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Gerald Plato*

Farmers in nine counties --three each in Illinois, Indiana, and Iowa-- were offered put option contracts on 1993 December corn futures by the Options Pilot Program (OPP). The contracts had a \$2.90 strike price and were traded on the Chicago Board of Trade (CBOT). This pilot program is part of the Food, Agriculture, Conservation, and Trade Act of 1990. Its purpose is to determine if options provide farmers reasonable price protection, if farmers would accept this form of price protection, and the effect of widespread farmer adoption.

Eligibility for signing up for the OPP program in the nine counties required fulfilling all provisions of the 1993 commodity program for corn. Participation in the OPP required a farmer to forgo the price protection provided by deficiency payments and commodity loans on that portion of the corn crop covered by the 290 put. Farmers were encouraged to participate by the offer of a \$0.15 per bushel incentive payment.

An eligible farmer could sign up for up to 10 of the 5,000 bushel put option contracts subject to the constraint of sufficient program yield and acreage. Any production not covered by the 290 put remained in the current program. Farmers were reimbursed for the cost of purchasing put options when they provided the USDA with purchase documentation. Farmers received the \$0.15 per bushel incentive payment when they provided the USDA with option sales documentation, if the sale was within two weeks of the November 19 option expiration date. If the option sale was prior to two weeks before option expiration, then forward pricing documentation of corn in the OPP was also required. The OPP was limited to 20 million bushels of corn, but farmers in the nine counties signed up for more than this amount. To limit the OPP to 20 million bushels, participants were selected by lottery from the farmers that signed up. Put option contracts for more than 17 million bushels were purchased.

The \$2.90 strike price equals the \$2.75 corn target price for the 93 crop plus \$0.15 for the expected price level difference between the CBOT December futures price and the U.S. average farm price of corn from September through January. This strike price provided a level of price protection similar to, but not the same as that provided by deficiency and loan payments. The two levels of price protection differ because of errors in predicting the difference between corn futures and farm prices from September through January. The timing of option sales also contributes to the protection difference since 290 put options may be sold when the difference between futures and the

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September-January farm price is above or below the average difference for the current marketing year.

Government payments to farmers to offset their cost of purchasing 290 put options is the major cost of the OPP and any future program that reimburses farmers for put option purchases. Consequently, the USDA is particularly interested in evaluating the performance of the 290 put option.

This paper examines several key aspects of the 290 put option. These are its liquidity; incentive for early exercise; influences of the 15 cent incentive payment and participant borrowing interest rate on 290 put selling; and potential profitable arbitrage among the prices of the 290 put, 290 call and futures. Preceding the examination is a description of participant trading using their daily purchases, sales and open interest and the total daily volume and open interest of the 290 put and 290 call markets. Motivations for non-participant purchases of the 290 put and 290 call are inferred from the description.

Data and Notation

Data on the OPP participants were provided by the Agricultural Stabilization and Conservation Service, of the USDA. It contains the following information on each participant transaction: date, number of contracts, price, and if a purchase or sale. This data set was used to examine trading behavior and 290 put liquidity.

The Commodity Futures Trading Commission provided two CBOT data sets. One contains the daily volume, open interest and closing prices of all December corn options and futures contracts. It also contains a record of early option exercise. This data set was used to examine the incentive for early exercise and to compare participant trading with the overall market trading. The other CBOT data set contains the transaction prices that resulted from a price change for December corn options (both puts and calls) and December futures. It also contains the time that each of these price changing transactions was made. These prices were used to check for profitable arbitrage involving the 290 put and to examine market liquidity.

Examination of the incentive for early exercise and arbitrage requires the riskless interest rate each trading day for the period remaining until option expiration. A data set of these riskless interest rates was estimated from the bid-ask discounts reported in the Wall Street Journal for t-bills expiring on November 18, 1993, one day before expiration on 1993 December corn options.¹ For convenience, the interest rates were expressed on an annual-continuously-compounded basis.

¹ The procedure used to convert bid-ask discounts into an annual interest rate is shown in Cox and Rubinstein, p. 255.

The notation for the data, program, and estimated variables used in the analysis are shown below.

$C(F,t,X)$ = call price on futures
 F = futures price
 F^* = critical futures price for a put²
 I = 15 cent incentive payment for participants
 $P(F,t,X)$ = put price on futures
 r = annualized t-bill interest rate
 t = time to option expiration in fraction of a year
 X = exercise or strike price

Participant and Total Market Trading

Daily purchases, sales, and open interest of the participants were constructed from their individual transactions data. This data was used to examine participant trading. The examination included comparisons with the total daily volume and open interest of the 290 put. First, an overall summary of participant trading provides a background to examining the daily data.

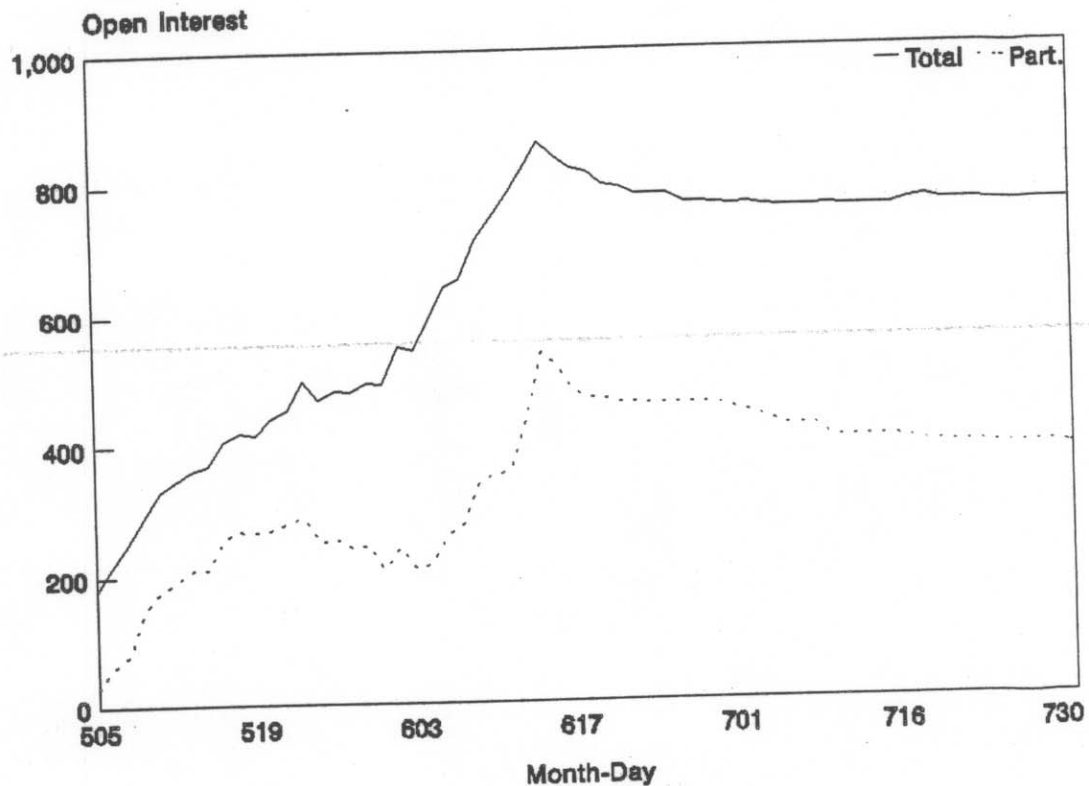
The participant data contains 1,153 farmers who purchased 3,449 contracts for a total of over 17 million bushels. The data begins on March 1 and ends on August 2. One hundred forty nine participants held open positions in 383 contracts on August 2. This represented 12.9 percent of all participants and 11.1 percent of the total contracts purchased. The peak trading period was the 30 trading days from May 4 through June 15. During this period participants purchased 3,364 contracts --97.5 percent of the total. They also sold 2,834 contracts --82.2 percent of the total purchased. Participant trading from June 1 through 15 was particularly active. Sixty eight percent of all participant contracts were purchased and 67.8 percent of all the contracts purchased were sold during this 11 trading day period.

Eight hundred forty two of the participants sold 2,515 contracts on the same day they purchased them. This represents 73.0 percent of all participants and 72.9 percent of all contracts purchased. Within 10 days of purchase 80 percent of all participants had sold 81 percent of the contracts purchased.

Figure 1 shows the daily open interest of participants and the total 290 put market from May 5 through August 2. Open interest of participants remained below 25 contracts until May 5. It then increased to 287 contracts on May 24, decreased somewhat to 206 contracts on June 3 and increased to a high of 535 contracts on June 15. Open interest decreased from its high to 383 contracts on August 2 without any day to day increases.

² The time value or value of holding a put to expiration is zero when the futures price is at or below the critical futures price.

**Figure 1. Participant and Total Open Interest in the 290 Put.
(May 5 through August 2)**



On May 4 non-participant open interest in the 290 put was 132 contracts larger than that for participants. On June 15 it was 325 contracts larger. Non-participant open interest then increased to 377 contracts above that for participants by August 2. These comparisons coupled with an open interest in the 280 put below 25 contracts from May 4 through August 2 indicate that non-participant open interest in the 290 put was stimulated by participant trading. The additional non-participant purchase of 290 puts may represent an attempt to take advantage of perceived mispricing. The small level of trading at this time in the 280 put implies there was little or no non-participant demand for buying 290 puts to hedge cash corn.

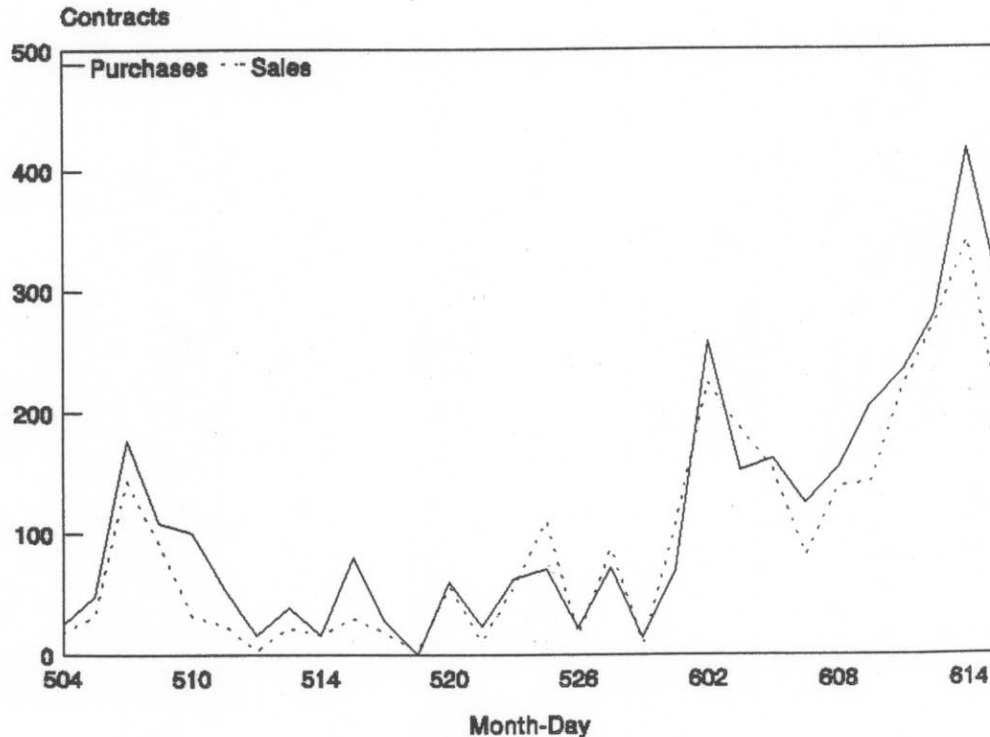
The 290 call would have been an integral part of arbitrage transactions involving the 290 put. From May 5 through June 15 the open interest in 290 calls increased from 4022 to 5529 contracts. This increase of 1507 contracts was over two times that for 290 puts. The 290 puts increased from 144 to 860 contracts during this period. In addition, from June 15 through August 2 the open interest in 290 calls increased by 1048 contracts while overall open interest in 290 puts decreased from 860 to 760 contracts. These comparisons suggest adequate open interest in 290 calls to arbitrage with 290 puts. They

also suggest that while the 290 put open interest was responding to participant demand, the 290 call open interest was largely responding to something other than demand for arbitrage with the 290 put. Rising prices during this period may have resulted in a desire to speculate on a price rise by buying inexpensive out of the money calls.

Figure 2 shows daily participant purchases and sales of the 290 put from May 4 through June 15. Daily participant sales closely matched daily purchases because most contracts were either sold on the same day purchased or sold within a few days.

Participant purchases of 290 puts were very small prior to May 4 and after June 15. Only eight contracts were purchased by participants after June 15. Participants only sold 86 contracts during the five trading days after the peak period. There were very few participant sales after this five day period. In addition, the volume in the 290 put market became very small after the sale of these 86 contracts. The daily volume over the remaining life of the option averaged slightly over 6 contracts per day. About one third of the remaining volume occurred during the last 10 days of the option.

Figure 2. Participant Purchases and Sales of the 290 Put.
(May 4 through June 15)



The total volume of 290 puts increased in response to participant purchases. For example, participant purchases in the five trading days before May 4 was 13 contracts while total put volume was 103 contracts. Participant purchases for the five day trading period beginning with May 4 was 459 contracts while total volume was 977 contracts. The data show a similar relationship at the end of the peak trading period. For the last five trading days ending on June 15 participants purchased 1,431 contracts while the total 290 put volume was 2,628 contracts. For the five trading days after June 15 participants purchased 8 contracts and while the total volume was 136 contracts.

Figure 3 compares participant purchases and sales of the 290 put with that for non-participants during the peak trading period from May 4 through June 15. On five trading days during this 30 trading day period, participants purchased and sold more contracts than nonparticipants.³ On two days there was an exact match. On six days the sum of purchases and sales for participants was 30 or less contracts below the sum for nonparticipants. These comparisons suggest that a large number of the transactions involved farmers on both sides. Pricing of the 290 put may have been facilitated during the peak trading period by participants buying and selling from one another because it reduced the number of puts that had to be written.

The 290 call volume was about 43 percent of the 290 put volume from May 4 through June 15. After June 15, the 290 call volume became large relative to the 290 put volume. For example, from June 16 through June 30 the 290 call volume was over 1,900 percent of the 290 put volume. These comparisons, like that for open interest, suggest adequate 290 call option trading to arbitrage with 290 puts. They also suggest that trading in the 290 call was responding to something besides arbitrage with 290 puts.

Market Liquidity

Same-day participant purchases and sales of the 290 put provide an opportunity to examine liquidity, because of the brief time period between the offsetting transactions.

Liquidity is the cost of buying and selling in a market. It includes fees and the difference between the current purchase and sale price.

The same day transactions also provide an opportunity to determine if liquidity was affected by the size of transaction. Participant transactions ranged from 1 to 10 contracts.

Table 1 shows the average buying minus selling price for the same day transactions by size of transaction. The average transaction cost from the differences between buying and selling prices are small implying that the 290 put market was liquid. Two of the

³ Two times the daily volume of 290 puts minus the sum of participant purchases and sales equals the number of contracts purchased and sold by nonparticipants.

Figure 3. Purchases Plus Sales of the 290 Put for Participants and Nonparticipants.
(May 4 through June 15)

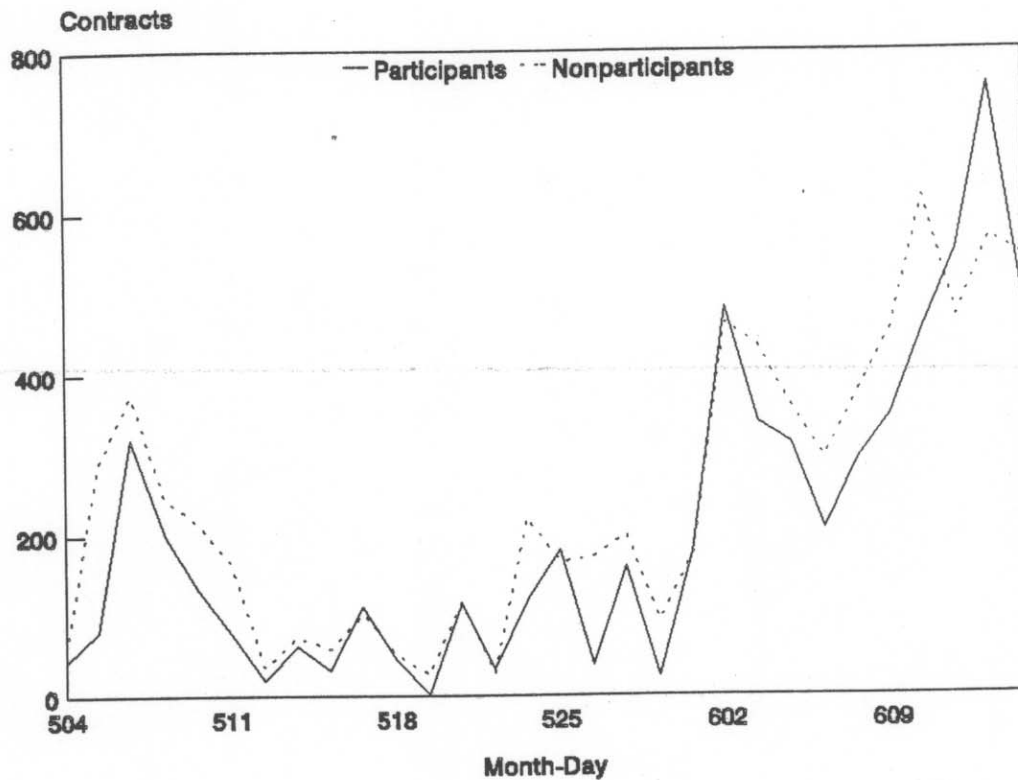


Table 1. Liquidity of the 290 Put for Same Day Participant Purchases and Sales by Size of Transaction.

Contracts per Transaction	Number of Farmers/Contracts		Buying minus Selling Price (cents/bu.)
1	295	295	0.18
2	212	424	0.31
3	80	240	0.14
4	81	324	0.09
5	45	225	0.14
6	41	246	0.19
7	20	140	0.26
8	23	184	0.09
9	13	117	0.23
10	30	300	0.20

¹ Number of farmers also equals the number of transactions.

Table 2. Average Absolute Values of Transaction Price Changes by Month for 93 December Corn Futures and for 290 Puts and 290 Calls on 93 December Futures.

	Average Absolute Price Change/ (cents/bu.)	Number of Observations
April		
290 call	0.25	112
290 put	0.56	96
futures	0.27	4645
May		
290 call	0.24	89
290 put	0.31	369
futures	0.24	4,940
June		
290 call	0.20	202
290 put	0.32	732
futures	0.27	7,281

Over 75 percent of the May and June price changes were at 2 ticks. The 290 put had a slightly larger average price change in May and June than did futures because it had fewer 1 tick price changes and because it had some greater than 5 tick price changes.

The 290 put in April had a bimodal price distribution. The two modes were at 2 and 4 ticks. Almost 80 percent of the price changes were at 2 and 4 ticks. The larger average price change for the 290 put in April than in May and June was due to larger percent price changes at 4 and greater than 5 ticks in April.

The distribution of the price changes for the 290 call was concentrated at 1 and 2 ticks.

Incentives for Early Exercise and Immediate Participant Selling

The opportunity cost of exercisable funds, that is, the value of interest income not earned on these funds, $(X - F)(1 - e^{-rt})$, increases as a put moves deeper into the money reducing its time value to zero at the critical futures price, F^* .⁵ It is rational to exercise when the time value becomes zero.

⁵ The early exercise premium --difference in price between an American put option and otherwise identical European put option-- approaches the opportunity cost on exercisable funds as the price of the underlying asset or financial instrument falls below the critical price (Whaley, March 1986, pp.131-2).

Table 3. Distribution of Transaction Price Changes by Number of Ticks for 93 December Corn Futures and for 290 Puts and 290 Calls on 93 December Futures.¹

Month and number ticks	290 call	290 put	futures
	----- percent -----		
April			
1	28	2	21
2	61	39	54
3	1	0	22
4	10	39	3
5	0	0	0
> 5	1	21	0
May			
1	46	8	35
2	40	76	43
3	2	0	15
4	8	10	7
5	0	0	0
> 5	3	5	0
June			
1	56	9	27
2	38	79	40
3	1	0	25
4	0	7	7
5	3	0	0
> 5	1	5	0

¹ Tick size equals 0.125 cents per bushel.

Time values of in-the-money options reflect the opportunity cost of the exercisable fun evaluated at the riskless interest rate. The riskless interest rate is appropriate because riskless arbitrage is important in determining the premiums of puts and calls relative to futures prices. The t-bill interest rate is a proxy for the riskless interest rate.

Critical closing futures prices each trading day were estimated for the 290 put. They were used to assess the incentive for early exercise by examining the distance between observed and estimated critical futures prices, especially, during the peak participant trading period.

Farmers in effect received a hybrid put. It had the premium of a 290 put but, exercisable funds of a 305 put, 305 - F, because of the 15 cent incentive payment.

Although not traded, critical closing futures prices were estimated for the 305 put. The estimates suggest that participants often purchased a put with exercisable funds that would normally have a zero time value.

The influence of higher than t-bill borrowing costs for participants on their 290 put selling is also examined in this section. In addition, the influence of the incentive payment on 290 put selling is examined.

The BAW procedure for estimating the prices of American options was used to estimate critical closing futures prices each trading day for the 290 and 305 puts (Barone-Adesi and Whaley). Critical-futures-price estimates are a byproduct of the BAW procedure.⁶

Estimates of the two critical futures prices each trading day requires estimates of the variance until option expiration. The implied variance until option expiration for each trading day was estimated using the Black option pricing model. The data used were the daily closing December futures prices, the daily closing prices of the closest to-the-money call option on December futures, and the daily t-bill interest rates discussed earlier.

Emphasis was given to comparing the estimated daily critical futures prices with the closing futures prices from May 4 through June 15. The comparison suggests that this peak participant trading period be divided into two parts.

From May 4 through May 28 the daily critical price estimates for the 290 put and for the 305 put were below their corresponding daily closing futures prices. The estimated critical futures prices for the 290 and 305 puts averaged 25.3 and 14.3 cents per bushel below the closing futures prices for this period. These are estimates of the amounts that the futures price on average would have had to decrease during this period to make the closing time values of the 290 and 305 puts equal to zero.

The critical price estimates for the 290 put are consistent with the observed time values. Both indicate that the futures price would have to decrease to make the time value zero and early exercise rational. Time values for the 290 put ranged from 1.75 to 3.75 cents per bushel for the period. Estimated time values for the 305 put ranged from 0.1 to 0.6 cents for the period.

From June 1 through 15 the estimated daily critical futures prices for the 305 put were all above their corresponding closing futures prices. The estimates averaged 5.4 cents per bushel above the closing futures prices for the period. This result implies that the 305 put would have had no time value during this period and that early exercise was rational. Estimates of the critical futures prices for the 290 put from June 1 through 15 were above their corresponding closing futures prices for the period implying that early

⁶ Gay et. al. and Overdahl used this procedure to estimate critical futures prices for use in examining the rationality of early exercise.

exercise was not rational. The estimates averaged 6.2 cents per bushel above closing futures for the period.

The critical price estimates for the 290 put are consistent with the observed closing time values except for June 11. On June 11 the 290 premium at the close had a zero time value. To be consistent with the zero time value, the critical futures estimate should have been equal to or greater than the closing futures price. Instead, the estimate was 3.8 cents less than the closing futures price. This result suggests that the critical futures prices are being underestimated. It also suggests that the incentive for early exercise is being understated.

The time values in the closing 290 put prices from May 4 through June 15 show that early exercise at the close was not rational, except for June 11. The critical futures price estimates suggest that participants, from June 1 through 15, purchased 290 puts with exercisable funds that would normally be associated with a zero time value and early exercise but were not except for the close on June 11. It is not rational to purchase options with zero time values and use them for price protection.

The largest closing time value for the 290 put from June 1 through 15 was only 1.25 cents per bushel. The small time values together with the critical futures price estimates for the 290 put suggest that a relatively small decrease in the futures price would have resulted in zero time value for the 290 put for the entire period.

There are two reasons that the time value of the 290 put was smaller for the June 1 through 15 period than for the May 4 through 28 period. First, the futures price was on average 8.4 cents per bushel smaller for the June 1 through 15 period. Second, the implied standard deviation of the futures price until option expiration was on average 12 percent smaller for the June 1 through 15 period.

Many of the participants may have had a sufficiently high opportunity cost for the 290 put premium relative to its time value to encourage immediate selling. Commercial banks were charging an 8.1 percent annual rate on average for current farm operating expense loans during the second quarter of 1993 (Walraven and Ricci). The annual rate for t-bills expiring on November 18 averaged 3.2 percent from May 4 through June 15. The amount that a participant's borrowing rate is above the t-bill rate, both adjusted for time to option expiration, times the premium is an additional opportunity cost for holding the put. It is rational to sell immediately if this opportunity cost is greater than the put's time value.

From May 4 through June 15 the opportunity cost of holding the 290 put due to borrowing at 8.1 percent rather than at the t-bill rate ranged from 1.19 to 1.47 cents per bushel. This opportunity cost was above the closing time values of the 290 put from June 1 through 15 except for three trading days.

Participant opportunity costs of holding the 290 put is increased by their borrowing rates, adjusted for time to expiration, times the 15 cent incentive payment. This added

opportunity cost also contributes to the incentive to sell early. From May 4 through June 15 this opportunity cost of holding the put based on the 8.1 percent borrowing rate ranged from .53 to .65 cents per bushel. The opportunity cost on the incentive payment plus the opportunity cost on the premium using the 8.1 percent borrowing rate was larger than the closing time values of the 290 put except for one trading day from June 1 through June 15. The influence of the incentive payment on selling can be eliminated by separating this payment from the selling decision.

Potential for Profitable Arbitrage

Three boundary conditions were used to check for profitable arbitrage. The one shown in (1) says that the exercise value of a put must be less than or equal to its premium. If not it, would be possible to buy the put, immediately exercise it, and earn a profit.

$$(1) \quad X - F \leq P(F,t,X) \text{ for } X > F \text{ (Put-Futures Boundary)}$$

It is important to check this boundary condition for a put with little time value. In this situation, a large number of sell orders could temporarily cause a put price to drop below its exercise value.

The two boundary conditions shown in (2) are known as the put-call parity for American options on commodity futures contracts.⁷

$$(2) \quad Fe^{-rt} - X \leq C(F,t,X) - P(F,t,X) \leq F - Xe^{-rt} \text{ (Put-Call Parity)}$$

The CBOT transaction prices discussed earlier were used to check for arbitrage boundary violations in (1) and (2). A 290 put and 290 call transaction pair was considered a time match for arbitrage testing if they occurred within 5 minutes of one another. In addition, a futures transaction was considered a time match for arbitrage testing with a put-call match if all three transactions occurred within a 5 minute period. A one minute period was also used to select transaction time matches.

There were generally more than one futures match for each put-call match because corn futures has a much larger volume than corn options. If there were more than one futures match, than the futures match with the smallest futures price along with the put price in the put-call match was used to check for arbitrage boundary violations in (1). In addition, if there were more than one futures price in a put-call match, than the largest futures price was used with both the put and call prices in the match to check for arbitrage boundary violations of the left inequality in (2). The smallest futures price along with the put and call prices in a put-call match was used to check for violation of the right inequality in (2).

⁷ The arbitrage transactions for the two inequalities in (2) are explained in Whaley (May-June 1986) and in Stoll and Whaley.

Table 4 shows that the number of put-call time matches is sensitive to the time period chosen. Increasing the time period from one to five minutes increased the number of put-call matches up to ten times. In addition, the average number of futures matches for the put-call-futures matches increased by at least three times.

The number of time matches shown in table 4 suggests that a one minute time period may be too small because the 290 puts and calls were not heavily traded. In addition, the small number of matches for April for the five minute period suggests there were not many potential arbitrage opportunities at this time. Increasing the time period above 5 minutes may pick up combinations of trades that are not arbitrage opportunities.

Table 4 shows that the put-futures boundary in equation 1 was only violated 3 times at the five minute time period and only 1 time at the 1 minute period. This is an extremely small number, since the 290 put had a small time value.

Table 4 also shows that the put-call parity boundary was only violated one time at the 5 minute period and none at the 1 minute period. Increasing to an 8 minute period resulted in 8 put-call parity violations in April. This number of violations is about 5

Table 4. Arbitrage tests Involving 93 December Corn Futures and the 290 Put and 290 Call on 93 December Corn Futures.¹

Month	Put-Call	Time Matches	Violations	
		Put-Call-Futures ²	Put-Call Parity	Put-Futures Boundary
.....one minute time interval.....				
April	8	8 (1.0)	0	0
May	35	32 (3.6)	0	0
June	101	92 (4.6)	0	1
.....five minute time interval.....				
April	83	79 (8.5)	0	0
May	213	201 (11.1)	0	0
June	492	476 (16.2)	1	3

¹ Results for one minute and five minute time matches are shown.

² Numbers in parentheses are the average number of futures prices per put-call-futures match.

percent of all time matches for the 8 minute period. However, an 8 minute period is most likely too long for arbitrage.

Summary

The performance of the 290 put option on 1993 December corn futures for the participants in the Options Pilot Program was examined. This put option was offered to eligible farmers in lieu of the price protection provided by the 1993 commodity program for corn.

The trading records of the pilot-program participants revealed that 97.5 percent of the 290 puts purchased by participants were purchased from May 4 through June 15. In addition, 82.2 percent of the contracts purchased by participants were sold during this period. The trading records also revealed that 80 percent of the participants sold their 290 puts within 10 days of purchase.

The participant trading records as well as the Chicago Board of Trade's transaction prices showed that the 290 put was liquid. For example, the 290 put selling price on average was only 0.19 cents per bushel less than the purchase price for those participants that purchased and sold 290 puts on the same day. This difference is about 1.5 price ticks. In addition, liquidity was not affected by the size of participant transactions.

The examination indicated that the futures price during the peak participant trading period was frequently only a few cents above the price at which the 290 put premium would have a zero time value. It also indicated that the opportunity cost of the put's exercisable funds from holding the put was frequently larger than the put's time value for many of the participants. Higher opportunity cost than time value may have been a major incentive for participants to sell rather than hold their puts. The opportunity cost of waiting for the pilot program's 15 cent per bushel incentive payment may also have contributed to selling rather than holding the 290 put.

Two arbitrage tests failed to detect inconsistent pricing among the 290 put, 290 call and futures.

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