Garch Option Pricing with Implied Volatility

by

N'Zue F. Fofana and B. Wade Brorsen

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Generalized autoregressive conditional heteroskedasticity (GARCH) provides a better fit to futures price data than the common assumption of identical independent normal distribution. GARCH option pricing models (OPM) with historical volatility have proven superior to the log-normality assumption of the Black option pricing model with historical volatility. Implied volatilities derived from GARCH OPM might therefore be expected to provide better guidance in investment decisions than those derived from the Black option pricing model. This paper estimates implied volatilities from GARCH OPM. The estimated implied volatilities are used to forecast option premia. Results are compared against forecasts of option premia using implied volatilities from Black's option pricing model. The GARCH implied volatilities are more stable than the Black implied volatilities. The GARCH option pricing model with implied volatility outperformed the Black option pricing model with implied volatility in terms of forecasting actual option premia.

Introduction

Black's option pricing model (OPM) is the dominant model of pricing options on futures contracts. Of the five variables in the Black model, only the standard deviation of returns is not observable. Typically, a Black option pricing model with implied volatility is superior, in predicting actual option prices, to a Black option pricing model with a volatility estimated from historical data (Hauser and Liu). Among models of historical data, generalized autoregressive conditional heteroskedasticity (GARCH) models have proven superior to the log-normality assumption of the Black model (Yang and Brorsen). A GARCH OPM with historical volatility has proven superior to the Black model with historical volatility (Myers and Hanson; Kang and Brorsen). Indeed, it is now evident that commodity futures prices exhibit time varying volatility and tend to have excess kurtosis (characteristics that are not taken into account by the log-normality assumption of the Black model). GARCH models with conditional student *t* distributions can capture both the time-varying volatility and the excess kurtosis (Yang and Brorsen). The GARCH models with historical volatility are still inferior to a Black model with implied volatility.

The purpose of this study is to determine whether a GARCH option pricing model with implied volatility provides a more accurate forecast of option premia than the Black model with implied volatility. Engle and Mustafa and Hanson, Myers, and Wang derive implied GARCH parameters from option premia, however, they did so conditional on an estimate of historical

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volatility. The present paper proposes an alternative approach. The alternative is to estimate the GARCH parameters on lagged variance (β) and lagged error (α) from historical data. Unconditional volatility will then be estimated given the GARCH parameters by minimizing squared errors. It is believed that this approach will prove superior, since estimates of α and β are relatively constant across studies. Unconditional volatility changes due to seasonality in variance among other factors. Initial volatility must be calculated in an arbitrary fashion when it is calculated from historical data. In the present research, initial volatility was set equal to 11.69098 (actual historical volatility). Implied volatilities with GARCH will be compared to implied volatilities estimated using Black's option pricing model. Moreover, implied volatilities from both GARCH and Black option models will be used to simulate actual market option prices. The performance of each model will then be determined.

Background

To estimate the implied GARCH parameters, Engle and Mustafa solved the following minimization problem:

(1) $$\min \sum_{j=1}^{J} \theta_j \, [P_{jt} - \hat{P}_{jt}(\omega_t, \alpha_t, \beta_t; \hat{h}_{t-1}^{\,2})]^2$$

where, θ_j represent relative weights and the j subscript indicates put and call options written on the same underlying futures contract but with different strike prices. For simplicity, they assumed equal weights. The symbols α_t, ω_t, and β_t represent the implied GARCH parameters, P_{jt} represent the actual premiums. The estimated option premium $\hat{P}_{jt}$ is a function of the GARCH parameters conditional on historical volatility $(\hat{h}_{t-1}^{\,2})$. The choice variables in the problem described in equation (1) are the GARCH parameters α, ω, and β. The approach we propose is

(2) $$\min \sum_{j=1}^{J} \theta_j \, [P_{jt} - \hat{P}_{jt}(\sigma^2_t; \hat{\alpha}, \hat{\beta}, \hat{h}_{t-1}^{\,2})\,]^2$$

here, we assume that $\theta_j = 1$. The estimated option premium $\hat{P}_{jt}$ conditional on the GARCH parameters ($\hat{\alpha}$ and $\hat{\beta}$) and the initial volatility $(\hat{h}_{t-1}^{\,2})$, is a function of the unconditional volatility σ^2_t. The choice variable in equation (2) is σ^2_t, since ($\hat{\alpha}$ and $\hat{\beta}$) are constant across studies and initial volatility is fixed at 11.69098. The choice variable is obtained as follows:

(3) $$\sigma_t^2 = \frac{\omega_t}{(1 - \hat{\alpha} - \hat{\beta})}$$

Procedures

GARCH with a conditional *t* distribution (henceforth GARCH-t) was estimated by maximum likelihood using the first differences of the natural logarithms of the daily closing prices of wheat at the Chicago Board of Trade. The first differences were rescaled by multiplying them by 100.

The GARCH-t process was defined to model well-documented market anomalies such as day-of-the-week effects in both the mean and variance equation (Chiang and Tapley; Junkus), seasonality in variance (Anderson; Kenyon et al.), and maturity in variance (Milonas). The general stochastic process can be written as follows:

(4) $$y_t = \mu + \epsilon_t$$

(5) $$\epsilon_t \sim t(0,\ h_t^2,\ \nu)$$

(6) $$h_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}^2$$

where $y_t = 100(\ln(P_t) - \ln(P_{t-1}))$, P_t is Chicago wheat futures price, h^2_t is conditional variance of futures price changes, and $t(0, h^2_t, \nu)$ is the student t distribution with variance h^2_t and degrees of freedom ν. In the approach proposed, h^2_{t-1} is the initial volatility, and the unconditional volatility is σ^2_t. The initial volatility was fixed at 11.69098; hence only a restricted version of equation (6) was used in estimating implied volatilities and simulating option premia. The mean and variance equations estimated are respectively:

(7) $$y_t = a_0 + a_1 D_{MON_t} + a_2 D_{TUE_t} + a_3 D_{WED_t} + a_4 D_{THU_t} + \epsilon_t$$

(8) $$\begin{aligned} h_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}^2 + b_1 D_{MON_t} + b_2 D_{TUE_t} + b_3 D_{WED_t} \\ &+ b_4 D_{THU_t} + b_5 \sin(2\pi K_t/252) + b_6 \cos(2\pi K_t/252) \\ &+ b_7 \sin(2\pi K_t/126) + b_8 \cos(2\pi K_t/126) + b_9 MATURITY_t. \end{aligned}$$

where D denotes dummy variable for each day of the week; thus, $D_{MON} = 1$ if Monday and 0 otherwise, $D_{TUE} = 1$ if Tuesday and 0 otherwise, $D_{WED} = 1$ if Wednesday and 0 otherwise, and $D_{THU} = 1$ if Thursday and 0 otherwise. The constant π is approximated as 3.14, and K_t in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, that is, 252 indicates a one-year cycle whereas 126 indicates a half-year cycle. $MATURITY_t$ denotes the time to maturity measured as the number of trading days prior to maturity. The GARCH-t process was estimated using the maximum likelihood module of the statistical software package GAUSS.

Parameter estimates of the GARCH-t process (used as starting values), market determined option premia, and initial futures prices are required to solve equation (2). Since the GARCH option pricing model does not have a closed form solution, a Monte Carlo approach (see Paskov for details on Monte Carlo algorithm) was used to approximate option premia, defined as $\hat{P}_{jt}$ in equation (2), which is then discounted back at the risk-free interest rate. The discount factor being:

(9)
$$d = e^{-rT}$$

where r is the risk-free rate of interest and T is the time to maturity. Two sets of random numbers were generated[2]: one from a t-distribution with v degrees of freedom and another from a standard normal distribution. Time was measured in number of trading days. The time-varying conditional variances were generated for T periods using parameter estimates from the GARCH. Then, with the conditional variances, the futures prices F_t are simulated for T periods to get the futures price at maturity. Denoting this price at maturity $\{F_t\}_i$, the simulated option premia are:

(10)
$$\hat{P}_{jt} = \begin{cases} d(\frac{1}{n})\sum_{i=1}^{n} \max[k - \{ F_t \}_i , 0] & \text{for call} , \\ d(\frac{1}{n})\sum_{i=1}^{n} \max[\{ F_t \}_i - k, 0] & \text{for put} , \end{cases}$$

where n = 1000 is the number of replications of this procedure, and k is the strike (or exercise) price of the option. Equation (2) was then solved using the OPTMUM module of GAUSS. Since GARCH processes account for both time-varying volatility and excess kurtosis, GARCH implied volatilities are expected to be more stable than those obtained using Black option pricing model. The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm was used and then switched to the Scaled BFGS algorithm after the 10th iteration. The line search method used was the cubic or quadratic method (known as STEPBT). Implied volatilities were also obtained from the Black option pricing model.

Black vs. GARCH OPM

To examine the ability of the GARCH OPM with implied volatility and the Black OPM with implied volatility to forecast actual option premia, implied volatilities resulting from the minimization problem, in the Monte Carlo approach defined earlier, were used to forecast next day Chicago wheat option premiums for given strike prices. Root mean squared errors were used to measure the forecasting performance of both GARCH and Black option pricing models. Root mean squared errors (RMSE) is defined as

(11)
$$\text{RMSE} = \left[\frac{\sum_{t=1}^{T} (AP - SP)^2}{T} \right]^{0.5} ,$$

where AP is actual Chicago wheat option premia, SP is simulated Chicago wheat option premia

[2]The random numbers are generated using the same seed (seed = 409473).

and VAR is the symbol for variance. The sign test was used to test whether the root mean squared errors from the two option pricing models were significantly different (see Steel and Torrie p. 538 for details on the sign test). The sign test has a chi-squared distribution with one degree of freedom under the null hypothesis that the root mean squared errors from both models are not different.

Data

The data used to estimate the GARCH model were from July 1987 to July 1993, and were created using Continuous Contractor from Technical Tools. The rollover date is the 15th day of the month prior to delivery. On March 28, 1994, closing option premia for six strike prices were quoted on the Chicago Board of Trade for July 1994 futures contracts providing six closing option premia (March 28, 1994 was chosen to be about two months before the option expired). Daily Chicago wheat option premia (both put and call options were considered) and futures prices were collected from the Wall Street Journal from March 28 till maturity date of the July contract (June 17, 1994). Only options near maturity were considered to minimize problems with non-synchronous trading. The risk free rate of interest was assumed constant throughout the simulation period at r = 3.71%.[3] Descriptive statistics of the log differences of Chicago wheat futures prices are summarized in table 1. Skewness, kurtosis, and the D'Agostino omnibus test[4] provide evidence of non-normality.

Table 1 summarizes descriptive statistics and tests for departures from normality. All three tests show strong support for non-normality. Indeed, they (the tests) reject the null hypotheses of zero skewness, zero kurtosis at the 5% significance level. Tables 2 and 3 summarize the actual Chicago wheat futures option premia, and futures prices.

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[3]The risk-free rate of interest is approximated to be the rate of return on treasury bills with the same maturity date as the option premia collected. Both rate of return on treasury bills and option premia were collected from the Wall Street Journal.

[4]The omnibus test combines both skewness and kurtosis. It is defined as:

$$K^2 = Z^2(\sqrt{b_1}) + Z^2(b2) \sim \chi^2_2$$

where $\sqrt{b_1}$ and b_2 are skewness and kurtosis respectively, and $Z(\sqrt{b_1})$ and $Z(b_2)$ are approximately standard normal with mean zero and variance one. K^2 is distributed as chi-squared with two degrees of freedom under the null hypotheses of zero skewness ($\sqrt{b_1} = 0$) and zero excess kurtosis ($b_2 - 3 = 0$).

Empirical Results

Table 4 presents parameter estimates of the GARCH-t(1,1) process, and test statistics of the significance of day-of-the-week effects in both mean and variance equations, and test statistics of the seasonality effect in the variance equation. The estimated GARCH parameters are all significant. The sum of the GARCH terms (α and β) is less than one which implies stationarity. Tests of the significance of day-of-the-week effects show that both mean and variance of Chicago wheat futures price movements differ by day-of-the-week. No significant seasonal pattern is found in the variance. The implied volatilities estimated are plotted in figure 1. The graph shows that GARCH implied volatilities are more stable than Black implied volatilities as hypothesized. In both cases (GARCH and Black), implied volatilities increase as maturity approaches, results consistent with the findings by Day and Lewis. Indeed, Day and Lewis argued that demand by option traders to close positions in expiring options and to open positions in the next expiration series creates a temporary upward bias in the option prices that is reflected in the estimates of implied volatilities.

Table 5 shows the forecasting performance measured by the root mean squared errors of both GARCH and Black option pricing models for put premiums. The root mean squared errors calculated from the actual premia and the simulated GARCH option premia are smaller than those calculated from actual premia and simulated Black option premia at all strike prices except for a strike price of $3.80. The sign test result (7.11 > 3.84 the critical value at the 5% significance level) suggests that they (GARCH OPM with implied volatility and the Black OPM with implied volatility) are significantly different. Hence, the GARCH option pricing model with implied volatility outperforms the Black option pricing model with implied volatility.

Table 6 shows the forecasting performance measured by the root mean squared errors of both GARCH and Black option pricing models for call premiums. The root mean squared errors calculated from the actual premia and the simulated Black option premia are smaller than those calculated from actual premia and simulated GARCH option premia at all strike prices except for a strike prices of $3.70 and $3.80. The sign test result (2 < 3.84 the critical value at the 5% significance level) suggests that they (GARCH OPM with implied volatility and the Black OPM with implied volatility) are not significantly different. Hence, the GARCH option pricing model with implied volatility is as good as the Black option pricing model with implied volatility in forecasting call option premiums. However, since the GARCH implied volatilities are more stable, the GARCH OPM with implied volatility should provide better guidance in investment decision making than the Black option pricing model with implied volatility would.

Summary and Conclusions

This paper estimated implied volatilities with the GARCH option pricing models. The GARCH-t process was used to model Chicago wheat futures price movements. Implied volatilities were found by minimizing squared errors using both GARCH and Black option pricing models. Implied volatilities estimated were then used to simulate actual Chicago wheat option premia. Root mean squared errors were calculated to assess the forecasting performance of both models. In both GARCH and Black models, implied volatilities estimated increase near maturity. However, the GARCH implied volatilities are more stable than those obtained using the Black option pricing model. Root mean squared errors computed suggest that the GARCH implied volatilities are better than the Black implied volatilities in forecasting futures option premia (at least for put options). Plans for future research will therefore be to let initial volatility be a choice variable to be estimated or possibly to use historical volatility.

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Table 1. Summary Statistics of Daily Chicago Wheat Futures Prices from July 1987 to July 1993[a].

Description	Statistics	Test Value
Sample size	1537	
Mean	0.00608	
Standard Deviation	0.011436	
Skewness	0.181795	2.900[b]
Kurtosis	6.586588	11.564[c]
Omnibus Test		142.140[d]

[a] Units are percentages $([\ln(P_t) - \ln(p_{t-1})]*100$.
[b] statistic has a z distribution under the null hypothesis of zero skewness. The critical value for a two sided test is 1.96 at a 5% significance level.
[c] statistic has a z distribution under the null hypothesis of zero excess kurtosis. The critical value for a two sided test is 1.96 at a 5% significance level.
[d] Chi-square statistic calculated to test the null hypothesis of normality. The critical value at the 5% significance level is 5.99.

Table 2. March 28, 1994 to June 17, 1994 Chicago Wheat Futures Option Premia (Put options).

Time to Maturity	Strike Prices (dollar/bushel)									Futures Prices
	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	
57	na	0.0400	0.0725	0.1175	0.1775	0.2500	0.3325	na	na	3.2950
56	na	0.0400	0.0750	0.1200	0.1825	0.2550	0.3400	na	na	3.2875
55	na	0.0400	0.0775	0.1250	0.1875	0.2600	0.3425	na	na	3.2825
54	na	0.0550	0.0975	0.1550	0.2225	0.3025	0.3900	na	na	3.2325
53	na	na	0.0800	0.1300	0.1950	0.2700	0.3550	0.4450	na	3.2700
52	na	na	0.0750	0.1200	0.1775	0.2500	0.3325	0.4200	na	3.2975
51	na	na	na	0.0975	0.1413	0.2075	0.2825	0.3675	0.4575	3.3600
50	na	na	na	0.1000	0.1475	0.2150	0.2900	0.3750	0.4650	3.3550
49	na	na	0.0625	0.1050	0.1600	0.2275	0.3100	0.3950	na	3.3300
48	na	0.0475	0.0850	0.1375	0.2025	0.2775	0.3650	na	na	3.2575
47	na	0.0438	0.0825	0.1350	0.1975	0.2750	0.3625	na	na	3.2600
46	na	0.0500	0.0875	0.1450	0.2100	0.2900	0.3775	na	na	3.2400
45	na	0.0700	0.1225	0.1888	0.2638	0.3525	0.4425	na	na	3.1675
44	na	0.0725	0.1263	0.1950	0.2725	0.3575	0.4525	na	na	3.1575
43	na	0.0624	0.1088	0.1738	0.2475	0.3325	0.4250	na	na	3.1875
42	0.0363	0.0750	0.1288	0.1975	0.2788	0.3675	na	na	na	3.1425
41	0.0338	0.0713	0.1213	0.1913	0.2700	0.3575	na	na	na	3.1525
40	0.0325	0.0725	0.1213	0.1975	0.2725	0.3675	na	na	na	3.1425
39	0.0325	0.0700	0.1275	0.1988	0.2788	0.3675	na	na	na	3.1400

na = not available.

Table 2. Continued.

Time to Maturity	Strike Prices (dollar/bushel)									Futures Prices
	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	
38	0.0275	0.0575	0.1050	0.1713	0.2500	0.3350	na	na	na	3.1750
37	0.0225	0.0525	0.0975	0.1625	0.2400	0.3225	na	na	na	3.1900
36	0.0175	0.0363	0.0738	0.1300	0.2000	0.2750	na	na	na	3.2450
35	na	0.0300	0.0538	0.1000	0.1575	0.2275	0.3125	na	na	3.3075
34	na	0.0250	0.0475	0.0900	0.1400	0.2025	0.2825	na	na	3.3475
33	na	0.0300	0.0625	0.1075	0.1675	0.2400	0.3250	na	na	3.3000
32	na	na	0.0488	0.0900	0.1425	0.2050	0.2850	0.3725	na	3.3475
31	na	0.0300	0.0600	0.1075	0.1650	0.2400	0.3225	na	na	3.3025
30	na	0.0363	0.0700	0.1225	0.1850	0.2625	0.3475	na	na	3.2700
29	0.0188	0.0400	0.0750	0.1288	0.1975	0.2725	na	na	na	3.2550
28	0.0200	0.0400	0.0850	0.1475	0.2175	0.3025	na	na	na	3.2175
27	0.0175	0.0388	0.0825	0.1438	0.2150	0.2975	na	na	na	3.2175
26	0.0138	0.0375	0.0800	0.1413	0.2125	0.2950	na	na	na	3.2200
25	0.0138	0.0400	0.0825	0.1413	0.2200	0.3075	na	na	na	3.2050
24	na	0.0275	0.0575	0.1100	0.1750	0.2525	0.3425	na	na	3.2650
23	na	0.0275	0.0600	0.1100	0.1775	0.2675	0.3475	na	na	3.2600
22	0.0125	0.0213	0.0700	0.1275	0.1975	0.2775	na	na	na	3.2400
21	0.0125	0.0325	0.0675	0.1300	0.2025	0.2875	na	na	na	3.2300
20	na	0.0250	0.0550	0.1075	0.1725	0.2550	0.3425	na	na	3.2675
19	na	na	0.0400	0.0788	0.1175	0.1825	0.2600	0.3500	na	3.3675
18	na	0.0225	0.0525	0.0950	0.1525	0.2300	0.3150	na	na	3.2975
17	na	0.0200	0.0500	0.0875	0.1450	0.2200	0.3050	na	na	3.3075
16	0.0125	0.0325	0.0700	0.1375	0.2125	0.2975	na	na	na	3.2175
15	0.0150	0.0388	0.0800	0.1575	0.2350	0.3275	na	na	na	3.1800
14	na	0.0200	0.0475	0.0950	0.1600	0.2400	0.3350	na	na	3.2750
13	na	0.0150	0.0375	0.0850	0.1475	0.2275	0.3175	na	na	3.2900
12	na	0.0113	0.0300	0.0725	0.1325	0.2125	0.3000	na	na	3.3100
11	na	0.0100	0.0300	0.0725	0.1325	0.2125	0.3000	na	na	3.3075
10	na	0.0088	0.0263	0.0638	0.1200	0.2000	0.2900	na	na	3.3175
9	na	na	0.0163	0.0463	0.0963	0.1700	0.2575	0.3500	na	3.3575
8	na	na	0.0100	0.0300	0.0750	0.1425	0.2200	0.3150	na	3.3900
7	na	na	0.0075	0.0350	0.0850	0.1575	0.2463	0.3450	na	3.3575
6	na	na	0.0050	0.0213	0.0575	0.1300	0.2175	0.3125	na	3.3875
5	na	na	0.0038	0.0088	0.0388	0.1050	0.1925	0.2900	na	3.4125
4	na	na	0.0025	0.0113	0.0550	0.1338	0.2263	0.3250	na	3.3750
3	na	na	0.0013	0.0050	0.0350	0.1050	0.1988	0.2950	na	3.4050
2	na	na	0.0013	0.0075	0.0600	0.1500	0.2475	0.3475	na	3.3525
1	na	na	0.0013	0.0013	0.0350	0.1350	0.2350	0.3300	na	3.3525

na = not available.

Table 3. March 28, 1994 to June 17, 1994 Chicago Wheat Futures Option Premia (Call options).

Time to Maturity	Strike Prices (dollar/bushel) 3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	Futures Prices
57	na	0.2325	0.1650	0.1150	0.0725	0.0500	0.0325	na	na	3.2950
56	na	0.2275	0.1600	0.1075	0.0700	0.0475	0.0300	na	na	3.2875
55	na	0.2225	0.1575	0.1063	0.0688	0.0450	0.0275	na	na	3.2825
54	na	0.1850	0.1238	0.0850	0.0550	0.0375	0.0225	na	na	3.2325
53	na	na	0.1475	0.0988	0.0650	0.0450	0.0275	0.0175	na	3.2700
52	na	na	0.1650	0.1175	0.0750	0.0525	0.0350	0.0250	na	3.2975
51	na	na	na	0.1550	0.1000	0.0700	0.0475	0.0325	0.0225	3.3600
50	na	na	na	0.1550	0.1013	0.0725	0.0500	0.0350	0.0250	3.3550
49	na	na	0.1900	0.1350	0.0863	0.0600	0.0425	0.0300	na	3.3300
48	na	0.2000	0.1400	0.0950	0.0588	0.0375	0.0250	na	na	3.2575
47	na	0.2025	0.1400	0.0950	0.0575	0.0375	0.0250	na	na	3.2600
46	na	0.1875	0.1275	0.0850	0.0500	0.0350	0.0225	na	na	3.2400
45	na	0.1375	0.0900	0.0588	0.0350	0.0225	0.0138	na	na	3.1675
44	na	0.1300	0.0850	0.0550	0.0300	0.0200	0.0125	na	na	3.1575
43	na	0.1500	0.0950	0.0638	0.0350	0.0225	0.0138	na	na	3.1875
42	0.1800	0.1175	0.0725	0.0413	0.0225	0.0125	na	na	na	3.1425
41	0.1850	0.1200	0.0750	0.0438	0.0250	0.0150	na	na	na	3.1525
40	0.1775	0.1150	0.0700	0.0400	0.0225	0.0125	na	na	na	3.1425
39	0.1750	0.1100	0.0638	0.0375	0.0213	0.0100	na	na	na	3.1400
38	0.2000	0.1300	0.0800	0.0475	0.0263	0.0138	na	na	na	3.1750
37	0.2125	0.1425	0.0875	0.0525	0.0300	0.0163	na	na	na	3.1900
36	0.2600	0.1800	0.1175	0.0750	0.0450	0.0238	na	na	na	3.2450
35	na	0.2300	0.1600	0.1075	0.0650	0.0400	0.0250	na	na	3.3075
34	na	0.2650	0.1900	0.1338	0.0888	0.0550	0.0338	na	na	3.3475
33	na	0.2300	0.1600	0.1075	0.0688	0.0425	0.0250	na	na	3.3000
32	na	na	0.1938	0.1350	0.0875	0.0550	0.0325	0.0250	na	3.3475
31	na	0.2300	0.1600	0.1100	0.0663	0.0425	0.0275	na	na	3.3025
30	na	0.2050	0.1400	0.0900	0.0550	0.0350	0.0225	na	na	3.2700
29	0.2700	0.1925	0.1225	0.0813	0.0500	0.0313	na	na	na	3.2550
28	0.2350	0.1575	0.1013	0.0625	0.0350	0.0225	na	na	na	3.2175
27	0.2325	0.1525	0.1000	0.0613	0.0325	0.0200	na	na	na	3.2175
26	0.2350	0.1550	0.1000	0.0600	0.0325	0.0175	na	na	na	3.2200
25	0.2200	0.1425	0.0850	0.0500	0.0250	0.0125	na	na	na	3.2050
24	na	0.1900	0.1200	0.0725	0.0388	0.0200	0.0100	na	na	3.2650
23	na	0.1850	0.1113	0.0688	0.0375	0.0188	0.0100	na	na	3.2600
22	0.2475	0.1675	0.1075	0.0650	0.0313	0.0200	na	na	na	3.2400
21	0.2400	0.1513	0.0950	0.0575	0.0325	0.0200	na	na	na	3.2300

na = not available.

Table 3. Continued.

Time to Maturity	Strike Prices (dollar/bushel) 3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	Futures Prices
20	na	0.1925	0.1200	0.0750	0.0040	0.0023	0.0010	na	na	3.2675
19	na	na	0.2025	0.1425	0.0900	0.0525	0.0300	0.0213	na	3.3675
18	na	0.2200	0.1475	0.0925	0.0525	0.0300	0.0175	na	na	3.2975
17	na	0.2275	0.1550	0.0925	0.0525	0.0275	0.0150	na	na	3.3075
16	0.2275	0.1500	0.0863	0.0525	0.0250	0.0138	na	na	na	3.2175
15	0.1900	0.1150	0.0600	0.0338	0.0150	0.0075	na	na	na	3.1800
14	na	0.1950	0.1150	0.0675	0.0350	0.0163	0.0100	na	na	3.2750
13	na	0.2050	0.1250	0.0725	0.0375	0.0175	0.0100	na	na	3.2900
12	na	0.2200	0.1400	0.0825	0.0425	0.0225	0.0125	na	na	3.3100
11	na	0.3100	0.2175	0.0800	0.0400	0.0200	0.0100	na	na	3.3075
10	na	0.2250	0.1425	0.0800	0.0388	0.0188	0.0075	na	na	3.3175
9	na	na	0.1688	0.0975	0.0475	0.0275	0.0100	0.0063	na	3.3575
8	na	na	0.2000	0.1200	0.0625	0.0313	0.0125	0.0075	na	3.3900
7	na	na	0.1650	0.0925	0.0400	0.0150	0.0050	0.0038	na	3.3575
6	na	na	0.1925	0.1100	0.0463	0.0175	0.0050	0.0025	na	3.3875
5	na	na	0.2138	0.1225	0.0500	0.0163	0.0050	0.0025	na	3.4125
4	na	na	0.1775	0.0875	0.0300	0.0088	0.0025	0.0013	na	3.3750
3	na	na	0.2063	0.1100	0.0375	0.0100	0.0038	0.0025	na	3.4050
2	na	na	0.1525	0.0600	0.0125	0.0025	0.0013	0.0013	na	3.3525
1	na	na	0.1625	0.0650	0.0050	0.0013	0.0013	0.0013	na	3.3525

na = not available.

Table 4. Parameter Estimates of GARCH-t(1,1) Process of Chicago Wheat Futures prices

	Estimated Coefficients	p-value[a]		Estimated Coefficients	p-value
Mean:			Variance:		
Intercept	-0.024	(0.322)	Intercept	0.052	(0.315)
DMON	0.040	(0.299)	Alpha	0.079*	(0.000)
DTUE	0.062	(0.205)	Beta	0.876*	(0.000)
DWED	0.125*[b]	(0.037)	DMON	-0.021	(0.450)
DTHU	-0.091	(0.097)	DTUE	0.142	(0.155)
			DWED	0.183	(0.109)
			DTHU	-0.243	(0.091)
			SIN252	0.013	(0.269)
			COS252	-0.014	(0.097)
			SIN126	-0.005	(0.323)
			COS126	0.002	(0.410)
			MATURITY	-0.011	(0.337)
Degrees of Freedom:					
df	7.505	(0.000)*			
Wald F statistics:					
Day of the week in mean	2.753*				
Day of the week in Variance	2.452*				
Seasonality in Variance	0.762				

[a] Numbers in parentheses are probability values. Hence a p-value < 0.05 indicates that the parameter estimated is significant.
[b] Asterisks indicate significance at the 5% level.

Table 5. Out of Sample Root Mean Squared Forecast Errors[a] of BLACK and GARCH-t Option Pricing for 1994 Chicago Wheat Put Options Root Mean Squared Errors[a].

Option Models	Strike Prices (dollar per bushel)								
	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80
BLACK	0.018	0.024	0.029	0.035	0.034	0.034	0.036	0.044	0.066*
GARCH-t	0.014[b]*	0.017*	0.023*	0.027*	0.024*	0.024*	0.028*	0.037*	0.069

[a] Root mean squared errors are in dollar per bushel.
[b] Asterisks indicate smallest root mean squared errors.

Table 6. Out of Sample Root Mean Squared Forecast Errors[a] of BLACK and GARCH-t Option Pricing for 1994 Chicago Wheat Call Options Root Mean Squared Errors[a].

Option Models	Strike Prices (dollar per bushel)								
	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80
BLACK	0.048[b]*	0.057*	0.064*	0.058*	0.047*	0.035*	0.027	0.025	0.024
GARCH-t	0.055	0.076	0.083	0.082	0.057	0.038	0.027	0.021*	0.016*

[a] Root mean squared errors are in dollar per bushel.
[b] Asterisks indicate smallest root mean squared errors.