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The Effect of Planting on the Volitility of Grain Futures Prices

David A. Hennessy and Thomas I. Wahl*

Abstract: Existing literature on commodity futures price volatility emphasizes time to expiration and the resolution of uncertainty. This paper stresses the supply and demand inflexibilities arising from decision making. A decision made on the supply (demand) side makes future supply (demand) responses less elastic. Therefore, a shock arising after a decision is made is more effective in changing the futures price than a shock before the decision is made. The results support the maturity hypothesis but do not conflict with the state variable hypothesis of futures price volatility.

Introduction

The existing literature on the determination of futures price volatility does not accommodate the possibility of production or consumption responses. In this paper we will show that if a futures contract is of sufficiently long duration that producers may receive and act upon evolving information, a pattern in futures price volatilities emerges as maturity approaches. This pattern is consistent with observations on the seasonality of volatilities and with the maturity hypothesis. However, unlike the present explanation of the seasonality phenomenon, the state variable hypothesis, seasonality is shown to arise not from the resolution of uncertainty but rather from increasingly constrained supply and demand functions as settlement date approaches. Contrary to the state variable hypothesis, we show that the resolution of supply and demand uncertainty may increase rather than decrease volatility. We demonstrate our theory by using a rational expectations model that incorporates production flexibility.

Since Telser suggested that futures price volatility may increase as the settlement day approaches, there has been a vigorous theoretical and empirical debate on the issue. Samuelson (1965) used an autoregressive price relationship to demonstrate the plausibility of Telser's conjecture. Also, commencing with his 1965 paper, Samuelson (1971, 1973, 1976) wrote a series of articles with the goal of reconciling the randomness of the price of claims on assets inferred by rational expectations considerations with the economic implausibility of the seemingly associated unbounded variation in prices. He proposed that, at the limit, as time to maturity increases, the settlement price becomes ergodic in distribution, i.e., independent of the initial futures price. For the futures price martingale property (Samuelson 1965) to be consistent with the ergodicity property, it is necessary that, as time to maturity increases, the volatility of futures price falls, eventually, to zero. Because, in its weakest form, Samuelson's proposition holds that volatility must only eventually fall with time to maturity, empirical investigations can never be absolutely conclusive. Rutledge tested and found only partial support for the hypothesis that futures price volatility is monotonic decreasing in time to maturity. An alternative hypothesis, the state variable hypothesis (Stein, Richard and

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Sundaresan), exists. The most persuasive version is that proposed by Anderson and Danthine in which volatility is high at times when much uncertainty about fundamentals is resolved and low when little uncertainty is resolved (Anderson). The two theories are not necessarily incompatible in that the ergodicity assumption can be interpreted as the restriction that as time to maturity increases, knowledge about fundamentals becomes impossible and so no uncertainty can be resolved.

Anderson suggested that the heterogeneities are seasonal in nature. He tested time series data for wheat (two exchanges), corn, oats, soybeans, soybean oil, live cattle, silver, and cocoa to find strong support for seasonal effects even for the nonagricultural commodities. He found weaker, but still statistically significant, support for the maturity effect. Choi and Longstaff also identified a seasonality structure in the volatility of soybean futures prices. Kenyon et al. identified a similar seasonality structure for corn and soybeans, while Milonas arrived at similar results in his study of the decomposition of the futures price heteroskedasticity. He ascribed the source of seasonality to the resolution of supply and demand uncertainty (as per Anderson and Danthine) and to the depletion of inventories as harvest approaches. In all these analyses grain futures price volatility tended to be high in the late spring and summer and low in the late fall and winter.

In this paper we propose a model that emphasizes the importance of production or demand flexibility. This factor has not been considered in the literature, though presumably Samuelson would include it in his "ultimate economic law" (1976, p. 120) that gave rise to his adherence to the ergodicity assumption. Our model is compatible with both the maturity and the state variable hypotheses and may explain more satisfactorily the observed patterns in the volatility data. The main body of this is paper comprised of two sections. In the first the theoretical model is developed for both discrete and continuous time flexibility. In the second section the model's implications are tested.

The Model

Following the approach of Black, we assume that the futures price follows geometric Brownian motion,

$$\frac{dF}{F} = \alpha_F dt + \sigma_F dz_F, \tag{1}$$

where F = futures price,
α_F = drift parameter,
σ_F = diffusion parameter, and
z_F = standard Wiener process.

Within this continuous time framework we partition time into n intervals; the interval $[\tau(1), T]$ when no production response is possible, $[\tau(2), \tau(1))$ where one factor can be altered to change production, and so on to $[0, \tau(n))$ where n factors can be altered to change production. We propose the Cobb-Douglas production function

$$Q_{s,T} = k_s \prod_{i=1}^{n} J_{\tau(i)}^{f(i)}, \tag{2}$$

where $Q_{s,T}$ = output quantity,
$J_{\tau(i)}$ = level of input i chosen at time $\tau(i)$,
k_s = production constant of proportionality, and
f(i) = flexibility = elasticity of output response to input I.

We assume that futures price and physicals price at maturity are identical,

(3) $$F_T = P_T,$$

where F_T = futures price at settlement time T, and
P_T = physicals price at settlement time T.

The demand equation is considered to be iso-elastic,

(4) $$Q_{d,T} = k_d \phi_T F_T^{-\epsilon},$$

where $Q_{d,T}$ = output demand at harvest time,
ϕ_T = stochastic demand shock evaluated at harvest,
k_d = demand constant of proportionality, and
ϵ = bsolute elasticity of demand.

Here ϕ_t evolves according to geometric Brownian motion,

(5) $$\frac{d\phi}{\phi} = \alpha_\phi dt + \sigma_\phi dz_\phi,$$

where α_ϕ = drift parameter,
σ_ϕ = diffusion parameter, and
z_ϕ = standard normal Wiener process.

Because of the iso-elasticity, this specification of how demand evolves is consistent with the futures price following geometric Brownian motion. Let the acres planted decision be the production decision closest to harvest. Now, given the acreage decision (i.e. over $[\tau(1), T]$), the sole determinant of the futures price is ϕ_T. Information on its eventual value is obtained by observing ϕ_t evolve. We will first solve the problem over $[\tau(1), T]$. Here J at time $\tau(1)$, $J_{\tau(1)}$, has been chosen. Supply is fixed at

(6) $$Q_{s,T} = k_{sr} J_{\tau(1)}^{f(1)},$$

where k_{sr} is a short-run constant that may depend on long-run factor choices. The futures price, F_t, depends only on ϕ_t,

(7) $$F_t = F(\phi_t).$$

An Ito expansion gives

(8) $$\frac{\partial F}{\partial t} + 0.5\,\phi_t^2 \sigma_\phi^2 \frac{\partial^2 F}{\partial \phi^2} = 0.$$

This equation must satisfy the boundary condition

$$F_T = \left(\frac{Q_{d,T}}{k_d \phi_T} \right)^{-1/\epsilon} \tag{9}$$

arising from equation (4). But by market clearance, and assuming no storage, supply equals demand. Therefore,

$$F_T = \left(\frac{k_s J_{\tau(1)}^{f(1)}}{k_d \phi_T} \right)^{-1/\epsilon} . \tag{10}$$

Solving (8) subject to (10) and the boundary condition that if the futures price ever becomes zero then it remains at zero thereafter, we get

$$F_t = \left(\frac{k_d}{k_s} \right)^{1/\epsilon} \phi_t^{1/\epsilon} J_{\tau(1)}^{-f(1)/\epsilon} e^{\frac{0.5(1-\epsilon)\sigma_\phi^2 (T-t)}{\epsilon^2}} . \tag{11}$$

This equation encapsulates the dynamic relationship between F_t and ϕ_t over the interval $[\tau(1), T]$. It can be shown that

$$\sigma_\phi^2 = \epsilon^2 \sigma_F^2. \tag{12}$$

To see this, log both sides of (10) and then take the variance of both sides. Thus, the volatility of futures price is constant over $[\tau(1), T]$. The acres planted decision, represented by $J_{\tau(1)}$, only contributes to determining the level of the futures price in the interval. We now step back into the interval $[\tau(2), \tau(1))$. In this case because the input choice at $\tau(1)$, $J_{\tau(1)}$, has yet to be made, we must solve an equilibrium model endogenizing the intervening input decision. As shown by Feder et al. in the absence of production uncertainty the production choice will be given by equating the (discounted) futures price with marginal cost,

$$F_{\tau(1)} e^{-r[T-\tau(1)]} = MC, \tag{13}$$

where MC = marginal cost, and
r = the discount rate.

This is because, in the presence of futures markets, speculation through altering the futures contracts position does not incur the production costs associated with speculation through altering the production position. To go further, we maximize over the production choice $J_{\tau(1)}$. Prior to $\tau(1)$, output is no longer exogenous because the planting decision has yet to be made. To endogenize we look at the discounted profit function,

$$PV[\pi] = F_{\tau(1)} e^{-r[T-\tau(1)]} k_s J_{\tau(1)}^{f(1)} - w J_{\tau(1)}, \tag{14}$$

where PV = present value operator,
π = profit, and
w = input price index.

Optimizing over $J_{\tau(1)}$, we find

$$J_{\tau(1)} = \left(\frac{f(1) k_s F_{\tau(1)} e^{-r[T-\tau(1)]}}{w} \right)^{\frac{1}{1-f(1)}}. \tag{15}$$

Substituting into (11), evaluated at $\tau(1)$, and solving for $F_{\tau(1)}$, we find

$$\begin{aligned} F_{\tau(1)} &= \left(\frac{k_d (w/f(1))^{\frac{f(1)}{1-f(1)}}}{k_s^{\frac{1}{1-f(1)}}} \right)^{\gamma} \phi_{\tau(1)}^{\gamma} e^{\left(\frac{0.5(1-\epsilon)\sigma_\phi^2}{\epsilon} + \frac{rf(1)}{1-f(1)} \right)[T-\tau(1)]} \\ &= B \phi_{\tau(1)}^{\gamma} e^{D[T-\tau(1)]}, \end{aligned} \tag{16}$$

where $\gamma = [1-f(1)]/[f(1)+\epsilon-f(1)\epsilon]$, and B and D are the obvious substitutions. Thus, we see that while F_t has a diffusion described by volatility σ_ϕ^2/ϵ^2 in the time interval $[\tau(1), T]$, the diffusion is described by volatility $\gamma^2 \sigma_\phi^2$ in the interval $[\tau(2), \tau(1))$. This is because no decisions are made in the interior of this interval. Note that $\gamma = 1/\epsilon$ when $f(1) = 0$; that is when there is no production flexibility in the $[\tau(2), \tau(1))$ time interval either. Differentiating γ with respect to f(1), we find

$$\frac{d\gamma}{df(1)} = -\frac{1}{[f(1)+\epsilon-f(1)\epsilon]^2} < 0. \tag{17}$$

Thus, because $f(1) > 0$, the inequality $\gamma < 1/\epsilon$ must always hold. This verifies, for these functional forms, the conjecture that volatility after the planting date exceeds that before the planting date. The underlying reason is that long-run elasticities always exceed short-run elasticities, and price variability is inversely related to the ability to respond, ex-ante, to available information.

We now introduce the concept of residual cumulative volatility. This is the instantaneous volatility integrated with respect to time remaining until maturity. In Black's model it is homogeneous linear in time, $\sigma_F^2 (T - t)$. In this model it is continuous, homogeneous, piecewise linear in time,

$$\sigma_{F,L}^2 (T-t) = \left(\frac{1}{\epsilon^2} + (\gamma^2 - \frac{1}{\epsilon^2}) L_{\tau(1)} \right) \sigma_\phi^2 (T-t), \tag{18}$$

where $L_{\tau(1)} = 1$ when $t < \tau(1)$, and zero otherwise. Merton (1973) has shown that the Black and Scholes model remains valid if the volatility is a function only of time. This is true also

of Black's options on futures model. After planting, while the rate of diffusion increases, the amount of cumulative diffusion that has occurred is continuous. Substituting into Black's call valuation model, we arrive at

$$C(F,E,T-t) = e^{-r(T-t)}[F_t N(d_1) - EN(d_2)],$$

(19)

$$d_1 = \frac{Ln(\frac{F_t}{E}) + 0.5\sigma_{FL}^2(T-t)}{\sigma_{FL}(T-t)^{0.5}},$$

$$d_2 = d_1 - \sigma_{FL}(T-t)^{0.5},$$

where N(.) is the cumulative normal distribution function and E is the strike price.

Introduction of inventories makes the problem dynamic in the inter-year sense. Samuelson (1971) showed the problem to be one requiring a dynamic programming solution. The ability to substitute intertemporally will reduce the volatilities so if the commodity (or product if the problem is posed as concerning a share in an industry adjusting to demand dynamics) is non-perishable, then equation (19) represents an upper bound on the value of the option. The impact of flexibility in determining the magnitudes of volatilities should not be affected much by the existence of inventories.

We now consider how a sequence of ex-ante supply decisions affects the volatilities. Let the last round of supply decisions involve variable costs, the second last round involve capital decisions, the third last involve region-wide applied research and so on. Let the technology be as described in equation (2) above. Returning to the expression $\gamma = [1-f(1)]/[f(1)+\epsilon-f(1)\epsilon]$ and noting that in the final time interval $\sigma_F = \sigma_\phi/\epsilon$, we can iteratively continue the mapping to find
futures price volatilities at earlier stages of production. For example if there are countably infinite stages of production and if $f = f(1) = \ldots = f(i)\ \forall\ i$, then

(20)

$$\sigma_{F,n}^2 = \frac{(1-f)^2\sigma_\phi^2}{(nf + \epsilon - f\epsilon)^2},$$

where $\sigma_{F,n}^2$ = futures volatility just prior to the n^{th} last production decision. At the limit, as n goes to infinity, volatility recedes to zero as suggested by Samuelson's ergodicity postulate. In general, for the f(i) strictly positive but not all equal, it is easy to show that $\sigma_{F,n}^2$ is monotonic increasing in n. To show that there exists a sequence of decreasing flexibilities such that volatility recedes to a strictly positive number is a trivial exercise in real analysis. To discern whether a given technology structure (i.e., a given sequence {.., f(i),.. }) is compatible with ergodicity is a problem in the theory of sequences and series. No sequence, the infinite sum of which is finite, is compatible.

If all the f(i) are equal to a constant, f, and if adjustment is assumed to occur not just at one or countably many time points, but continuously, then f may be considered the constant elasticity of adjustment, and

$$\sigma_{F,T-t} = \frac{(1 - f)\sigma_\phi}{f(T - t) + \epsilon - f\epsilon}. \tag{21}$$

In this case the residual cumulative volatility, obtained by integrating with respect to time, is

$$\int_0^T \sigma^2_{F,T-t}\, dt = \frac{(1 - f)^2 \sigma^2_\phi}{f}\left[\frac{1}{(1 - f)\xi} - \frac{1}{fT + (1 - f)\epsilon}\right]. \tag{22}$$

At the limit, as T goes to infinity, this is

$$\int_0^\infty \sigma^2_{F,\infty-t}\, dt = \frac{(1 - f)\sigma^2_\phi}{f\epsilon}, \tag{23}$$

i.e. residual cumulative volatility is decreasing in demand elasticity and in f, but increasing in variability of the demand shock.

Agricultural futures market uncertainty is frequently caused by post-planting weather uncertainty. The reasoning applied to the demand side above applies equally well to the supply side. However, as there would then be no one-to-one correspondence between the input decision and output, applying production rule (13) to obtain a closed form solution would be problematic. Closed form solutions can be obtained if additional assumptions are made.[1]

Testing the Flexibility Hypothesis

An obvious economic instance of reduced flexibility occurs when farmers make the crop acreage allocation decision. For the principal crops in the US, planting must occur in a fairly well defined time interval. After planting, the producer has very limited influence over the ultimate crop size. Given the high liquidity of US corn, soybean, and wheat futures markets, a comparison of pre-planting and post-planting volatilities would seem an ideal test of the flexibility hypothesis.

To test the flexibility hypothesis, we analyzed daily U.S. corn, soybean and wheat futures prices over the January 2, 1985, to March 1, 1994, period (table 1). The first contract following harvest was used, and the series rolled over at expiration to the corresponding contract for the following year. Indicator variables were used to represent planting decisions. However, the fall planting decision for winter wheat could not be analyzed because winter wheat generally is seeded more than a year prior to contract expiration, and futures price data prior to planting was not consistently available. Winter wheat suffers from winterkill the extent of which is not known with certainty until spring replanting. Hence, an April-to-

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[1]For example, if individual output bears a log-log relationship with aggregate output, then quantity uncertainty can be symbolically mapped into price uncertainty. In this case the solution is virtually identical to the demand side uncertainty solution presented above. The proof is available from the authors upon request.

September indicator variable is used for both winter and spring wheat. May-to-December and May-to-November indicator variables are used for corn and soybeans, respectively, to reflect later planting dates.

For a given month the annualized volatility is calculated as

$$\sigma^2_{i,j} = 365\ \mathrm{Var}\left[\mathrm{Log}(F_{i,j,t}/F_{i,j,t-1})\right], \tag{24}$$

where $F_{i,t,j}$ is the futures price for contract j in day t of month i. To synchronize with futures contract expiration dates, months are shifted to end on the eighth last business day. Using these five series, a seemingly unrelated regression system was estimated using the specification

$$\sigma^2_{i,j} = \alpha_0 + \alpha_1 D + \xi_{i,j}, \tag{25}$$

where D is one between spring planting and contract expiration and zero otherwise, and $\xi_{i,j}$ is the stochastic error term. The results are presented in table 2 below. The t statistics are in brackets. As can be seen, the volatilities all increase after spring planting. Surprisingly, the winter wheat volatilities are just as responsive to spring planting as the spring crop volatilities. A positive but weaker response had been expected because inflexibility is built into production at fall planting. However, it appears that the replanting effects are strong for winter wheat. Figures 1 and 2 depict the results. In figure 1 corn and soybean volatilities rise just after planting, fall in mid-season, and rise again toward harvest. The residuals to our specification may be explained by the Anderson and Danthine state variable hypothesis. In figure 2 (the wheats) volatilities rise after spring planting, spike in June, and increase again towards expiration. Again, the volatility might be explained by the state variable hypothesis.

Summary and Conclusions

The intent of this paper is to reconcile the two main theories of the time pattern of futures price volatilities. While our theory agrees that a pattern that could be termed seasonal can arise, it tends in general to support a modified version of Samuelson's maturity hypothesis. While we do not deny the possibility of heterogeneous volatility arising out of the resolution of supply and demand uncertainty, our model emphasizes the flexibility, or quasi-fixity, of supply and demand. Even though our model has been structured so that uncertainty is resolved uniformly over time, volatility is not uniform over time. Indeed, if one considers the resource allocation decision to be a resolution of uncertainty, volatility is seen to rise upon resolution of supply uncertainty rather than fall as suggested by the state variable hypothesis. Were the source of uncertainty on the supply side rather than the demand side, the same qualitative result would pertain. For example, flour millers would fix ex-ante the amount of a less variable factor they wish to employ over the coming season and any subsequent, unforeseen supply and demand shocks will meet against a less elastic demand curve. Further, volatility is seen to be a decreasing step function with respect to time to maturity. Aggregating over different decision dates, we arrive at a monotonic decreasing function, in accord with Samuelson's maturity hypothesis. Our results complement the conclusions of Anderson and Danthine. Whereas they look at a situation where all production and demand decisions except the market clearing "decision" have been made, this paper considers the effect of decision-making on volatilities. While our approach explains the trend component, their approach explains the non-trend component.

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Table 1: Description of Data and Variables

Crop	Indicator Variable	Contract Expiration
Corn, Chicago Board of Trade	1: May-December 0: Otherwise	December
Soybeans, Chicago Board of Trade	1: May-November 0: Otherwise	November
Hard Red Winter Wheat, Chicago Board of Trade	1: April-September 0: Otherwise	September
Soft Red Winter Wheat, Kansas City	1: April-September 0: Otherwise	September
Hard Red Spring Wheat, Minneapolis	1: April-September 0: Otherwise	September

Table 2: System Regression Results

Contract	α_0	α_1
Corn, Chicago Board of Trade	0.0288 (2.39)	0.0584 (3.90)
Soybeans, Chicago Board of Trade	0.0274 (2.52)	0.0522 (3.63)
Hard Red Winter Wheat, Chicago Board of Trade	0.0270 (2.69)	0.0580 (4.06)
Soft Red Winter Wheat, Kansas City	0.0230 (2.31)	0.0533 (3.75)
Hard Red Spring Wheat, Minneapolis	0.0172 (1.71)	0.0584 (4.11)

$F_{(10,540)} = 8.12$ Adjusted $R^2 = 0.116$

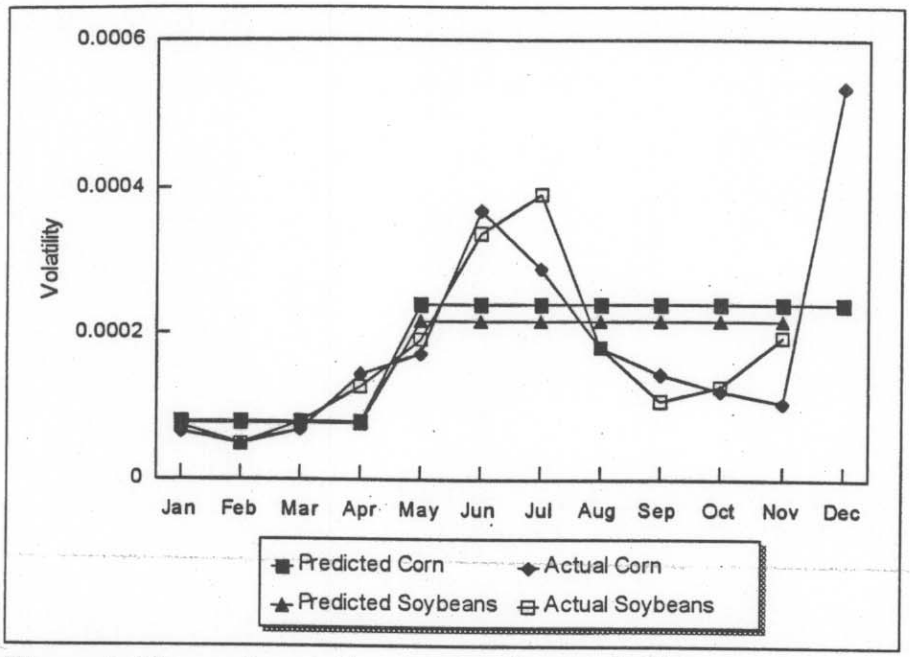

Figure 1. Time patterns of corn and soybean volatilities.

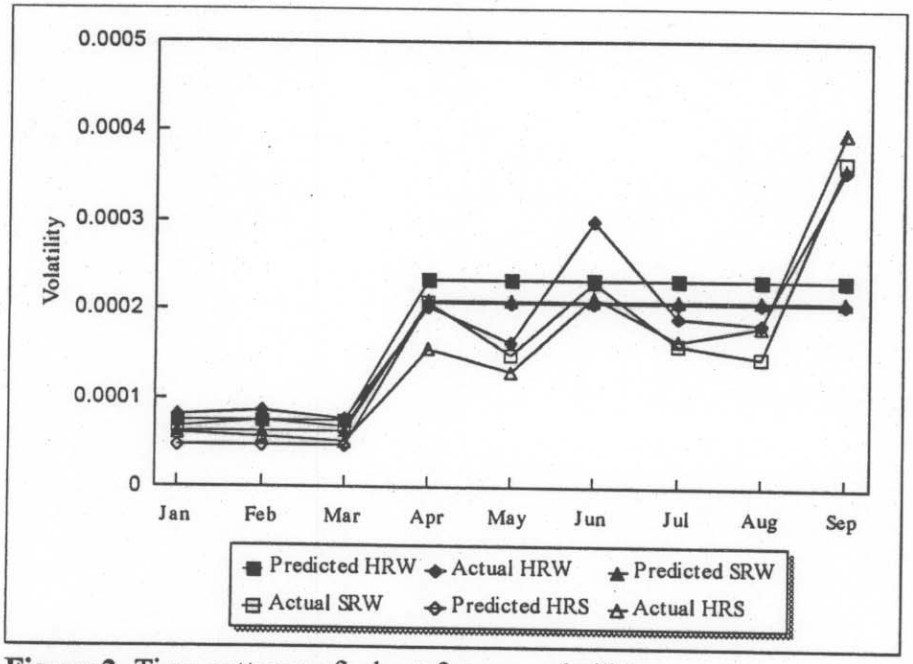

Figure 2. Time patterns of wheat futures volatilities.