capture the probabilistic features of the data were all considered in arriving at the final specification of the STAR models. The most appropriate models were deemed to be: STAR(2,2,13) for live cattle, STAR(2,2,28) for live hogs, STAR(2,2,6) for corn and STAR(2,2,7) for soybeans. Table 2 lists the parameter estimates and results of misspecification tests based on the scaled residuals proposed for these models by Spanos

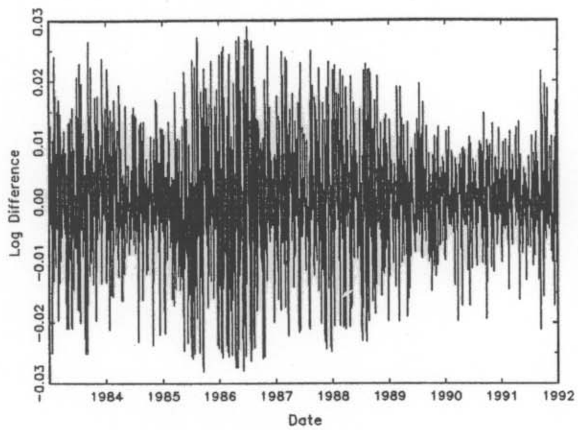

Figure 1. Log Difference T-plot, Cattle.

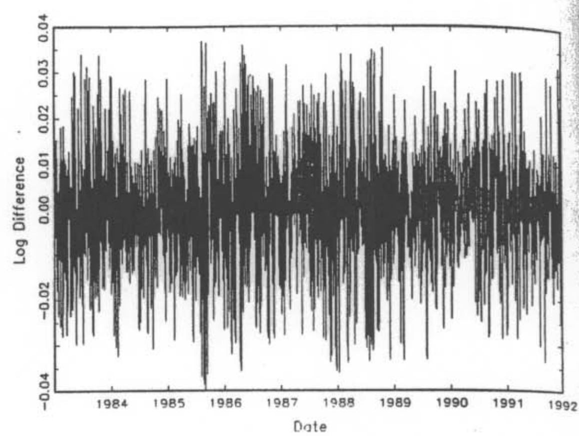

Figure 2. Log Difference T-plot, Hogs.

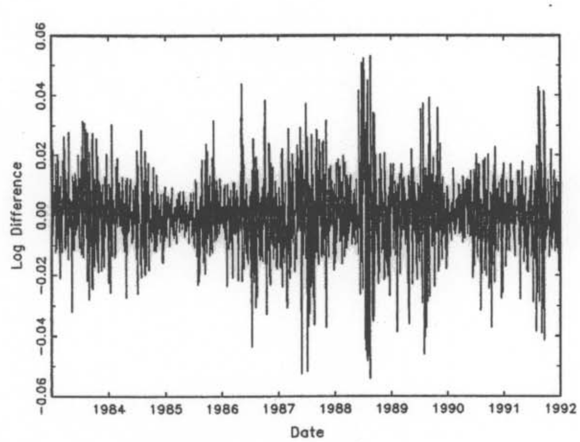

Figure 3. Log Difference T-plot, Corn.

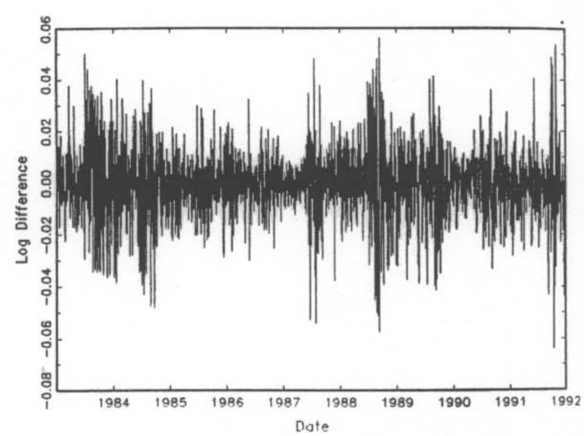

Figure 4. Log Difference T-plot, Soybeans.

(1993).[2]

The skewness and kurtosis tests are the D'Agostino third and fourth moment tests which are approximately distributed N(0,1) under the null hypothesis of normality (D'Agostino and Stephens). The validity of the assumptions that allowed the reduction of the joint distribution of observables can be checked with tests of the conditional mean and variance. In this model, the conditional mean is specified to be linear in the conditioning variables and the parameters of the mean are assumed stable (not time dependent). Also, $y \equiv (y_{p+1},...,y_T)$ is assumed to be an asymptotically independent sample drawn sequentially

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[2]These scaled residuals are defined as $\hat{u}_t/\hat{\omega}_t$ - $\hat{\omega}_t\epsilon_t$ where $\hat{u}_t$ are the unscaled residuals and ϵ_t is a simulated, i.i.d. St(0,1,ν) series. Some uncertainty remains as to the appropriateness of using these residuals for comparisons with the traditional ARCH/GARCH scaled residuals ($\hat{u}_t/\hat{\omega}_t$).

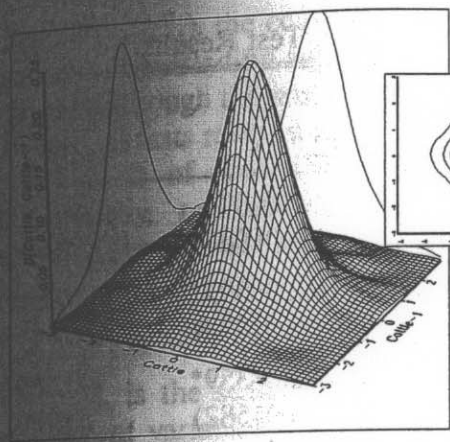

Figure 5. $\hat{D}(CAT, CAT_{t-1})$

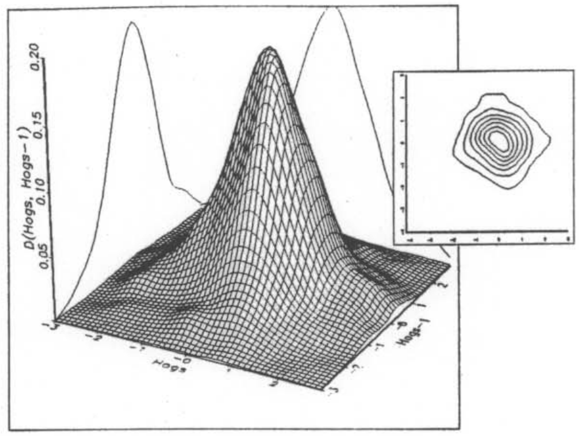

Figure 6. $\hat{D}(HOG, HOG_{t-1})$

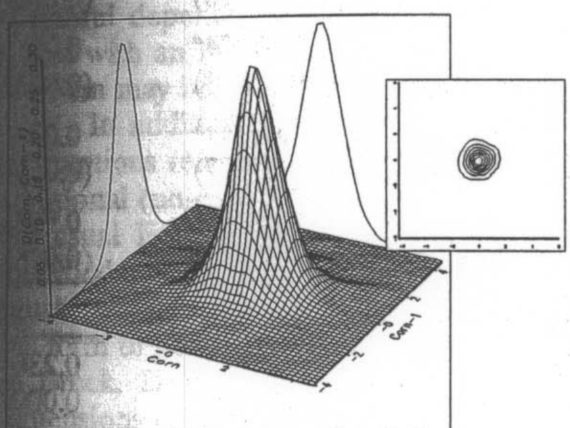

Figure 7. $\hat{D}(COR, COR_{t-1})$

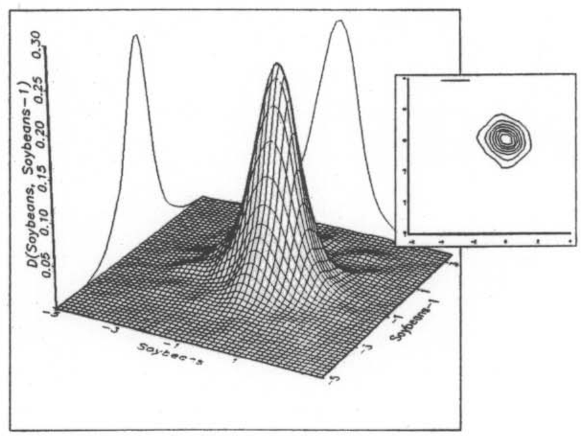

Figure 8. $\hat{D}(SOY, SOY_{t-1})$

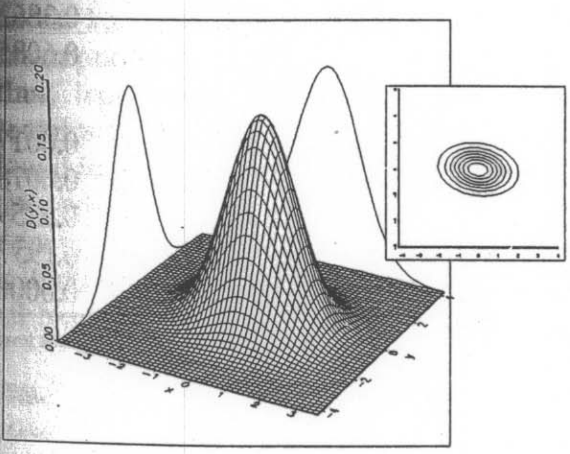

Figure 9. Reference Bivariate Normal.

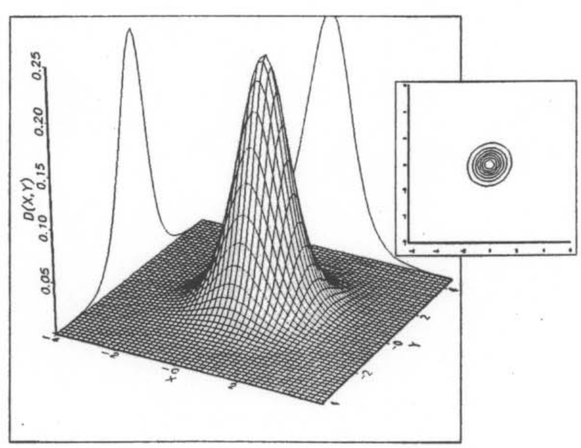

Figure 10. Reference Student's t Distribution, 6 df.

Table 2. STAR Parameter Estimates and Misspecification Test Results.[a,b]

Commodity:	Cattle	Hogs	Corn	Soybeans
Star Model:	(2,2,13)	(2,2,28)	(2,2,6)	(2,2,7)
μ	0.04679* (0.01075)	0.03893* (0.01317)	-0.06892 (0.01086)	-0.00120 (0.00136)
β_0	0.04767* (0.01075)	0.04082* (0.01385)	-0.006971 (0.01098)	-0.00129 (0.01454)
β_1	0.00460 (0.02075)	-0.00051 (0.02187)	0.08359* (0.01582)	-0.03968 (0.02012)
β_2	-0.02339 (0.02040)	-0.04807* (0.02180)	-0.09507* (0.01574)	-0.03122 (0.01975)
q_0	0.07633* (0.00238)	0.03574* (0.00113)	0.16841* (0.00516)	0.14334* (0.00573)
q_1	-0.00087 (0.00172)	0.00001 (0.00078)	-0.01274* (0.00273)	0.00681* (0.00337)
q_2	0.00131 (0.00171)	0.00176* (0.00078)	0.01541* (0.00377)	0.00405 (0.00348)
σ^2	0.64729* (0.02326)	1.46505* (0.04625)	0.68389* (0.02389)	0.99583* (0.03681)
--- P-values for Misspecification Tests---				
Skewness:	0.0687*	0.4226	0.1612	0.2364
Kurtosis:	0.0000*	0.0015*	0.0000*	0.0000*
Joint Mean				
Overall:	0.3099	0.9216	0.0048*	0.6713
Trend:	0.3389	0.7985	0.2015	0.6649
KG2:	0.1027	0.8334	0.0005*	0.3863
Autocorrelation:	0.9877	0.5120	0.6596	0.6085
Variance[c]				
HC(2):	0.2247	0.9474	0.3211	0.0017*
HC(4):	0.1397	0.2923	0.1286	0.0078*
BDS(2):	0.2896	0.8133	0.0003*	0.2259
BDS(4):	0.0228*	0.4944	0.0000*	0.1957
ML(12):	0.0000*	0.7506	0.0000*	0.0000*

[a]asterisks indicate parameter estimates greater than two standard errors or rejection of the null hypothesis at the five percent level for a misspecification test.
[b]all data series were multiplied by 100 to facilitate convergence.
[c]HC=modified ARCH; BDS=Brock, Dechert and Scheinkman; ML=McLeod-Li.

from $D(y_t|Y^0_{t-1})$, the conditional distribution of the observables. If the conditional mean is properly specified no significant autocorrelation will remain in the residuals.

Although the preceding assumptions can be checked individually, the validity of these individual tests rests on all of the other assumptions being correct (McGuirk, et. al, 1993a). Testing all of the assumptions regarding the conditional mean jointly, rather than individually, reduces the number of maintained hypotheses required. The joint misspecification test of the conditional mean relies on the auxiliary regression:

$$\hat{u}_t = \beta' X_t + \Gamma_P' \Psi_t^P + \Gamma_F' \Psi_t^F + \Gamma_I' \Psi_t^I + v_t$$

where $\hat{u}_t$ is the scaled residual at time t, X_t contains the independent variables (lagged dependent variables in the present model), Ψ_t^P contains linear and quadratic time trend variables, Ψ_t^F is a Kolmogorov-Gabor second order polynomial (KG2) of the original regressors, and Ψ_t^I contains the lagged residuals, $\hat{u}_{t-1}$ and $\hat{u}_{t-2}$. β and the Γ's are parameter vectors and v_t is a white noise error term. In this testing scheme, Ψ_t^P models parameter instability in the mean, Ψ_t^F allows non-linearity in the conditioning variables and Ψ_t^I permits temporal dependence. The null hypothesis of a correctly specified conditional mean is tested with an F-test for $\Gamma_P = \Gamma_F = \Gamma_I = 0$. If rejection of the null is indicated, source of the problem may be detected by examining the significance of the Γ's individually.

In addition to the assumptions regarding the conditional mean, we also examine assumptions relating to the conditional variance. Several tests are performed to check for additional (un-modeled) dynamic heteroskedasticity. For the ARCH and GARCH models, the usual Engle (1982) ARCH test is used (2 and 4 lags). For the STAR model, the modified ARCH test described in McGuirk, et. al. is used. This modified test includes the squares and cross-products of the regressors (lagged y_t's) under the null to be consistent with the form of heteroskedasticity implied by the Student's t distribution. The test attributable to Brock, Dechert and Scheinkman (BDS) was conducted at two different embedding dimensions (2 and 4) to check for i.i.d. residuals. The McLeod-Li portmanteau test results for 12 lags are reported as additional evidence of higher order residual dependence.

The STAR models pass many of the misspecification tests. Exceptions include tests for normality and the assumption of no second-order dependence in the residuals. However, excess kurtosis in the residuals is expected from the STAR models since the distributional assumptions underlying the model dictate that the residuals are distributed Student's t (and thus, leptokurtotic). The McLeod-Li test indicates that some non-linear dependence remains in the cattle, corn and soybean models. No evidence of non-linear dependence is found in the hog model, but the data plots suggest that this series contained little non-linear dependence to begin with. The modified ARCH test finds un-modeled second-order dependence in only the soybean model while the BDS test indicates non-linear dependence in both the cattle and corn models. The conditional mean appears correctly specified for all but the corn model.

Several of the STAR conditional variance parameter estimates are significant; the results range from all parameters being significant in the corn model to one significant conditional variance parameter estimate in the cattle model. Only the hog and corn models have significant coefficients in the conditional mean.

Tables 3 and 4 report the comparable results from the more usual ARCH and GARCH formulations. Seven lags were included in the conditional variance of the ARCH

Table 3. Normal ARCH Parameter Estimates and Misspecification Test Results.

Parameter	Cattle	Hogs	Corn	Soybeans
β_0	0.00046*	0.00036	-0.00009	-0.00036
	(0.00017)	(0.00025)	(0.00020)	(0.00023)
β_1	0.01997	0.01758	0.07573*	-0.02713
	(0.02079)	(0.01267)	(0.02277)	(0.02221)
β_2	-0.03081	-0.04041*	-0.05749*	-0.00745
	(0.02242)	(0.01244)	(0.02215)	(0.02212)
α_0	0.00003*	0.00012*	0.00004*	0.00005*
	(0.00000)	(0.00001)	(0.00000)	(0.00001)
α_1	0.03376	0.02838*	0.14300*	0.07278*
	(0.02058)	(0.01329)	(0.02881)	(0.02583)
α_2	0.08644*	0.00001	0.09470*	0.08501*
	(0.02501)	(0.00026)	(0.02649)	(0.02699)
α_3	0.12049*	0.02325	0.11188*	0.09670*
	(0.02621)	(0.01407)	(0.02859)	(0.02507)
α_4	0.08994*	0.03696*	0.14321*	0.10744*
	(0.02430)	(0.01270)	(0.02965)	(0.02905)
α_5	0.07984*	0.07398*	0.11758*	0.17821*
	(0.02316)	(0.01412)	(0.02638)	(0.03276)
α_6	0.13199*	0.06020*	0.06687*	0.12531*
	(0.02710)	(0.01280)	(0.02686)	(0.03031)
α_7	0.07943*	0.05574*	0.06585*	0.06795*
	(0.02773)	(0.01540)	(0.02526)	(0.02470)
--- P-values for Misspecification Tests---				
Skewness:	0.0001*	0.0735	0.1586	0.0984
Kurtosis:	0.0001*	0.0564	0.0000*	0.0000*
Joint Mean				
Overall:	0.4954	0.0004*	0.2758	0.6843
Trend:	0.8268	0.2010	0.2574	0.6882
KG2:	0.1585	0.9925	0.1162	0.5173
Autocorrelation:	0.5891	0.0000*	0.7399	0.4634
Variance[b]				
ARCH(2):	0.7278	0.3359	0.9459	0.8718
ARCH(4):	0.7033	0.5647	0.7591	0.7041
BDS(2):	0.6826	0.0734	0.6987	0.7791
BDS(4):	0.2974	0.8743	0.9271	0.5639
ML(12):	0.0017*	0.0008*	0.6807	0.2162

[a]asterisks indicate parameter estimates greater than two standard errors or rejection of the null hypothesis at the five percent level for a misspecification test.

[b]ARCH=ARCH test of Engle (1982); BDS=Brock, Dechert and Scheinkman; ML=McLeod-Li.

Table 4. Normal GARCH(1,1) Parameter Estimates and Misspecification Test Results.

Parameter	Cattle	Hogs	Corn	Soybeans
β_0	0.00034 (0.00017)	0.00027 (0.00024)	-0.00007 (0.00020)	-0.00019 (0.00024)
β_1	0.01528 (0.02311)	0.01409 (0.02167)	0.06725* (0.02255)	-0.03722 (0.02388)
β_2	-0.01136 (0.02303)	-0.03761 (0.02269)	-0.05976* (0.02160)	-0.02247 (0.02340)
α_0	0.00000* (0.00000)	0.00000* (0.00000)	0.00000* (0.00000)	0.00000* (0.00000)
α_1	0.07207* (0.00770)	0.03903* (0.00660)	0.10011* (0.00892)	0.09124* (0.01016)
γ_1	0.92148* (0.00792)	0.94884* (0.00607)	0.88663* (0.00807)	0.89300* (0.00861)
--- P-values for Misspecification Tests ---				
Skewness:	0.0000*	0.0053*	0.3372	0.1497
Kurtosis:	0.0000*	0.0156*	0.0000*	0.0000*
Joint Mean				
Overall:	0.6013	0.1176	0.2826	0.4063
Trend:	0.7636	0.1720	0.2031	0.7015
KG2:	0.2534	0.9864	0.1204	0.5358
Autocorrelation:	0.6181	0.0263*	0.9227	0.1273
Variance[b]				
ARCH(2):	0.0708	0.0558	0.0630	0.3887
ARCH(4):	0.1046	0.0892	0.4282	0.5707
BDS(2):	0.0034*	0.3516	0.8950	0.4641
BDS(4):	0.0048*	0.0503	0.9634	0.4023
ML(12):	0.0810	0.0301*	0.9484	0.0891

[a]asterisks indicate parameter estimates greater than two standard errors or rejection of the null hypothesis at the five percent level for a misspecification test.

[b]ARCH=ARCH test of Engle (1982); BDS=Brock, Dechert and Scheinkman; ML=McLeod-Li.

models based on previous work in which similar time series exhibited a very long lag structure (McGuirk, et. al., 1993b). The misspecification results reported in these tables are for the tests described above using scaled residuals (residual divided by the square root of the conditional variance estimate). The normal ARCH(7) results are mixed. The McLeod-Li test indicates un-modeled non-linear dependence for cattle and hogs, and the conditional mean appears misspecified in the hog model. All models except the hog model indicate non-normality (specifically excess kurtosis). With respect to parameter estimates, nearly all of the ARCH conditional variance parameters are significant as well as both lags in the conditional mean of the corn model.

The GARCH(1,1) models produced results similar to those of the ARCH models. Excess kurtosis is detected in the residuals of all four GARCH models. No dynamic heteroskedasticity is detected by the ARCH tests although the p-values for these tests are relatively smaller than those for the ARCH model conditional variance tests. Some evidence of first-order autocorrelation was detected in the hog model, but this model still passes the overall conditional mean test. Only the hog model fails the McLeod-Li test for second-order autocorrelation while the BDS test detects non-linear dependence in the cattle model.

The STAR and GARCH models produce widely-differing conditional variance estimates as illustrated by Figures 11 and 12 for the corn data. Estimates for the other commodities are similar and are not presented. The STAR conditional variance is very similar to the variance estimate produced by recursive least squares[3]. The smoothed nature of the STAR conditional variance stands in contrast to the volatile conditional variance estimate of the GARCH model. Plots of the scaled residual series for the corn STAR and GARCH models are given in Figures 13 and 14. By comparing these plots with the original data plots in Figure 3, it is apparent that the both the GARCH and STAR models were relatively successful in eliminating non-linear dependence in the data.

Conclusions

An alternative model of futures returns, the Student's t Autoregressive (STAR) model was examined. Unlike previous time series models used with futures data, this model is derived by making probabilistic reduction assumptions regarding the joint distribution of the observable variables. The Student's t model proposed by Spanos was used to model futures returns in an effort to explain the observed excess kurtosis and non-linear dependence in these data.

Continuous series' on the nearby futures contract were created for live cattle and hogs, corn and soybeans. T-plots and bivariate density estimates showed that the data was indeed leptokurtoic and non-linearly dependent, suggesting the STAR model may be appropriate for these data. These plots revealed a much larger degree of non-linear dependence in the grains than in the livestock futures returns.

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[3] The model is estimated by OLS for an initial sample size of 200, then the sample size is increased by one and re-estimated. This process is repeated until the end of the sample is reached, retaining the variance estimates along the way.

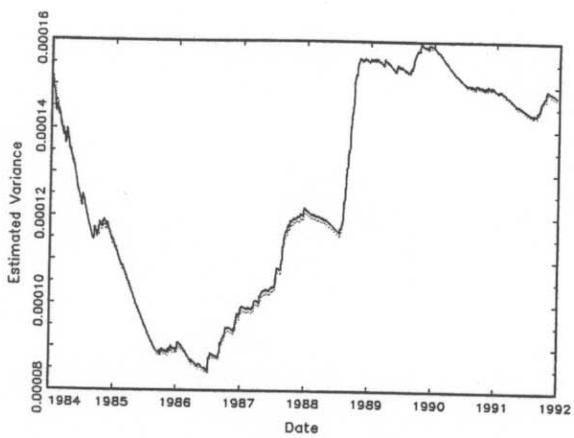

Figure 11. STAR Model Conditional Variance Estimate and RLS Variance Estimate, Corn.

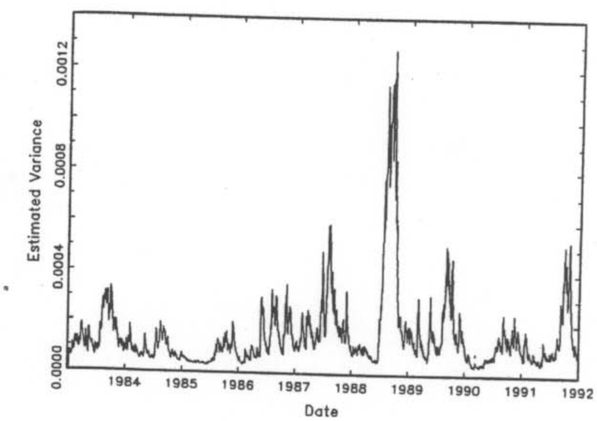

Figure 12. GARCH Model Conditional Variance Estimate, Corn.

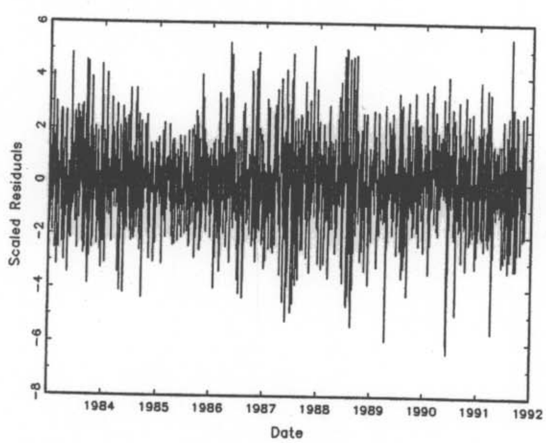

Figure 13. STAR Model Scaled Residuals, Corn.

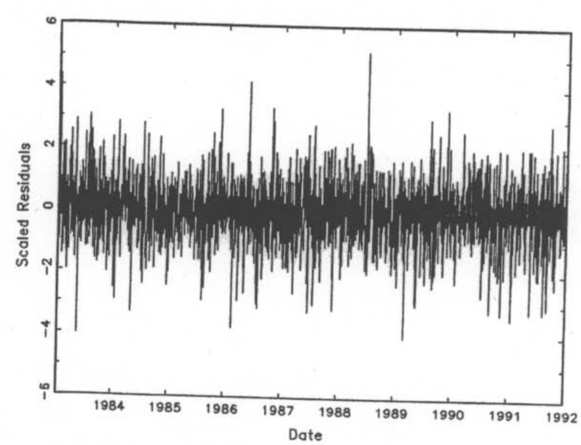

Figure 14. GARCH Model Scaled Residuals, Corn.

A battery of misspecification tests indicate that the STAR model does an adequate job of accounting for the probabilisitic characteristics of the livestock futures prices that were examined. There are, however, mixed results with respect to how well these models accounted for non-linear dependence (especially for the grain series). The modified ARCH tests finds no variance misspecification in three of the four STAR models while the McLeod-Li and BDS tests indicate otherwise. Each of the STAR models pass at least one variance misspecification test. Evidence from this analysis favors the position that the STAR specification is an appropriate representation of the probabilisitic features of livestock futures price change data.

The more familiar ARCH and GARCH specifications seem to perform on par with the STAR model. Conflicting results with respect to remaining non-linear dependence arises for the livestock versions of these models. The grain models pass all of the conditional mean and variance tests, suggesting that the ARCH and GARCH specifications may be more appropriate for these storable commodities. Excess kurtosis in the scaled residuals of these models is in conflict with their underlying normality assumptions.

Nearly all of conditional variance parameter estimates for the ARCH(7) and GARCH(1,1) models were highly significant. This suggests the presence of a long memory in the conditional variance of futures prices. If this is the case, then the STAR model, with its conditional variance structure that uses the entire history of the series, is quite possibly a better representation of the actual data generating process. The "moving-average" nature of the STAR conditional variance produces a much smoother variance estimate than does the ARCH or GARCH model. Thus, the STAR model captures long-run changes in conditional variance while the ARCH and GARCH models give estimates of the shorter-run fluctuations in the variance.

The ability of a model to incorporate all of the systematic information in a data series is crucial to any attempt at hypothesis testing. Tests of futures market efficiency are a common application where a statistically adequate model is imperative. While none of the time series models examined were shown to clearly dominate the others (on statistical grounds) for use in modeling futures prices, the STAR model was found to perform at least as well as the more widely used ARCH-type models for some commodities and thus merits consideration as useful tool for the analysis of futures prices.

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