A Flexible Dynamic Inverse Demand System: An Application to U.S. Meat Demand

by

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In recent years there has been considerable interest in systems of inverse demand equations for agricultural commodities. Prior studies have, however, tended to give dynamic considerations short shrift, working instead with either first-difference or first-order autoregressive models. This study addresses squarely this issue by developing a general vector time-series model for expenditure shares in the context of an inverse demand system. Importantly, minimal constraints are placed on the model's short-run structure; consistent theoretical behavior is, however, incorporated in the model's long-run structure by using an Inverse AIDS (IAIDS) demand system. The resulting framework is used to model consumer meat expenditure decisions in the U.S. with favorable results. The model is then used to test for several more restrictive specifications such as autoregressive and partial adjustment models. In each case these more commonly used models are rejected. Also, the general model is superior in several regards to a first-difference specification.

1. INTRODUCTION

In recent years there has been renewed interest in systems of inverse demand equations for food and agricultural products (Chambers and McConnell; Barten and Bettendorf; Huang). In such systems prices are defined as dependent variables, while quantities consumed are treated as exogenous or right-hand-side variables (Anderson; Weymark). Interest in inverse demand systems stems from the fact that many food products, unlike most manufactured goods and services, involve relatively long production lags so that quantities available in the short run are essentially fixed. Furthermore, many food items are highly perishable, and therefore storable for only brief periods of time.

In light of this emerging interest in inverse demand models, several authors have explored ways in which estimable models can be specified that maintain essential elements of the theory. Christensen, Jorgenson, and Lau, for example, develop an Inverse Translog (ITL) demand system, which was subsequently employed by Christensen and Manser to estimate of a system of inverse meat demand equations. Alternatively, Chambers and McConnell and Barten and Bettendorf developed an inverse differential demand system analogous to the Rotterdam demand model. Recent advances in modelling inverse demand systems, however, utilize the distance function (Deaton), and include the Inverse Almost Ideal Demand System (IADS) of Moschini and Vissa, and Eales and Unnevehr (1994), and the Inverse Lewbel Demand System (ILDS) of Eales. In general, results show that inverse demand systems can provide reasonable estimates of short-run demand flexibilities for food items, and in particular for meats in the U.S.

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While considerable progress has been made in the application of inverse demand systems, further work is required. Importantly, models estimated to date have largely ignored the potential for market and price dynamics to influence consumption decisions. This is in spite of the fact that dynamic adjustments in consumer demands for meats are well documented (Wohlgenant and Hahn, Kesavan et al.). Factors such as short-run inventory adjustments, habit persistence, and sticky prices can all result in significant dynamic behavior in short-run consumer demand and price formation. The result is that consumers are likely unable to adjust to equilibrium every time period. Previous studies have tended to either ignore the potential for dynamic adjustments in demand and persisted in working with static models, or have otherwise attempted to capture their effects in relatively restrictive ways.

The objective of this paper is to combine recent advances in modelling systems of inverse demand equations with dynamic demand specifications. Specifically, the framework advanced originally by Anderson and Blundell (1982, 1983) is used to estimate a flexible dynamic inverse demand system for quarterly U.S. meat demands. The Anderson-Blundell (A-B) approach has considerable appeal because it places minimal restrictions on dynamic adjustments that can occur in short and intermediate runs, while at the same time allowing for a fully specified inverse demand system (specifically, the IAIDS) to be embedded in the model's long-run structure. A further advantage of their approach is that restrictions implied by theory can be imposed on the model's long-run structure, where, if anything, we expect such restrictions to hold (Paris, Caputo, and Holloway). Finally, the A-B dynamic model nests within it several popular but more restrictive dynamic specifications, including autoregressive and partial adjustment models, as well as a static specification. Further, these specifications can be tested against the more general alternative.

In recent years there have been numerous meat demand studies, including Moschini and Meilke; Eales and Unnevehr (1988, 1993, 1994); Eales; Alston and Chalfant (1991, 1993); Brester and Wohlgenant; and Kesavan et al. Of these, only Kesavan et al. develop a dynamic model similar in spirit to the one presented here. Moreover, to our knowledge the A-B approach, although well established in the general economics literature, has not been used previously to estimate a dynamic demand system (either direct or indirect) for a set of agricultural commodities. We report here the first known attempt to do so.

The specification of the IAIDS model used to characterize long-run demand behavior is given in the next section. In section 3 we turn to modelling short-run dynamics in the context of an error correction model, where error correction terms are identified by IAIDS demand equations. The approach is then applied to a model of quarterly U.S. meat demand for the period 1960-93 in section 4. To facilitate comparison, the general dynamic model is contrasted with a model specified on first differences of the data, a common but potentially restrictive approach to capturing short-run dynamics in meat demand estimation (e.g., Moschini and Meilke; Eales and Unnevehr (1988, 1993); Alston and Chalfant (1991, 1993); Brester and Wohlgenant). Finally, results are summarized and conclusions are presented in section 5.

2. THE IAIDS MODEL AND LONG RUN PREFERENCES

As noted in the introduction, the modelling strategy pursued here assumes the long-run structure of a dynamic model of meat expenditures is consistent with economic theory. We consider this structure first. Anderson and Blundell (1983) assumed that consumer expenditures could, in the long run, be characterized by a direct AIDS demand system. Our approach follows A-B's general setup, but with the caveat that long-run preferences are now specified according to the IAIDS demand system.

Let $\underline{\mathbf{w}}$ denote a $\underline{n}$ x 1 vector of budget shares on $\underline{n}$ goods, $\underline{\mathbf{q}}$ a $\underline{n}$ x 1 vector of quantities, and Q a measure of scale effects. In general, the long-run inverse demand structure may be written as:

$$\underline{\mathbf{w}} = f(\underline{\mathbf{q}}, Q, \underline{\boldsymbol{\theta}}), \tag{1}$$

where $\underline{\boldsymbol{\theta}}$ is a vector of underlying parameters that characterizes consumer preferences. By using the IAIDS demand system of Moschini and Vissa and Eales and Unnevehr (1994) to describe (1) yields:

$$\underline{\mathbf{w}} = \Pi(\underline{\boldsymbol{\theta}})\underline{\mathbf{x}}, \tag{2}$$

where $\underline{\mathbf{x}}$ = is a $\underline{l}$ x 1 vector of (transformed) quantities and scale effects, and Π is an appropriately dimensioned matrix function of the parameters, $\underline{\boldsymbol{\theta}}$. Specifically, a single equation from (2), representing the i-th budget share, is given by:

$$w_i = \alpha_i + \Sigma_j \gamma_{ij} \ln q_j + \beta_i \ln Q, \tag{3}$$

where $w_i = \pi_i q_i$, π_i being the normalized price of the i-th good (nominal price p_i normalized by total outlay); q_j denoting the per capita quantity of the j-th good consumed; and lnQ a suitable *per capita* quantity aggregator index. The specification of lnQ, derived from the consumer's underlying distance function representation, is:

$$\ln Q = \alpha_0 + \Sigma_j \alpha_j \ln q_j + \tfrac{1}{2} \Sigma_i \Sigma_j \gamma_{ij} \ln q_i \ln q_j , \tag{4}$$

which can be interpreted as a translog quantity aggregator index.

The long-run IAIDS structure is therefore given by (3) and (4), $i = 1,\ldots,n$, which includes nonlinear cross-equation parameter restrictions. This structure conforms to the description of long-run preferences in (2), where $\underline{x}$ consists of an intercept, log quantity terms, and transformed log quantity terms. The vector of underlying preference parameters can be obtained by placing suitable nonlinear restrictions on Π. Furthermore, in any fully integrable inverse demand system the usual properties of adding up, homogeneity, and symmetry apply. For the long-run IAIDS model these restrictions imply: $\Sigma_i \alpha_i = 1$, $\Sigma_i \gamma_{ij} = 0$, $\Sigma_i \beta_i = 0$ (adding up); $\Sigma_i \gamma_{ij} = 0$ (homogeneity); $\gamma_{ij} = \gamma_{ji}$ (symmetry). See Eales and Unnevehr (1994) for further details on the derivation and properties of the IAIDS model.

3. A FLEXIBLE DYNAMIC MODEL OF CONSUMER EXPENDITURES

In recent years there has been considerable interest in estimating error correction models, where short-run dynamics are modeled simultaneously with the model's implied long-run structure (Engle and Granger; Hoffman and Rasche). The flexible dynamic framework used here to model consumer expenditures on meat has its foundation in the error correction paradigm. The main difference is, of course, that we embed a theoretically consistent specification of consumer behavior in the long-run structure, the IAIDS model.

For illustrative purposes, consider a general first order dynamic model of the form:

$$\Delta \underline{\mathbf{w}}_t = A^* \Delta \underline{\mathbf{x}}_t - B^* (\underline{\mathbf{w}}_{t-1} - \Pi(\underline{\boldsymbol{\theta}}) \underline{\mathbf{x}}_{t-1}) + \underline{\epsilon}_t , \tag{5}$$

where Δ is a first difference operator such that $\Delta z_t = z_t - z_{t-1}$, A^* is a conformable short-run coefficient matrix, B^* is an appropriately dimensioned speed-of-adjustment or *error correction* matrix, $\underline{\epsilon}_t$ is a $n \times 1$ vector of independently and identically distributed mean-zero random error terms, and $\underline{t}$ is a time index, $\underline{t} = 1,\ldots,T$. Define the parameter matrices in (5) such that $B^* = (I - C_1)$, $A^* = A_1$, and $\Pi(\boldsymbol{\theta}) = B^{*-1}(A_1 + A_2)$. It then follows that an equivalent representation of (5) is:

$$\mathbf{w}_t = A_1\mathbf{x}_t + A_2\mathbf{x}_{t-1} + C_1\mathbf{w}_{t-1} + \boldsymbol{\epsilon}_t. \tag{6}$$

The two dynamics specifications, (5) and (6), are observationally equivalent. While (6) may be the more familiar specification of a dynamic model, the relative advantage of (5), however, is that the model's long-run parameters—in this case, the parameters of the IAIDS model—are specified directly. In other words, model (5) facilitates directly imposing behavior consistent with theory on the long run structure.

Without further restrictions, neither (5) or (6) are estimable. This is because: (1) an intercept term is included in both $\mathbf{x}_t$ and $\mathbf{x}_{t-1}$; and (2) because adding up implies $\boldsymbol{\iota}'\mathbf{w}_t = 1$ for all $\underline{t}$, where $\boldsymbol{\iota}$ is a conformable vector with all elements equal to one. To derive an estimable form of (5), let $\tilde{\mathbf{x}}_t$ denote $\mathbf{x}_t$, only with the constant term deleted, and let superscript $\underline{n}$ denote the operator that deletes the $\underline{n}$-th row of any matrix or vector. It then follows that an estimable form of (5) is given by:

$$\Delta\mathbf{w}_t = A\Delta\tilde{\mathbf{x}}_t - B(\mathbf{w}^n_{t-1} - \Pi^n(\boldsymbol{\theta})\mathbf{x}_{t-1}) + \boldsymbol{\epsilon}_t. \tag{7}$$

As Anderson and Blundell (1983) note, there is a loss of identification in the error correction matrix, B. Specifically, they show that the $\underline{n} \times (\underline{n} - 1)$ elements of B are related to the n^2 elements of B by

$$b_{ij} = b^*_{ij} - b^*_{in}, \qquad i = 1,\ldots,\underline{n}, \text{ and } j = 1,\ldots,\underline{n} - 1.$$

Without additional information pertaining to the error correction parameters, the b^*_{ij}s cannot be recovered from estimated b_{ij} parameters. In any event, the adding up restriction does, however, imply no loss of information regarding the model's long run structure; all elements in $\boldsymbol{\theta}$ can be recovered by applying appropriate restrictions on Π^n. Finally, adding up restrictions on the complete model in (7) implies $\boldsymbol{\iota}'\Delta\mathbf{w}_t = 0$. As Anderson and Blundell (1982) demonstrate, this identity results in additional restrictions on elements of A and B. The column sums of these matrices must all be zero.

Model (7) is a flexible specification of dynamics in consumer expenditures. Nested within this specification are several commonly used dynamic setups, including autoregressive and partial adjustment models. A purely static model is also nested within dynamic system (7). Specifically, a first-order autoregressive model of the type estimated by, for example, Eales and Unnevehr (1994) and Eales, can be obtained by imposing the restrictions:

$$a_{ij} = \Pi_{ij+1}(\underline{\theta}), \qquad i = 1,\ldots,\underline{n} - 1,\ j = 1,\ldots,\underline{k} - 1, \tag{8}$$

on (7), where a_{ij} and $\Pi_{ij+1}(\underline{\theta})$ are, respectively, the ij-th elements of A and $\Pi^{\underline{n}}$. Likewise, a partial adjustment model can be deduced from (7) by imposing the restrictions:

$$a_{ij} = \Sigma_{\underline{k}}\, b_{i\underline{k}}\Pi_{\underline{k}j+1}(\underline{\theta}), \qquad i = 1,\ldots,\ \underline{n} - 1,\quad j = 1,\ldots,\ \underline{k} - 1, \tag{9}$$

where a_{ij} and $\Pi_{ij+1}(\underline{\theta})$ are as defined in (8). Finally, a purely static model, where adjustments to equilibrium occur instantaneously every period, can be derived from (7) by combining the restrictions in (8) with restrictions on B. Specifically, a static model can be obtained by enforcing:

$$a_{ij} = \Pi_{ij+1}(\underline{\theta}), \qquad i = 1,\ldots,\underline{n} - 1,\ j = 1,\ldots,\underline{k} - 1,$$

$$b_{ij} = 1\ i = j,\quad b_{ij} = 0,\ i \neq j,\ \text{and}\ b_{\underline{n}j} = -1, \qquad j = 1,\ldots,\underline{n} - 1. \tag{10}$$

The restrictions implied in (8)-(10) provide a basis for conducting statistical tests of the dynamic structure of a system of expenditure equations. Of the above specifications, the autoregressive model is most frequently employed in meat demand studies (e.g., Eales and Unnevehr (1984), and Eales). Importantly, as illustrated by the restrictions in (8), autoregressive models constrain short-run and long-run effects to be identical. This is a strong assumption, and moreover, such restrictions are typically imposed without the benefit of formal statistical support. Lastly, it is possible to impose restrictions on (7) to obtain a share equation system specified entirely in first-difference form.[1/] But as Anderson and Blundell (1982) indicate, likelihood ratio or other asymptotically equivalent tests are invalid in this situation so that formal tests for a first-difference alternative are not available.

4. AN APPLICATION TO MEAT EXPENDITURE DECISIONS IN THE U.S.

The data used in the application are quarterly time-series data on consumers' expenditure on four meat items in the U.S. for the period 1960-1993.[2/] Specifically, the four meats included are beef, pork, chicken, and turkey. The data were obtained from standard USDA sources, and all quantities are expressed in per capita terms. Prior to estimation, all data were deseasonalized by regressing each price and quantity series on a set of three trigonometric seasonal indices, a fifth-order polynomial trend, and interaction terms between trend and seasonal indices to allow for gradual shifts in seasonality.[3/] The resulting deseasonalized data, used in all subsequent estimations, are summarized in figures 1 and 2 and table 1.

First, augmented Dickey-Fuller (ADF) test statistics reported in table 1 indicate that the null hypothesis that all meat expenditure share and quantity data contain a unit root cannot be rejected at any reasonable levels.[4/] This result has implications for subsequent model specification, and among other tings, suggests that modelling short-run behavior by using data in levels is inappropriate.

Turning to the data themselves, as figure 1 illustrates, the most dramatic change in meat consumption over the past 30 years has been for chicken, with per capita consumption in 1960 at about 5.5 pounds (on a quarterly basis) but increasing to slightly under 20 pounds by 1993. Moreover, per capita chicken consumption surpassed pork in 1986 and beef in 1990. At the same time, beef consumption increased from slightly over 15 pounds per capita in 1960 to a maximum of 24 pounds in the first quarter of 1976 (table 1), and has since returned to early 1960s levels. At the same time, pork consumption has neither grown nor declined dramatically (table 1), although consumption did drop significantly in the mid 1980s. Lastly, per capita turkey consumption remains low, but has experienced some growth in recent years.

Alternatively, figure 2 and table 1 show that expenditure shares on these meats have remained relatively more stable than have quantities consumed. For example, beef's share of total meat expenditures has hovered between 50 and 59 percent, reaching its absolute peak in 1975. Likewise, expenditures on pork have declined gradually over the sample period from approximately 35 percent in the early 1960s to 27 percent in the mid-1990s. The opposite pattern is observed for chicken, with its share of total meat expenditures growing from around 10 percent in the early 1960s to just over 18 percent in recent years. Lastly, turkey's share of total meat expenditures has remained rather constant at around 3 percent, although modest growth has been noted in recent years. The stability of expenditure shares may suggest that much of the shifts in quantities consumed are largely offset by relative price changes.

As the preceding discussion suggests, considerable changes have occurred in meat expenditures in the U.S. over the past thirty years. What implications do these changes have for modelling meat demands, and might meat expenditure equations be better characterized in dynamic context? To address these and related questions, we estimate a general dynamic flexible model similar to that outlined in the previous section. Specifically, we estimate dynamic share equations that include fourth-order error correction terms of the type:[5/]

$$\Delta\underline{\mathbf{w}}_{\underline{t}} = A\Delta\underline{\mathbf{x}}_{\underline{t}} - \sum_{k=1}^{4} B_k(\underline{\mathbf{w}}^{n}_{\underline{t}-k} - \Pi^{n}(\underline{\boldsymbol{\theta}})\underline{\mathbf{x}}_{\underline{t}-k}) + \underline{\epsilon}_{\underline{t}}, \qquad (11)$$

where

$$\underline{\Pi}^{n}_{i}(\underline{\boldsymbol{\theta}})\underline{\mathbf{x}}_{\underline{t}-k} = \alpha_i + \Sigma_j\, \gamma_{ij} \ln q_{j\underline{t}-k} + \beta_i \ln Q_{\underline{t}-k}, \quad i,j = 1,\ldots,3, \quad k = 1,\ldots,4, \qquad (12)$$

and

$$\ln Q_{\underline{t}-k} = \Sigma_j\, \alpha_j \ln q_{j\underline{t}-k} + \tfrac{1}{2}\Sigma_i \Sigma_j\, \gamma_{ij} \ln q_{i\underline{t}-k} \ln q_{j\underline{t}-k}, \quad i,j = 1,\ldots,3, \quad k = 1,\ldots,4. \qquad (13)$$

In the dynamic system specified in (11)-(13) $\underline{\mathbf{x}}_{\underline{t}} = (1, \ln q_{1\underline{t}}, \ln q_{2\underline{t}}, \ln q_{3\underline{t}}, \ln q_{4\underline{t}}, \ln Q_{\underline{t}})'$, where $q_{1\underline{t}} = BFQ_{\underline{t}}$, $q_{2\underline{t}} = PKQ_{\underline{t}}$, $q_{3\underline{t}} = CKQ_{\underline{t}}$, $q_{4\underline{t}} = TKQ_{\underline{t}}$; $\underline{t} = 5,\ldots,136$; $\underline{\Pi}^{n}_{i}(\underline{\boldsymbol{\theta}})$ denotes the i-th row of the long-run IAIDS parameter matrix, $\Pi^{n}(\underline{\boldsymbol{\theta}})$; and $\underline{\epsilon}_{\underline{t}}$ is a mean-zero, joint normally distributed error vector, Ω denoting the contemporaneous covariance matrix of $\underline{\epsilon}_{\underline{t}}$.

Adding up, homogeneity, and symmetry restrictions imposed on parameters in $\Pi^{n}(\underline{\boldsymbol{\theta}})$ in (11) and (12) are as defined previously. With these restrictions in place, there are 60 free parameters to be estimated in model (11). Because $\underline{\iota}'\Delta\underline{\mathbf{w}}_{\underline{t}} = 0$ for all $\underline{t}$, it follows that $\underline{\iota}'\Omega = 0$ for all $\underline{t}$. That is, the contemporaneous covariance matrix is singular. To avoid this problem the equation for turkey was omitted during estimation (both in the short-run structure, as well as in the long-run IAIDS model). Full information maximum likelihood estimates of the parameters implied in the flexible dynamic meat expenditure model in (11)--with all restrictions implied by theory imposed--were obtained by using the Davidon-Fletcher-Powell algorithm as implemented in the FORTRAN-based program GQOPT. The estimates, along with asymptotic standard errors, are reported in table 2.

In terms of the estimated long-run IAIDS demand structure, 18 of the 24 estimated parameters reported in table 2 are more than twice their asymptotic standard errors. Regarding the estimate of the short-run coefficient matrix A, eight of 16 estimated a_{ij} parameters exceed their asymptotic standard errors by more than two. Alternatively, only five of the 48 estimated parameters in the B_k matrices reported in table 2 are more than twice their asymptotic standard errors. This result is likely due to multicollinearity because a likelihood ratio (LR) test of the restrictions implied by $B_2 = B_3 = B_4 = 0$ yielded a test statistic of 59.293, which is extreme in the asymptotic $\chi^2(33)$ distribution.

As outlined in equations (8), (9), and (10), the flexible dynamic model has nested within its structure, respectively: (1) a fourth-order autoregressive model; (2) a first-order partial adjustment model; and (3) a static IAIDS model. The LR test statistic for restrictions implied by a fourth-order autoregressive model equals 247.812, the value of an asymptotic $\chi^2(39)$ distribution under the null hypothesis, strongly rejecting the fourth-order autoregressive model. Likewise, the LR test statistic for the first-order partial adjustment model is 224.723, an extreme value in the asymptotic $\chi^2(36)$ distribution. The first-order partial adjustment model is also rejected in favor of the general dynamic specification. Finally, the LR test statistic for a static IAIDS model is 896.460, which for all practical purposes has a p-value of zero in the asymptotic $\chi^2(48)$ distribution. Of these three specifications, the fourth-order autoregressive model is most similar to the types of IAIDS models estimated previously for meat demands (e.g., Eales and Unnevehr (1994), and Eales).[6/] Importantly, our results show that the autoregressive approach to dealing with dynamics in meat demand estimation may well be too restrictive.

Although the first-difference model cannot be tested statistically against the general dynamic model, it is possible to compare estimation results. Estimates of a first-difference model--obtained for the same sample period--are recorded in table 3. Compared with the long-run IAIDS structure in table 2, the biggest discrepancies seem to occur for estimates of β_i scale parameters. The first-difference model implies larger β_i estimates (in absolute terms) for pork and turkey and smaller β_i estimates for beef and chicken (again, in absolute terms) than does the flexible dynamic model. This outcome is consistent with Anderson and Blundell's (1982), who found large divergences in estimated expenditure effects between a general dynamic AIDS model and a more restrictive static AIDS model. In addition to scale effects, there is also considerable variation in estimates of cross-quantity effects between the two models.

Further comparisons between the two models can be made by examining the estimated residuals. Several diagnostic measures are reported in table 4. For the dynamic model, Box-Pierce Q(12) statistics indicate only the pork share equation has any remaining significant autocorrelation. Likewise, for the first-difference model, only the chicken equation has significant autocorrelation. Importantly, $\bar{R}^2$s for share equations in the dynamic model are, in every case, larger than their counterparts in the first-difference model (table 4). In fact, $\bar{R}^2$s for beef, pork, and chicken equations in the dynamic model are more than twice those estimated for their counterparts in the first-difference model. Of course residual root mean squared-errors exhibit a parallel pattern, being lower for all equations in the dynamic model than in the first-difference model. On balance, the flexible dynamic model does a good job of explaining the data, and provides a better fit than does a first-difference specification.

Additional insights can be obtained by examining own-price, cross-price, and scale flexibilities for both models. These estimates, along with asymptotic standard errors, are recorded in table 5.[7] To start, all own-price flexibilities for the dynamic model are negative and are less than one in absolute terms, suggesting that meat demands are <u>flexible</u> (i.e., own-price flexibilities are between zero and minus one). Furthermore, the magnitudes, at least for beef, pork, and chicken, are comparable to those reported by Eales and Unnevehr (1994) and Eales. Interestingly, long-run cross-price flexibilities computed with the dynamic model suggest that pork and beef, and chicken and turkey are gross <u>q</u>-substitutes. Perhaps of greater interest, however, is that no beef cross-price flexibilities is significantly different from zero (table 5), a result that stands in contrast to that reported by Eales. Scale flexibilities computed for the dynamic model are all negative and in each case are significant. In general, scale flexibility for beef is larger than previous estimates and scale flexibilities for pork and chicken are smaller than prior estimates.

Cross-price and scale flexibilities for the first-difference model reported in table 5 are generally in closer agreement with those reported by Eales and Unnevehr (1994) and Eales, at least with respect to beef, pork, and chicken. When compared with the estimates of the dynamic model, with the exception of turkey there is not much discrepancy in own-price estimates. Estimated cross-price flexibilities are, however, generally larger in absolute terms than those for the flexible dynamic model. Likewise, there is a noticeable difference in scale beef flexibilities for beef implied by the two models. Interestingly, own- and cross-price flexibilities for turkey are mostly insignificant in the first-difference model, and the associated scale flexibility is implausibly large and positive (albeit significantly different from zero). Flexibilities obtained under the general dynamic specification generally appear more acceptable.

As a final comparison of the two approaches to modelling meat demand, eigenvalues of the Antonelli substitution matrix, along with associated standard errors, were computed at the means of the sample data.[8/] To assure existence of a well-defined distance function, not only must estimated share equations satisfy homogeneity and symmetry, but the estimated Antonelli substitution matrix must be negative semi-definite as well.[9/]

The negativity results, reported in table 6, are striking. In neither case is the dominate eigenvalue, λ_1, significantly different from zero. For the dynamic model, however, the remaining three eigenvalues are significantly less than zero, while for the first-difference model only the final value, λ_4, is significantly less than zero. Moreover, in 10,000 draws on the parameter vector, the long-run IAIDS specification embedded in the general dynamic model did not fail the negativity requirement on the estimated Antonelli matrix <u>once</u>, as indicated by Prob = 1.0 for the dynamic model in table 6. Alternatively, in the same number of draws, Prob = 0.18 for the first-difference model, implying the negativity condition is satisfied only 18 percent of the time. On the basis of these results alone, it appears the long-run IAIDS structure associated with the general dynamic model provides a more satisfactory representation of consumer preferences than does the first-difference model.[10/]

5. CONCLUSIONS

In recent years there has been renewed interest in estimating inverse demand systems for meat and other agricultural commodities. While progress has been made, dynamic considerations in most prior applications of inverse demand systems have been largely overlooked. This paper has addressed this issue by specifying and estimating a flexible dynamic model of consumers' meat expenditures in an inverse demand context. This was accomplished by adopting Anderson and Blundell's (1982, 1983) approach, which allows modelling of short-run expenditure dynamics in a fully flexible manner, while at the same time maintaining a theoretically consistent structure in the model's long-run response. Specifically, long-run demands were specified according to the Inverse Almost Ideal Demand System (IAIDS) advanced recently by Eales and Unnevehr (1993, 1994) and Moschini and Vissa. Importantly, the specified model had nested within its framework several commonly used specifications for demand dynamics, including autoregressive and partial adjustment setups.

A fourth-order general dynamic model was estimated by using quarterly time-series data on meat expenditures and quantities in the U.S. The estimated model fit the data well, appeared to do an adequate job of capturing dynamics in meat expenditure decisions, and yielded reasonable parameter and flexibility estimates. Of interest is that more restrictive specifications associated with: (1) an autoregressive model; (2) a partial adjustment model; and (3) a static model all were rejected in favor of the general specification. Inasmuch as these models, and in particular the autoregressive model, are among the most frequently employed dynamic demand specifications, results here suggest that future work should pay much more attention to the nature of underlying market dynamics.

Finally, the general dynamic model was compared with a first-difference model, a specification which has been employed frequently in studies that use time-series data. The general dynamic model was found to be superior in a several regards to the first-difference model, including model fit, flexibility estimates, and, perhaps most importantly, in its ability to satisfy negativity conditions arising from consumer theory. Given that these conditions are often the most difficult to impose in empirical applications, results reported here suggest that the dynamic specification could also be a useful vehicle for conducting policy and welfare analysis.

ENDNOTES

1/ In particular, a first difference model can be obtained from (7) by imposing:

$$a_{ij} = \Pi_{ij+1}(\underline{\boldsymbol{\theta}}), \qquad i = 1,\ldots,\underline{n} - 1,\ j = 1,\ldots,\underline{k} - 1,$$

$$b_{ij} = 0, \qquad i,j = 1,\ldots,\underline{n} - 1.$$

2/ In the empirical application the first four observations are used to initialize the model's dynamic lag structure.

3/ Specifically, the seasonal indices are given by $\cos(2\pi t/2)$, $\sin(2\pi t/4)$, and $\cos(2\pi t/4)$, $t = 1,\ldots,136$.

4/ Indeed, the same general results regarding unit roots hold for the log quantity data as well. ADF test statistics for a unit root in the log quantity terms are, respectively, −1.89 (ln QBF), −2.68 (ln QPK), −1.81 (ln QCK), −2.05 (ln QTK), all of which are less than, in absolute terms, the critical value of −3.13 at the 10 percent level.

5/ The fourth-order specification in (11) was determined largely on the basis of preliminary testing; the specification makes sense, however, because previous studies involving quarterly time-series data often find significant autocorrelation at annual frequencies (e.g., Kesavan et al.).

6/ Specifically, models estimated by Eales and Unnevehr (1994) and Eales employ a first-order autoregressive model with the autocorrelation terms constrained to be the same across equations (this is done to ensure invariance with respect to the equation omitted; see, e.g., Berndt and Savin). When this specification was tested against the general dynamic model, an LR test statistic of 249.284 was obtained, a value which also is extreme in the asymptotic $\chi^2(47)$ distribution.

7/ Flexibilities were obtained by using the formulae reported in Eales and Unnevehr (1994), evaluated at the means of the sample data. Standard errors were obtained by employing the Monte Carlo simulation techniques introduced by Geweke and employed in a demand systems context by Chalfant, Gray, and White. Specifically, the estimated parameter covariance matrix was used to obtain 10,000 draws on the $\boldsymbol{\theta}$ parameter vector. Flexibilities were then computed after each draw. Resulting sample standard deviations were used as estimates of asymptotic standard errors.

8/ The same Monte Carlo method used to compute standard errors for flexibilities was used to compute standard errors for eigenvalues, as well as the associated probability the Antonelli substitution matrix is quasi-concave.

9/ Due to adding up, the Antonelli matrix can never be of full rank, and hence can never be strictly concave. The implication is the dominate eigenvalue of the Antonelli matrix will—within acceptable roundoff error—equal zero.

10/ Of course all results regarding eigenvalues of the Antonelli matrix are conditional on symmetry and homogeneity being imposed.

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TABLE 1

Descriptive Statistics for IAIDS Meat Demand Variables: Variations in Quantities and Budget Shares (sample 1961–93, N = 132)

Augmented Dickey–Fuller regression equation for a unit root:

$$\Delta y_{t} = \alpha + \delta t + \rho y_{t-1} + \sum_{i=1}^{8} \zeta_{i} \Delta y_{t-i} + \varepsilon_{t}, \qquad t = 1, \ldots, 136.$$

Variable	Sample Average	Standard Error	Min	Max	ADF[a]
Quantities:					
Beef Quantity (BFQ):	19.29	2.00	15.39	24.04	-1.91
Pork Quantity (PKQ):	14.44	1.40	11.71	17.82	-2.66
Chicken Quantity (CKQ):	11.29	3.81	5.58	19.69	-0.07
Turkey Quantity (TKQ):	2.41	0.98	4.52	1.22	-1.63
Budget Shares:					
Beef Budget Share (WBF):	0.55	0.02	0.50	0.59	-1.07
Pork Budget Share (WPK):	0.30	0.02	0.26	0.35	-2.32
Chicken Budget Share (WCK):	0.12	0.03	0.09	0.18	-0.27
Turkey Budget Share (WTK):	0.03	0.01	0.02	0.05	-1.38

Note: Quantities are in pounds *per capita*, budget shares are in percent. Min denotes minimum value in the sample data and Max denotes the corresponding maximum value. Likewise, ADF denotes the augmented Dickey–Fuller unit root test statistic on ρ.

a. Critical value at the $\alpha = 0.10$ level is -3.13.

TABLE 2

Estimated Coefficients for the Dynamic IAIDS Model of Meat Demand, 1961-1993

Commodity i	Long-Run IAIDS Parameters $\Pi(\underline{\theta})$						A-Matrix Parameters			
	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	β_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}
Beef	-0.095 (1.301)	1.455 (0.424)	-0.548 (0.179)	-0.727 (0.236)	-0.180 (0.078)	1.904 (0.478)	0.149 (0.021)	-0.003 (0.042)	0.003 (0.034)	-0.080 (0.076)
Pork	5.071 (0.770)	-0.548 (0.179)	0.786 (0.121)	-0.179 (0.091)	-0.058 (0.034)	-0.843 (0.290)	-0.058 (0.025)	0.153 (0.041)	0.047 (0.028)	-0.148 (0.076)
Chicken	4.028 (0.804)	-0.727 (0.236)	-0.179 (0.091)	0.728 (0.146)	0.179 (0.065)	-0.864 (0.291)	-0.054 (0.017)	-0.002 (0.030)	0.048 (0.023)	-0.058 (0.056)
Turkey	0.996 (0.270)	-0.180 (0.078)	-0.058 (0.034)	0.179 (0.065)	0.059 (0.042)	-0.198 (0.097)	-0.036 (0.033)	-0.148 (0.043)	-0.098 (0.015)	0.286 (0.078)

	B_k-Matrix Parameters, k = 1,...,4											
	b_{i11}	b_{i21}	b_{i31}	b_{i12}	b_{i22}	b_{i32}	b_{i13}	b_{i23}	b_{i33}	b_{i14}	b_{i24}	b_{i34}
Beef	0.629 (0.295)	0.256 (0.324)	0.558 (0.310)	0.180 (0.324)	0.345 (0.362)	-0.036 (0.323)	-0.698 (0.353)	-0.691 (0.384)	-0.477 (0.337)	-0.198 (0.279)	-0.223 (0.316)	-0.357 (0.317)
Pork	-0.418 (0.236)	-0.202 (0.264)	-0.312 (0.246)	-0.015 (0.268)	-0.114 (0.293)	-0.054 (0.267)	0.237 (0.258)	0.438 (0.288)	0.261 (0.264)	-0.037 (0.221)	-0.066 (0.246)	-0.157 (0.242)
Chicken	0.147 (0.194)	0.292 (0.207)	0.258 (0.203)	-0.137 (0.222)	-0.151 (0.245)	0.040 (0.218)	0.403 (0.230)	0.193 (0.251)	0.178 (0.221)	0.142 (0.199)	0.150 (0.216)	0.388 (0.224)
Turkey	-0.357 (0.069)	-0.346 (0.074)	-0.505 (0.073)	-0.028 (0.070)	-0.080 (0.077)	0.051 (0.073)	0.058 (0.067)	0.061 (0.074)	0.038 (0.070)	0.092 (0.057)	0.139 (0.062)	0.125 (0.063)

Note: Asymptotic standard errors are given in parentheses. All α_i, γ_{ij}, and β_i parameter estimates and associated standard errors have been multiplied by 10 to facilitate presentation. All results were derived by omitting the equation for turkey.

TABLE 3

Estimated Coefficients for the First-Difference IAIDS Model of Meat Demand, 1961-1993

Commodity i	α_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	β_i
Beef	1.843 (1.976)	1.406 (0.300)	-0.308 (0.124)	-0.283 (0.154)	-0.816 (0.351)	-0.247 (0.378)
Pork	0.532 (1.382)	-0.308 (0.124)	0.777 (0.247)	-0.282 (0.153)	-0.188 (0.334)	-1.032 (0.470)
Chicken	4.176 (2.756)	-0.283 (0.154)	-0.282 (0.153)	0.495 (0.367)	0.069 (0.402)	-0.517 (0.335)
Turkey	3.450 (0.966)	-0.816 (0.351)	-0.188 (0.334)	0.069 (0.402)	0.934 (0.246)	1.796 (0.462)

Note: Values in parentheses are asymptotic standard errors.

TABLE 4

Residual Diagnostics of the Dynamic and First-Difference IAIDS Models (sample 1961–93, N = 132)

Equation	Autocorrelations 1	2	3	4	5	6	7	8	Box-Pierce χ^2_{12}	(p-value)	$\bar{R}^2$	RMSE
Dynamic Model												
Beef	0.11	0.03	0.09	−0.08	0.11	0.16	−0.13	−0.13	18.03	(0.115)	0.40	0.0052
Pork	0.07	0.09	0.04	−0.01	0.07	0.22	−0.14	−0.10	26.91	(0.008)	0.41	0.0043
Chicken	0.01	−0.03	0.07	−0.19	0.05	0.03	−0.14	−0.10	17.05	(0.148)	0.36	0.0037
Turkey	−0.15	0.12	0.07	0.01	0.05	0.01	0.11	−0.11	21.02	(0.050)	0.74	0.0011
First-Difference Model												
Beef	−0.14	0.04	−0.18	−0.07	−0.04	0.27	−0.14	−0.01	20.74	(0.054)	0.13	0.0083
Pork	0.06	0.06	−0.09	−0.17	−0.06	−0.05	−0.21	−0.07	17.37	(0.136)	0.17	0.0056
Chicken	0.06	−0.11	0.07	−0.33	−0.09	−0.01	−0.15	−0.01	27.55	(0.006)	0.17	0.0044
Turkey	−0.27	0.06	−0.03	0.09	−0.01	0.14	0.08	−0.02	18.13	(0.112)	0.58	0.0075

Note: RMSE denotes root mean-squared error.

TABLE 5

Comparison of Price and Scale Flexibilities: Dynamic IAIDS and First-Difference IAIDS Models

	BFQ	PKQ	CKQ	TKQ	Scale
			Dynamic Model		
Beef P:	−0.710*	0.079	−0.016	−0.004	−0.652*
	(0.055)	(0.042)	(0.013)	(0.005)	(0.088)
Pork P:	−0.199*	−0.887*	−0.149*	−0.041*	−1.275*
	(0.052)	(0.064)	(0.016)	(0.008)	(0.096)
Chicken P:	−0.661*	−0.516*	−0.640*	0.089*	−1.727*
	(0.160)	(0.088)	(0.058)	(0.043)	(0.242)
Turkey P:	−0.614*	−0.510*	0.351*	−0.865*	−1.638*
	(0.229)	(0.124)	(0.162)	(0.136)	(0.312)
			First-Difference Model		
Beef P:	−0.776*	−0.075*	−0.060*	−0.134*	−1.046*
	(0.044)	(0.033)	(0.030)	(0.053)	(0.069)
Pork P:	−0.296*	−0.937*	−0.176*	0.069	−1.339*
	(0.096)	(0.125)	(0.064)	(0.137)	(0.154)
Chicken P:	−0.483*	−0.476*	−0.702*	0.232	−1.430*
	(0.157)	(0.190)	(0.260)	(0.232)	(0.281)
Turkey P:	0.817	2.534	1.630*	−0.198	4.782*
	(1.508)	(1.710)	(0.730)	(0.977)	(1.490)

Note: Values in parentheses are asymptotic standard errors. An asterisk indicates the flexibility exceeds twice its standard error.

TABLE 6

Estimated Eigenvalues of the Antonelli Matrix for the Dynamic and First-Difference IAIDS Models at the Sample Means, 1960–1993

	Eigenvalues				
	λ_1	λ_2	λ_3	λ_4	Prob.
Dynamic:	−0.0001 (0.0048)	−0.0275* (0.0036)	−0.0668* (0.0089)	−0.3278* (0.0256)	1.0000
First-Difference:	0.0038 (0.0381)	−0.0024 (0.1010)	−0.0797 (0.0481)	−0.2689* (0.0989)	0.1813

Note: Values in parentheses are asymptotic standard errors. "Prob." denotes the estimated probability that the Antonelli substitution matrix is negative semi-definite when evaluated at the sample means. An asterisk indicates the estimated eigenvalue is significantly less than zero at the 0.01 level.

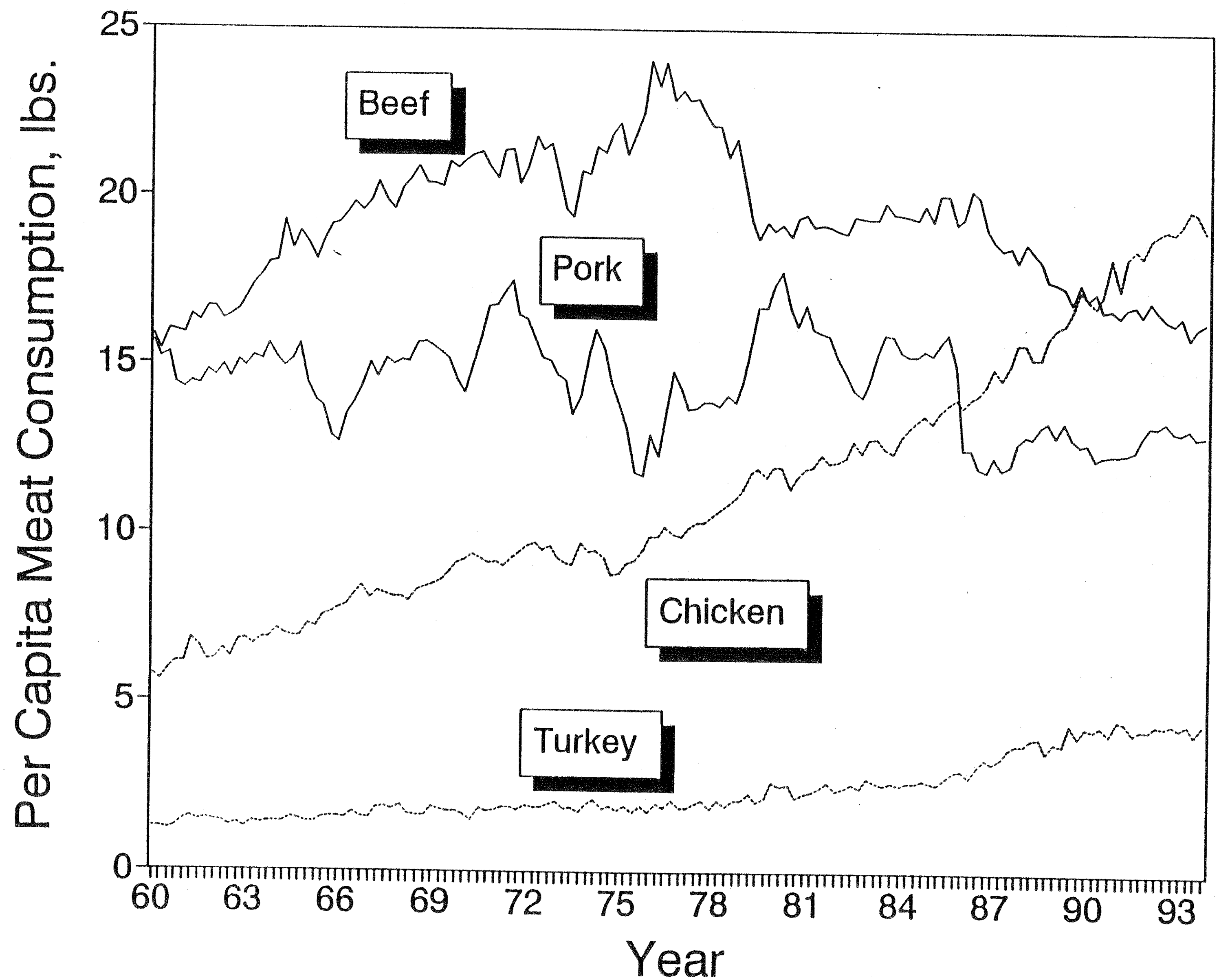

Figure 1. Quarterly *Per Capita* Consumption of Beef, Pork, Chicken, and Turkey, 1960-93.

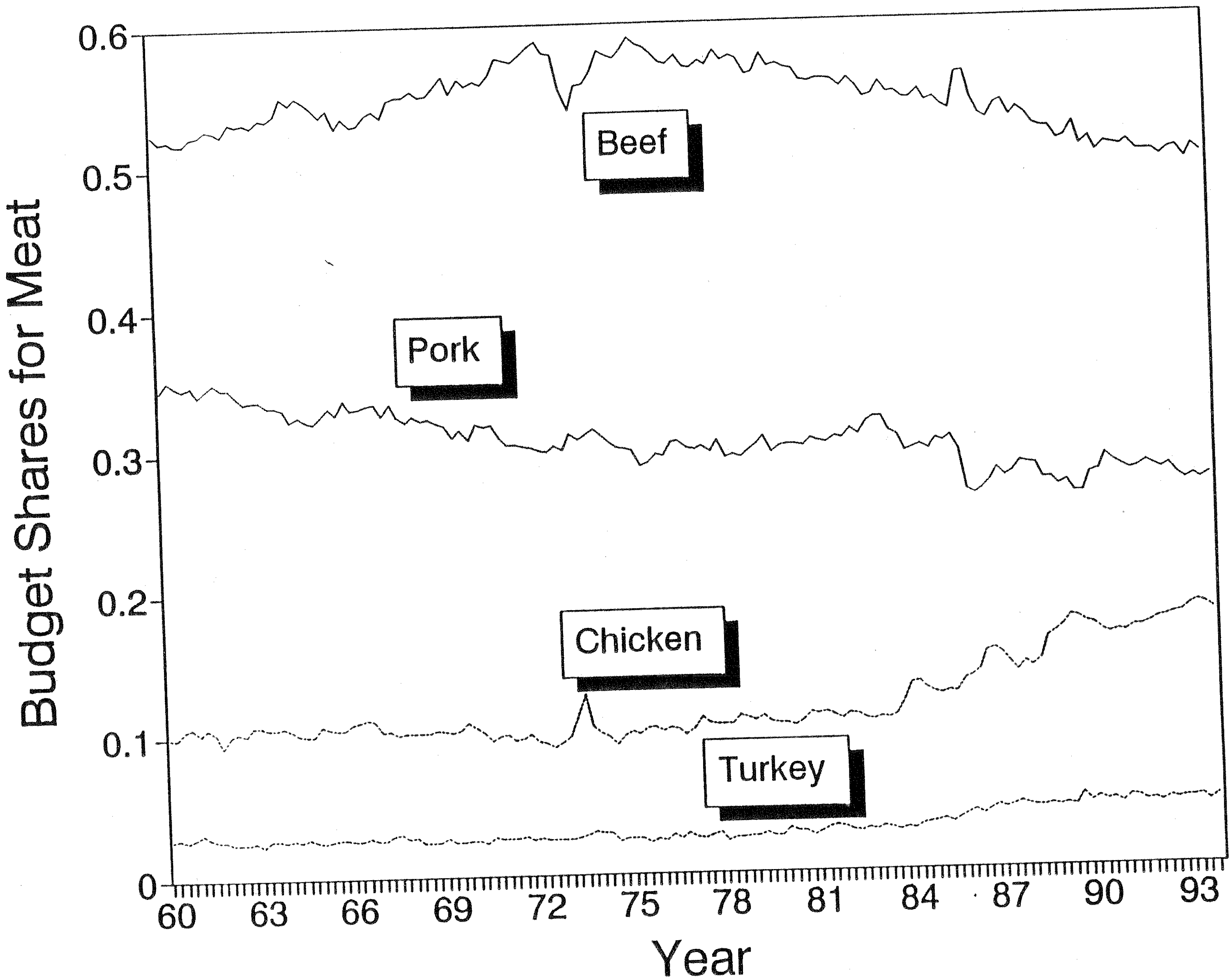

Figure 2. Shares in Total Meat Expenditure for Beef, Pork, Chicken, and Turkey, 1960-93.

Figure 3. Testing the Dynamic Structure of the Model