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#### Suggested citation format:

Sherrick, B. J., P. Garcia, and V. Tirupattur. 1994. "Empirical Tests of Distributional Assumptions for Option Pricing Models." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

## Empirical Tests of Distributional Assumptions for Option Pricing Models

Bruce J. Sherrick, Philip Garcia, and Viswanath Tirupattur\*

#### Introduction

Option markets provide useful public information about the probability distributions of prices of the underlying asset (Gardner). In fact, options's payoffs are uniquely determined functions of the outcome of the underlying asset's price. Interactions among option market participants result in collective expressions of expectations about the future price distribution in current option prices. Hence, option pricing models can potentially be used to recover probabilistic information about the prices of the underlying assets without extensive surveys or direct elicitation. Importantly, using options premiums for probability assessments enables the recovery of ex ante distributional parameters. The most common example is the use of Black's or Black and Scholes' option pricing models for obtaining ex ante estimates of the standard deviation of the underlying asset prices. A limitation of the recovery of implied volatilities in this manner is the dependence of the derived estimates on the pricing model imposed. If the pricing model is incorrect, the derived estimates may not be reliable.

Under no-arbitrage restrictions, Cox and Ross derive a general asset pricing theory in which they show that the current price of any asset can be viewed as being equivalent to the stream of expected payoffs discounted to the present at the appropriate rate. Under fairly general conditions, expectations can be made with respect to an artificial distribution, called a risk-neutralized valuation measure (RNVM). Except for requiring the prices to be non-negative, economic theory provides little guidance regarding the distributional form of RNVM. In practice, the appropriate representation of futures price distributions is largely left as an empirical issue. The lognormal distribution is used most frequently because of its relative simplicity and its correspondence to the Black and Black-Scholes option pricing models. However, known biases of the Black option pricing model, and empirical investigations suggest that other distributions may be useful in characterizing expected prices.

The purpose of this paper is to assess whether alternative parameterizations of the RNVM perform better than lognormal distributions using daily data on soybean options over the period 1986-91. This study considers direct measures of fit of the option pricing models conditioned on a different distribution functional form as evidence of performance of the imposed distributional conditions. This approach offers additional insights by providing daily estimates of expected future price distributions which are useful in assessing the evolution of information and expectations. Further, because the approach makes no use of the actual futures prices in determining the option-based estimates of the futures price distributions, important comparisons can be made between the two markets and the information related in

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their respective prices.

The remainder of this paper is organized as follows. First, a brief discussion of option pricing theory under no-arbitrage conditions is provided. Next, data and methods are discussed including the rationale for the choice of Burr type III distribution as an alternative candidate distribution. Results are then presented including comparisons of pricing errors under alternate parameterizations, comparisons of the location of the implied distributions to the futures prices, comparisons of implied to resulting price variability, and two versions of a popular market timing test. Summary remarks and suggestions for future research then follow.

### Option Pricing Theory under No-Arbitrage

Cox and Ross show that the minimal assumption that no arbitrage opportunities exist in an economy implies the existence of an artificial distribution such that an equilibrium asset price is the properly discounted expected value of its payoffs. This artificial distribution against which the expectation is taken is the RNVM. Within this framework, the current prices of put and call options can be described by the following,

(1) 
$$V_{p} = b(T) \int_{0}^{\infty} \max(x_{p} - Y_{T}, 0) g(Y_{T}) dY_{T},$$

(2) 
$$V_c = b(T) \int_{0}^{\infty} \max(Y_T - x_c, 0) g(Y_T) dY_T,$$

where V<sub>p</sub> and V<sub>c</sub> are the prices of a put and call option respectively with x as the strike price expiring at time T, Y<sub>T</sub> is the random price of the underlying asset at expiration, b(T) is the discount factor and g(.) is the probability density function of the asset price at maturity. This approach for option pricing does not require any restrictive assumptions about the dynamics of price change, or any other information about the time path of prices. The only assumption required is that no arbitrage opportunities exist which is more general and less restrictive than the assumptions of the Black-Scholes type models. Under this approach, the assumption of a lognormal RNVM results in option pricing formulas for puts and calls on futures contracts which are equivalent to the familiar option pricing formulas derived by Black and others (Fackler and King; Garven).

Knowledge of the RNVM and the discount rate are sufficient to compute option premiums. Conversely, given observed option premiums and a discount rate, a RNVM can be recovered. The only requirement for the latter is that an assumption needs to be made regarding the distributional characterization for the underlying asset prices. The most common parameterization of the RNVM is lognormal. However, several authors have pointed out that lognormality is not an accurate description of reality and have suggested the use of alternative candidate distributions, or mixtures of distributions (Hall, Brorsen, and

Irwin; So). Stable Paretian, Burr XII, Burr III, gamma, Weibull and exponential are among the several alternative candidate distributions suggested in the literature (Fackler; McCulloch; Sherrick, Irwin and Forster). Others have suggested modifications to the data generating process to impose conditional dependence or otherwise admit non-independent price changes (Myers and Hanson). On the other hand, the use of lognormality is also appealing in that it is consistent with the widely used equilibrium option pricing models along the lines of Black-Scholes and Black; and facilitates the hedging arguments presented in these models. In this study, the performance of the candidate distribution which seems to be most representative of the data is tested against the lognormal which forms a convenient benchmark.

#### **Data and Methods**

The data for the analysis consisted of daily closing futures prices and option premia for options on 22 soybean futures contracts from 1988-91 obtained from the Chicago Board of Trade. After careful scrutiny, trades with no volume, observations inconsistent with monotonic strike-premium patterns, and extraneous entries were eliminated. The resulting sample is more fully described in table 1. On average, there were 11.8 options per day with 61.7 percent of the sample being calls. To estimate parameters for the expected price distributions, a risk free rate that corresponds to b(T) was also needed. The rate used in this study is the daily three-month treasury-bill yield obtained from the Federal Reserve Bank of Kansas City compounded over the relevant interval until expiration.

The data were first analyzed using moment-ratio diagrams of the observed data as suggested by Rodriguez. Moment-ratio diagrams are useful for preliminary analyses of data and can help identify an attractive candidate as an alternative to the lognormal distribution. Essentially, these diagrams, drawn in the skewness and kurtosis plane, indicate boundaries and regions of relative moments admitted under different distribution parameterizations. For example, the skewness and kurtosis of a normal distribution are 0 and 3 respectively. Hence, the normal distribution plots as a single point in the moment-ratio diagrams. By contrast, the Pearson distributions admit a wide array of relative moments, and hence, occupy a region rather than a point in the skewness-kurtosis plane. Plotting empirical moments highlights data characteristics and identifies distributions that are unable to generate similar data. Thus, although the procedures do not prescribe a single acceptable distribution, they do give some guidance as to which distributions are not consistent with the sample moments.

Table 2 provide results of the examinations of the daily soybean futures price data both in detrended levels and log relative returns for each contract. The moment ratio diagrams indicate that data in detrended levels fall in the regions with positive skewness and varying kurtosis, but often outside the range admitted by any distribution as a set of independent draws from a single distribution. As a complementary test, normality was rejected using Jarque-Bera's statistic in all cases. Under returns (changes in log prices), normality was rejected for about a third of the 22 contracts, further indicating that the assumption of lognormality of prices may introduce inaccuracies. The implication of these results are that higher moment flexibility is a desirable property if modelling price data.

Previous studies as well as the moment ratio charts of the data suggest the use of a distribution which is allows a wide range of skewness and kurtosis values. Because much o the data fell in the space covered by Burr type III distribution and because of the non-normality of the data, particularly in terms of skewness and kurtosis, Burr III was chosen as the alternative candidate distribution whose performance is evaluated against the bench mark of lognormal.

Although Burr distributions have received limited attention in modelling of prices, they have been shown to be useful in describing other economic data with zero support, particularly in the insurance industry as a candidate for loss distributions. Further, it is important to note that Burr III covers all the space regions in the skewness-kurtosis plane occupied by Pearson types IV, VI, and bell-shaped curves of type I, gamma, Weibull, normal, lognormal, exponential, and logistic distributions. Further details about this distribution are provided in Tadikamalla.

The Burr-III cumulative distribution function (CDF) with parameters  $\alpha$ ,  $\lambda$ , and  $\tau$  is:

(3) 
$$F_{BR}(Y \mid \alpha, \lambda, \tau) = (1 + (Y_{\tau})^{-\lambda})^{-\alpha} \qquad \text{for } \alpha, \lambda, \tau > 0; Y \ge 0,$$

and thus the density, or PDF is:

(4) 
$$f_{RR}(Y \mid \alpha, \lambda, \tau) = \alpha \lambda \tau (Y_{\tau})^{-(\lambda-1)} (1+Y_{\tau})^{-\lambda})^{-(\alpha+1)}.$$

The cumulative distribution function for the lognormal distribution with parameters  $\mu$  and is:

(5) 
$$F_{LN}(Y \mid \mu, \sigma) = N((\ln Y - \mu)/\sigma),$$

where N(•) is the cumulative normal density function. The lognormal density is:

(6) 
$$f_{LN}(Y \mid \mu, \sigma) = (2\pi)^{-1/2} (\sigma Y)^{-1} \exp[-(\ln Y - \mu)^2/(2\sigma^2).$$

Using the no-arbitrage pricing model described in (1) and (2), observed option pre can be used to recover the parameters of the candidate price distributions by numerically searching for the values of parameters that result in implied option premia closest to the observed option premia. Intuitively, this procedure is similar to that of recovering implied volatility except that the dimension of the choice variable vector corresponds to the numb of parameters of the candidate distribution (two in the case of lognormal distribution and three in the case of Burr type III distribution). Further, it should be reiterated that the underlying asset prices are not needed in the estimation process.

The implied set of distributional parameters under alternative distributional assumptions were obtained by minimizing squared error between observed and model op premia. Specifically, the following equation was solved for each day's data and for each contract to obtain the parameter vector,  $\beta$ , under each of the assumptions of lognormal a

Burr type III distributions for g(Y<sub>T</sub>),

(7) 
$$Min[\sum_{i=1}^{n} ((V_c - b(T) \int_{x_i}^{\infty} g(Y_T | \beta)(Y_T - x) dY_T)^2) + \sum_{j=1}^{m} ((V_p - b(T) \int_{0}^{x_j} g(Y_T | \beta)(x - Y_T) dY_T)^2)].$$

This procedure simply minimizes the sum of squared differences between the model premia conditional upon the parameters of the distribution and the observed option premia. Daily samples of "m" puts and "n" calls were used subject to the requirement that (m+n) be greater than the number of parameters of each of the distributions. Equation (7) was solved one day at a time using non-linear optimization methods using the *Gauss-VMI* programming language. To keep the problem computationally manageable, the process was restricted to the last 150 trading days of each contract's life. Thus, implied distributional parameters were obtained for each day under Burr and lognormal distributional assumptions for each of the twenty two contracts. Figures 1 and 2 display a sample of the distributions as implied in option prices for the November 1990 soybeans futures contract at several points in the contract life under alternative distributional assumptions.

#### Results

The relative performance of the two sets of distributions is next evaluated using a battery of tests. First, a comparison of average pricing errors between observed and option prices implied under alternative distributional assumptions is given. Next, comparisons of the differences, between observed futures prices and the mean of the distribution implied under alternative distributional assumptions are provided. Variances are also compared between implied distributions and resulting data realizations. Finally, simple market timing tests are performed using the approach suggested by Henriksson and Merton. These and a few related issues are addressed in turn.

Average Pricing Errors: If prices were exact and continuous, and if the pricing model held exactly for every single option, parameters could be recovered that resulted in zero error between the implied and observed prices. However, model and market imperfections introduce errors between model implied and observed prices. Hence, the first test examines simple measures of error between the estimated option prices and observed option prices.

Summary statistics of the option pricing errors are reported in table 3. Although the results are available daily, they are only reported over roughly monthly intervals for presentation brevity. The entries in table 3 were calculated as follows. For each of the

<sup>&</sup>lt;sup>1</sup>The numeric search routine employed was set to terminate when the relative gradient of the objective function to each parameter was less than 10e-6, or a large number of iterations was exceeded with no change in the objective function. The difference in dimension of the parameter vector in the two distributions also introduces differences in the performance of the numeric search.

contracts examined, the average daily error between all options and their implied counterparts was computed under both distributional assumptions. Then, over the interval reported in the table, the mean and variance of those average errors was calculated. Entries in the table indicate the number of contracts for which each distribution had lower average pricing errors. The total number of contracts is less than 22 in cases where there were insufficient trading histories in particular contracts. The magnitude of the daily errors averaged only 0.14 and 0.16 cents respectively under Burr III and lognormal assumptions. Their magnitudes over different periods from maturity are also reported in table 3.

It may be seen that the Burr has lower average pricing errors closer to maturity which is the period of most heavy trading. Lognormal relatively performs better 120-150 days to maturity. Even though the Burr displays more cases with a lower average pricing errors, the economic significance of the improvement may not be great enough to justify the complication of moving to a more flexible distribution. On the other hand, if the user finds the movement to a more flexible distribution to be fairly low cost, there are many cases which result in smaller average pricing errors.

Actual Futures Prices and Implied Means: Because the futures prices are not used in the recovery of the implied distribution, meaningful comparisons can be made between the options and futures markets with regard to the information related in their respective prices. Both the current futures price and the mean of the option-implied distribution can be taken as estimates of the future price. Hence, simple comparisons were made under the two distributions to determine which more nearly agreed with the current futures price. Table 4 summarizes these results, reporting only those intervals that were reported in the previous section. Again, the more flexible Burr marginally outperforms the lognormal by numbers of cases. However, these results are tempered by the fact that neither distribution seems to display large differences between the implied means and the futures prices. The average absolute difference between the implied mean and the observed futures price over all contracts was very low, averaging only 0.351 and 0.391 cents under Burr III and lognormal assumptions, respectively.

These results admit at least two interpretations: if the futures is taken to be an unbiased estimate of future price, the option-implied means are unbiased as well; or, if the futures prices are not related to future prices, the options markets likewise contain similar information. Figure 3 graphically displays for one of the contracts the close relationship between the futures prices and the daily implied means from each distribution.

Variance Estimates and Resulting Price Variability: If the implied distributions reflect market expectations of end-of-period prices, and if prices are from a variance-additive-intime data generating process, resulting price movements can be compared to forecasted variance from the options implied distributions. Even if prices are not from a variance-additive-in-time data generating process, the dispersion of the implied distributions should reflect the relative probabilities of various outcomes to represent no-arbitrage prices.

For each day, the implied variance for the remaining interval until expiration was computed under both distributions. Then, the actual variance of resulting futures prices was

computed for the same interval. The absolute difference in value between the lognormal estimate and the futures versus the absolute difference between the Burr and the actual was computed. The distribution with the smaller absolute difference is then recorded as the closer candidate for that interval. The results across all contracts are tabulated in table 5 over the same intervals as presented earlier. Figure 4 graphically depicts these somewhat surprising results demonstrating that the more flexible Burr very frequently outperforms the lognormal at depicting ex ante price variability. Note that although the Burr is "better" than the lognormal in a vast majority of cases, the results do not imply that either are necessarily "good" indicators of resulting price behavior. However, the relative improvement by the Burr points out the potential value in using more flexible distributions when higher ordered moments are of interest.

Henriksson-Merton Market Timing tests: As a simple test of the economic importance of moving to more flexible functional forms, simple market timing tests were constructed using the approach of Henriksson and Merton and implemented in a regression framework following Breen, Glosten and Jagannathan. To illustrate the market timing tests, consider a market direction variable  $M_{t+i}$  such that:

(8) 
$$M_{t+i} = 1 \quad \text{if} \quad PA_{t+1} > PA_{t} \\ M_{t+i} = 0 \quad \text{if} \quad PA_{t+1} \leq PA_{t},$$

where  $PA_{t+i}$  is the actual price for period t+i and  $PA_t$  is the actual price for period t. Next, define a forecast direction (signal) variable  $Z_{t+i}$  such that:

(9) 
$$Z_{t+i} = 1 \quad \text{if} \quad PF_{t+1} > PA_{t} \\ Z_{t+i} = 0 \quad \text{if} \quad PF_{t+1} \leq PA_{t},$$

where  $PF_{t+i}$  is the forecasted price for the time period t+i. Then, specify the following regression:

$$Z_{t+i} = \alpha + \beta M_{t+i} + \epsilon,$$

where  $\epsilon$  is the IID error term. Breen, Glosten and Jagannathan showed that the forecasts have market timing if the  $\beta$  coefficient is significantly greater than zero.

It is important to note that the comparison of interest is in the relative performance of the two distributions, not whether they exhibit particular levels of significance. Hence, adjustments for transactions costs and the like are not considered as they would not affect the conclusions about relative performance.

Within each contract, two versions of the HM test were run under each distributional assumption. Test 1 uses the difference between the implied distribution mean and the contemporaneous futures price to construct the signal for the expected direction of price movement. A futures price that is below the mean of the implied distribution results in a forecast for an upward movement in futures price. Conversely, a lower implied mean relative to the futures price generates a signal for a downward movement in the futures price.

The resulting futures price movements were recorded for both short-term strategies (one day holding period) and for the remaining life of the contract (reported in the table). Because there are a large number of days in test 1 with very small differences in implied price and futures price, the test is somewhat diluted by meaningless signals. Further, under these skewed distributions, the mean is a somewhat incomplete measure of central tendency. Hence, test 2 is formulated with a signal generated by differences between the median of the implied distribution and the contemporaneous futures price with the market direction variable being same as in test 1.

A summary of the results is presented in table 6. The total incidence of positive market timing is arbitrarily tabulated at p-values of .05 for both tests and for both distributions. Missing entries occur when either the signal or outcome variable was collinear with the constant because the final price was outside the entire remaining trading range, and thus generated all entries of either 0 or 1. Note that in spite of the somewhat ad hoc nature of the second test formulation, its market timing performance jumps dramatically compared to test 1. Under either test, the Burr outperformed the lognormal, but again, the performance was not too remarkable under either formulation.

# Summary and Concluding Remarks

Simple no-arbitrage option pricing models were used to recover market-based estimates of ex ante futures price distributions. Sample data properties were examined with results that led to the selection of the Burr distribution as a alternative candidate to compare to the lognormal. Daily data were used to generate daily estimates under both lognormal and distributional assumptions to assess the potential benefits in moving to more flexible distributional specifications.

A battery of tests was performed including: comparisons of average pricing errors; comparisons of implied mean prices with futures prices; assessment of the predictability of resulting futures price variability; and simple market timing performance of the options relative to the futures market. In each case, there is evidence of at least marginal improvement in moving to the more flexible specification. However, these benefits come at traditional models. Thus, the economic significance remains somewhat unclear as well.

Implications of the results depend on the context of the use of the information. For example, if an estimate of the implied futures prices was needed during a day with a futures price limit-move, but during which options continue to trade, either distribution will likely give very accurate results in recovering an implied futures price. However, for higher moment estimates, more flexible distributions are just that — more flexible and are less likely day, a more induced inaccuracies in estimating higher moments. Thus, on that same limit-move day, a more flexible distribution will probably provide a more accurate estimate of the remaining volatility for the life of the contract.

The merits of moving toward more flexible distributional specifications appear greatest under conditions that are likely to involve more complicated information. In the

present context, the greatest likelihood of detecting additional economic value of more flexible specifications is under conditions with large amounts of information impacting the market-implied distributions. These conditions exist during heavily traded intervals close to maturity when much information is summarized in option prices and there are less chances for important probabilistic information to remain undetected.

These procedures and the implied distributions provide a rich background for additional work. Future tests should consider the use of more sophisticated trading strategies based on differences in implied distributions and the data. For example, the data indicate that the options-based estimates of variance are often too low far from maturity. Hence, a strategy of buying option straddles far from maturity may prove profitable. Further research should also consider other distributional candidates and matching of data characteristics to distributions for option pricing models.

Table 1. Descriptive Statistics of the Sample Data

Contract	# of options observations	% of obs.	calendar days total	# of trade days	Avg # of strikes per day
Mar 88	437	59.95	46	34	12.85
May 88	938	58.96	109	78	12.03
Jul 88	1773	60.97	165	117	15.15
Aug 88	1705	59.30	191	130	13.12
Sep 88	1875	62.45	226	146	12.84
Nov 88	4719	54.74	291	205	23.02
Jan 89	2022	56.63	204	162	12.48
Mar 89	2141	60.91	177	157	13.64
May 89	2115	66.90	196	147	14.39
Jul 89	3071	65.29	266	191	16.08
Aug 89	1674	68.70	226	170	9.85
Sep 89	1446	67.23	185	196	7.37
Nov 89	3790	57.55	325	243	15.60
Jan 90	1311	59.95	204	164	7.99
Mar 90	1450	59.31	249	179	8.10
May 90	1383	65.87	206	159	8.70
Jul 90	2400	68.54	283	222	10.81
Aug 90	1527	69.48	234	171	8.93
Sep 90	1269	72.66	233	184	6.90
Nov 90	2840	59.33	340	249	11.41
Jan 91	1375	55.20	235	167	8.23
Mar 91	1498	61.42	205	157	9.54
verage	1944	61.70	218	165	11.79

Table 2. Skewness, Kurtosis Statistics and Normality Tests for the Sample Data

	-	Detrende	d Prices	Log Price Changes		
Contract	√b <sub>1</sub> *	b <sub>2</sub> *	Jarque-Bera Statistic**	√b₁*	b <sub>2</sub> *	Jarque-Bera Statistic**
Mar 88	2.314	2.225	31.20	0.0012	1.922	1.597
May 88	3.296	2.249	143.06	0.0014	2.324	1.467
Jul 88	51.632	4.054	51989.28	0.0043	3.169	0.139
Aug 88	30.989	3.646	16047.35	0.0081	2.964	0.007
Sep 88	55.096	2.948	51098.92	0.0066	2.560	0.806
Nov 88	79.680	4.303	21696.97	0.0069	3.723	4.448
Jan 89	35.423	5.220	26168.17	0.0021	12.180	435.570
Mar 89	11.079	2.697	2537.20	0.0062	3.072	0.028
May 89	3.962	2.562	351.61	0.0008	3.559	1.732
Jul 89	6.554	2.586	1290.01	0.0036	3.625	2.911
Aug 89	2.589	2.381	142.85	0.0006	4.273	8.440
Sep 89	12.854	3.392	3084.93	0.0035	3.258	0.308
Nov 89	4.109	3.479	607.07	0.0039	3.981	8.593
Jan 90	31.120	5.533	18754.60	0.0192	8.790	160.654
Mar 90	20.057	402.27	10298.28	0.0042	13.624	681.967
May 90	2.277	2.275	126.66	0.0016	3.005	0.0002
Jul 90	12.072	3.934	9646.36	0.0014	3.375	
Aug 90	16.024	3.853	6295.61	0.0011	4.210	1.113 8.911
Sep 90	10.219	3.353	5612.62	0.0004	3.144	
Nov 90	17.195	4.484	11009.41	0.0003	3.636	0.110
Jan 91	5.715	2.210	678.23	0.0216	15.016	3.744
Mar 91	9.881	3.221	2227.86	0.0013	3.079	739.99 0.036

 $<sup>^{\</sup>circ} Vb_1$  and  $b_2$  are scaled versions of the third and fourth moments.

<sup>&</sup>quot;Under the null hypothesis of normality, the Jarque-Bera statistic has an asymptotic  $\chi^2_{(2)}$  distribution. The critical value at 95% significance level is 5.99.

Table 3. Comparison of Option Pricing Errors

Time to Maturity	Burr III (contracts with lowest error)*	Avg. option pricing error** (\$)	Lognormal (contracts with lowest error*)	Avg. option pricing error** (\$)	Total # of
30	20	0.00045	2	0.00135	22
60	17	0.00066	4	0.00138	21
90	13	0.00117	8	0.00164	21
120	8	0.00199	12	0.00157	20
150	7	0.00289	13	0.00195	20

<sup>\*</sup> Number of contracts with the lowest mean absolute average option price error over the time intervals.

Table 4. Comparison of Implied Mean and Observed Futures Price

Ti 4	Burr III  (# of contracts)*		Average	Lognormal (# of contracts)*		Average	
Time to Maturity	Mean	Var.	futures price error**	Mean	Var.	futures price error**	Total # of contracts
30	15	15	0.306	7	7	0.323	22
60	10	13	0.242	11	8	0.295	21
90	13	11	0.297	8	10	0.368	21
120	11	17	0.290	9	3	0.423	20
150	12	16	0.601	8	4	0.631	20

<sup>\*</sup> Number of contracts with the lowest mean absolute average futures price error and lowest variance of the average futures price error over the time intervals.

<sup>\*\*</sup> Computed as the unweighted average of mean absolute error of option premia versus implied option premia across all contracts.

<sup>\*\*</sup> Computed as the unweighted average of mean absolute difference between the observed futures price and futures price implied by the model across all contracts.

Table 5. Comparison of Implied Variance by Distribution and "Observed" Variance

Time to Maturity	Burr III (# of contracts)*	Lognormal (# of contracts)*	Total # of contracts
30	22	0	22
60	21	0	21
90	21	0	21
120	19	1	20
150	19	1	20

<sup>\*</sup> Number of contracts with smallest difference between the implied variance relative to "observed" variance.

Table 6. p-Values of Market Timing (Henriksson - Merton) Tests

C	Bu	rr III	Lognormal		
Contract	Test 1°	Test 2**	Test 1°	Test 2**	
Mar 88	0.261	0.608	0.521	0.004	
May 88	0.135	0.000	0.921		
Jul 88	0.230	-	0.699		
Aug 88	0.020	0.642	0.969	0.000	
Sep 88	0.082	0.447	0.123	0.037	
Nov 88	0.405	0.002	0.880	0.000	
Jan 89	0.856	0.005	0.486	0.350	
Mar 89	0.818	-	0.800	0.424	
May 89	0.368	-	0.090	-	
Jul 89	0.698	-	0.249	-	
Aug 89	0.460	0.006	0.568	0.264	
Sep 89	0.845	0.051	0.719	0.015	
Nov 89	0.331	0.000	0.992	0.641	
Jan 90	0.376	0.156	0.115	-	
Mar 90	0.967	0.000	0.925	-	
May 90	0.577	0.662	0.858	-	
Jul 90	0.316	0.029	0.877	0.001	
Aug 90	0.761	0.000	0.795	0.044	
Sep 90	0.230	0.346	0.797	0.256	
Nov 90	0.564	0.196	0.159	0.399	
Jan 91	0.000	0.007	0.639		
Mar 91	0.595	0.000	0.740	0.624	
ases of gnificant					
larket Timing	2 of 22	10 of 18	0 of 22	7 of 14	

<sup>\*</sup> Signal generated using the mean.
\*\* Signal generated using the median.

Figure 1. Option-Based PDFs for November 1990 Soybean Futures

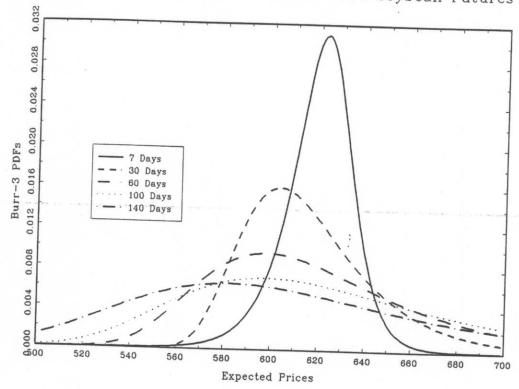
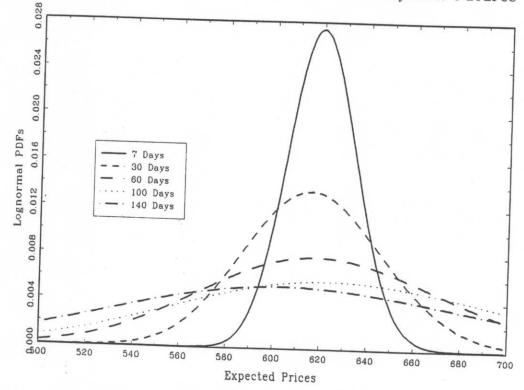
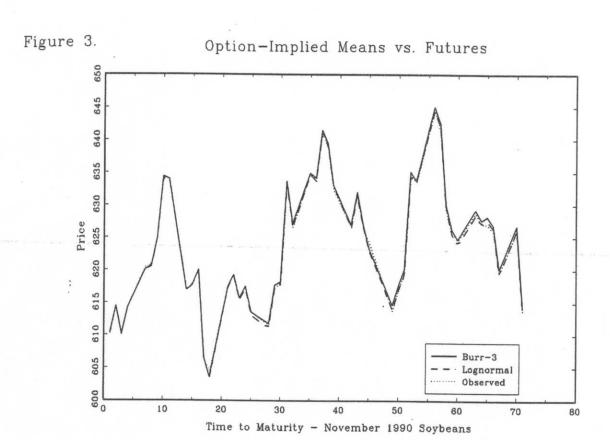
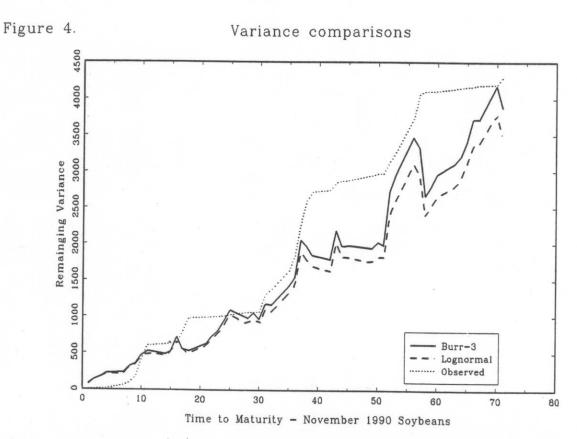


Figure 2. Option-Based PDFs for November 1990 Soybean Futures







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