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by

Xiongwen Rui and Mario J. Miranda

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This paper continues Deaton and Laroque's search for a variant of the nonlinear rational expectations commodity storage model that can explain the observed behavior of primary commodity prices. Using numerical functional equation methods and Monte Carlo simulation techniques, we demonstrate that the failure of Deaton and Laroque's model to explain high autocorrelation in primary commodity prices is attributable to the assumption of a constant returns to storage technology. Our findings suggest that a classical cost of storage function with "convenience yield" can give a satisfactory explanation for the high autocorrelation.

Introduction

Primary commodity prices historically have been characterized by significant volatility, serial correlation, and positive skewness. This can be seen in table 1, which summarizes the behavior of world prices for thirteen primary commodities between 1900 and 1987. Across the thirteen commodity price series, coefficients of variation averaged 0.37; first- and second-order autocorrelation coefficients averaged 0.80 and 0.61; and coefficients of skewness averaged 0.97. A reasonable model of primary commodity price formation should be able to explain all of these stylized facts.

In recent years, the most widely analyzed structural model of price formation in markets for primary commodities has been the nonlinear rational expectations commodity storage model. The nonlinear rational expectations storage model traces its origins to the "supply of storage" theory developed by Working, Kaldor, and Williams. Supply of storage theory maintains that commodity stocks are held up to the point at which the expected appreciation in the commodity price equals the marginal cost of storage. The nonlinear rational expectations supply of storage model has been used extensively to analyze the stabilizing effects of stockholding (Williams and Wright; Miranda and Helmberger). Empirical tests of the model, however, have been limited (Miranda and Glauber).

In recent work, Deaton and Laroque (1992, 1994) have put the nonlinear rational expectations commodity storage model to a series of empirical tests. Using generalized method of moments and pseudo maximum likelihood estimation methods, Deaton and Laroque examined the model's ability to explain the behavior of thirteen commodity prices between 1900 and 1987 (see table 1). They found that their variant of the nonlinear rational expectations storage model could explain the skewness and variability of commodity prices. Their model, however, could not explain high autocorrelation in commodity prices.

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TABLE 1. World Commodity Prices Facts 1900-1987, 1980 Dollars

	First-order a-c	Second-order a-c	Coeff. of variation	Skewness γ_1
bananas	0.91	0.82	0.17	0.15
cocoa	0.83	0.66	0.54	0.99
coffee	0.80	0.62	0.45	1.66
copper	0.84	0.64	0.38	1.02
cotton	0.88	0.68	0.35	0.35
jute	0.71	0.45	0.33	0.61
maize	0.76	0.53	0.38	1.18
palm oil	0.73	0.48	0.48	3.24
rice	0.83	0.61	0.36	0.55
sugar	0.62	0.39	0.60	1.49
tea	0.78	0.59	0.26	0.04
tin	0.90	0.76	0.18	0.42
wheat	0.86	0.68	0.38	0.87
Average	0.80	0.61	0.37	0.97

Note: The skewness measure γ_1 is $m_3/(m_2)^{1.5}$, m_g is the g-th central moment.
Sources: Deaton and Laroque.

In this paper, we take a fresh look at Deaton and Laroque's findings and continue their search for a variant of the nonlinear rational expectations commodity storage model that can explain the historical behavior of primary commodity prices. Our analysis demonstrates that Deaton and Laroque's negative findings are attributable to their specific assumption regarding the cost of storage relationship. Deaton and Laroque assume a "constant returns to storage" technology, which effectively implies that the marginal cost of storage is decreasing over positive stock levels. This formulation, however, differs from the classical supply of storage theory, which posits an increasing marginal cost of storage. We find that by replacing Deaton and Laroque's cost of storage function with one that conforms to the classical theory, the nonlinear rational expectations commodity storage model is able to capture the high autocorrelations in prices that Deaton and Laroque found so illusive.

Our paper is arranged as follows. We first discuss the modern nonlinear rational expectations commodity storage model under two alternative cost of storage formulations: Deaton and Laroque's constant return to storage formulation and the classical convenience yield formulation. In the second section, we discuss how numerical functional equation solution methods can be used to solve the nonlinear rational expectations commodity storage model accurately and efficiently. In the third section, we demonstrate how Deaton and Laroque's negative findings can be reversed by replacing their constant returns cost of storage function with a classical convenience yield cost of storage function. We conclude the paper with discussion of model extensions and future plans to estimate the convenience yield storage model from historical data using full information maximum likelihood techniques.

Two Storage Models

The centerpiece of the modern theory of storage is the competitive intertemporal arbitrage equation:

$$(1) \qquad \frac{1}{1+r} E_t p_{t+1} - p_t = c_t$$

The intertemporal arbitrage equation asserts that, in equilibrium, expected appreciation in the commodity price p_t, discounted at the interest rate r, must equal the unit cost of storage c_t. Dynamic equilibrium in the commodity market is enforced by competitive expected-profit-maximizing storers. Whenever expected appreciation exceeds the unit cost of storage, the attendant profits induce storers to increase their stockholdings until the equilibrium is restored. Conversely, whenever the cost of storage exceeds expected appreciation, the attendant loses induce storers to decrease their stockholdings until the equilibrium is restored. Intertemporal arbitrage through speculative stockholding links price across time, inducing serial dependence in prices even when production and consumption are serially independent.

The modern storage model is completed through the addition of supply and demand functions, a cost of storage function, market clearing conditions, and a theory of how price expectations are formed. Denote available supply at the beginning of period t by s_t, quantity consumed in period t by q_t, stocks at the end of period t by x_t, and new production or harvest in period t by y_t. In this paper, as in Deaton and Laroque's original papers, we will work with only the simplest version of the modern storage model. Specifically, we assume:

- available supply is the sum of initial stocks and new production:

  $$(2) \qquad s_t = x_{t-1} + y_t$$

- available supply is either consumed or stored:

  $$(3) \qquad s_t = q_t + x_t$$

- the market clearing price is a decreasing function $p(\cdot)$ of the quantity consumed:

  $$(4) \qquad p_t = p(q_t)$$

- and the unit cost of storage is a function $c(\cdot)$ of the quantity stored:

  $$(5) \qquad c_t = c(x_t)$$

Finally, we assume that harvests y_t are exogenous, stochastic, and independently and identically distributed over time, and that expectations are formed rationally in the sense of Muth.

Variants of the modern storage model differ as to the cost of storage function assumed. Models that draw literally from the classical supply of storage literature assume that the unit cost of storage $c(\cdot)$ comprises a marginal physical cost of storage and a marginal "convenience yield". The marginal convenience yield represents the amount processors are willing to pay to avoid the cost of revising their production schedules and the "option value" of being in a position to take advantage of potential price increases. The classical cost of storage function is illustrated in figure 1. If stock levels are high, the marginal convenience yield is zero and the unit storage cost equals the physical storage cost. As stock levels approach zero, however, the marginal convenience yield rises, eventually resulting in a negative unit storage cost (Kaldor; Working; Williams). The classical cost of storage function has received strong empirical support for commodities with futures markets (Brennan).

In contrast to classical supply of storage theory, Deaton and Laroque posit a "constant returns to storage" technology. Constant returns to storage means that one unit of commodity stored in the current period will yield $(1-\delta)$ units at following period. In Deaton and Laroque's formulation, no costs of storage are incurred by the storer other than the loss of a constant

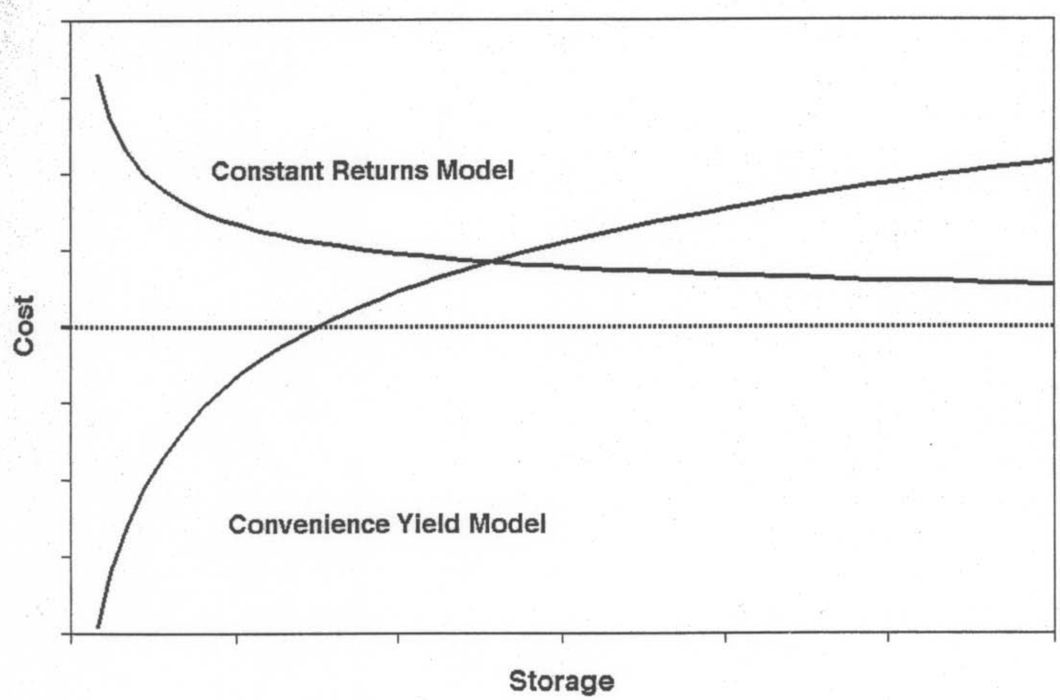

Figure 1. Unit Cost of Storage

proportion $\delta \in (0,1)$ of stocks between periods. This formulation effectively implies that the unit cost of storage is a constant multiple of the expected future price:

$$(6) \qquad c(x_t) = \frac{\delta}{1+r} E_t p_{t+1}$$

The expected price, however, falls with the level of stocks. Thus, under constant returns to storage, the unit cost of storage is a decreasing function of the stock level x_t. The constant returns to storage cost function is illustrated in figure 1. The main difference between this cost function and the classical convenience yield cost function is evident: the former is decreasing, the latter is increasing.

Solution Method

The commodity price p_t in the nonlinear rational expectations storage model (1)-(5) is a first-order Markovian stochastic process. The key to establishing this fact and deriving the Markov probability transition rule is to derive the equilibrium price function, $\lambda(\cdot)$, which gives the equilibrium price implied by the model for a given supply s. The equilibrium price function is characterized by a functional equation. The functional equation stipulates that for every realizable supply s

$$(7) \qquad \lambda(s) = p(s - x)$$

where x solves

$$(8) \qquad \frac{1}{1+r} E_y \lambda(x+y) - p(s-x) = c(x)$$

Although the equilibrium price function $\lambda(\cdot)$ can be shown to exist under mild regularity conditions (Scheinkman and Schectman), it generally does not possess a closed-form expression and cannot be derived using standard algebraic techniques. The function, however, can be approximated to any degree of accuracy using numerical functional equation methods. One particularly efficient computational technique is the Chebychev polynomial projection method. The method calls for the following discretization strategy: First, $\lambda(\cdot)$ is approximated using a finite linear combination of the Chebychev polynomials $\phi_0, \phi_1, \phi_2, \ldots, \phi_n$:

$$(9) \qquad \lambda(s) \approx \sum_{j=0}^{n} a_j \phi_j(s)$$

Second, to fix the n+1 coefficients of the polynomial approximant, (7)-(8) are asked to hold exactly, not over the entire interval of possible supply points, but rather only at the n+1 Chebychev nodes $s_0, s_1, s_2, \ldots, s_n$ of the interval. Third, Gaussian Quadrature principles are used to approximate the harvest distribution with an m-point discrete distribution that assumes values y_1, y_2, ..., y_m with probabilities w_1, w_2, ..., w_m, respectively. The Chebychev polynomial projection method is described and illustrated in Judd 1991, 1992 and in Miranda 1994.

The polynomial projection method replaces the original infinite-dimensional functional equation problem with a finite-dimensional nonlinear equation problem. Specifically, the approximation to $\lambda(\cdot)$ in (9) is derived by solving the 2n+2 equations

$$(10) \quad \sum_{j=0}^{n} a_j \phi_j(s_i) = p(s_i - x_i)$$

$$(11) \quad \delta \sum_{k=1}^{m} \sum_{j=0}^{n} w_k a_j \phi_j(x_i + y_k) - p(s_i - x_i) = c(x_i)$$

for the 2n+2 unknowns a_i, x_i, $i=0,1,2,\ldots,n$.

The equation system (10)-(11) can be solved using any nonlinear rootfinding technique, such as the Newton or Broyden methods (Judd 1991; Atkinson, Press et al.), or can be solved using the following successive approximation algorithm:

0. Initial Step: Select the degree of approximation n and guess the values of a_0, a_1, a_2, ..., a_n.

1. Solution Step: Holding the coefficients a_0, a_1, a_2, ..., a_n fixed, find, for each $i=0,1,\ldots,n$, the x_i that solves the nonlinear equation system (11).

2. Update Step: Holding the stock levels x_0, x_1, x_2, ..., x_n fixed, find the coefficients a'_0, a'_1, a'_2, ..., a'_n that solve the linear equation system (10).

3. Convergence Check: If $|a'_j - a_j| < \tau$ for all j and some convergence tolerance τ, set $a_j = a'_j$ for all j and stop; otherwise set $a_j = a'_j$ for all j and return to step 1.

Once an approximation for the equilibrium price function $\lambda(\cdot)$ is constructed, the Markov probability transition rule followed by equilibrium market prices can be described in terms of the exogenous driving process, namely the harvest process y_{t+1}. Given the price p_t in period t and the harvest y_{t+1} in period t+1, the price in period t+1 will be:

$$(12) \quad p_{t+1} = \lambda[\lambda^{-1}(p_t) - p^{-1}(p_t) + y_{t+1}]$$

Here, $\lambda^{-1}(p_t)$ equals initial supply s_t in period t, $p^{-1}(p_t)$ equals consumption q_t in period t, and their difference $s_t - q_t$ equals carryout x_t in period t; by construction, price in period t+1 equals $\lambda(s_{t+1})$ where $s_{t+1} = x_t + y_{t+1}$.

Similar methods can be used to derive the equilibrium price function for Deaton and Laroque's constant returns to storage model. Deaton and Laroque, however, used a finite element linear spline approximation technique; the interested reader is referred to their original paper. In the case of a constant returns to storage assumption, the Markov transition rule takes a slightly different form:

$$(13) \quad p_{t+1} = \lambda[(1-\delta)\{\lambda^{-1}(p_t) - p^{-1}(p_t)\} + y_{t+1}]$$

Note that the λ function depends on the storage cost specification and thus will differ between the convenience yield model and the constant returns to storage model.

Comparing Storage Models

Using Monte Carlo simulation methods, we compared the distributional properties of commodity prices under the two competing models of commodity price formation: Deaton and Laroque's constant return to storage model and the classical convenience yield model. In our analysis, both models share the same demand and harvest supply structures. The only difference lies in the cost of storage relationship that is assumed. The difference in the cost of storage relationships will guarantee distinct equilibrium price functions $\lambda(\cdot)$ and thus distinct equilibrium price paths as determined by (12) and (13).

Monte Carlo simulations were performed using the same model parameterizations as employed by Deaton and Laroque: two demand functions, linear and iso-elastic, and two harvest distributions, normal and log-normal, were simulated. The equilibrium price functions for the two models were derived first. We then took an initial stock level x_0 and generated a price series by randomly drawing a sequence of random harvests y_{t+1} using a pseudo-random generator, and calculating prices from the transition formulas (12) and (13).

Table 2 reports the simulated results from Deaton and Laroque model. In the first four cases, the demands are linear and harvests are normally distributed; in the last four, the demands are iso-elastic and strictly convex, and harvests are log-normally distributed. Notice how the constant storage depreciation rate δ is varied. The results in table 2 coincide with those reported by Deaton and Laroque. At low harvest shock volatility, there is little storage and storage does little to smooth the price series. The autocorrelations generated by storage is as low as 0.10 and skewness is also relatively low, though positive. At high harvest volatilities, carryover becomes more prominent and autocorrelation in price is more significant.

The most important feature of the results reported by Deaton and Laroque, which was confirmed in our simulations, is that there appears to be an upper limit to the autocorrelations that can be generated by the constant returns to storage model. The limit is far below the autocorrelations observed in actual price series. Even with a zero depreciation rate of storage and a very high harvest volatility, the constant returns to storage model could generate an autocorrelation no higher than 0.47.

As seen in table 3, the results obtained using the convenience yield model where sharply different in one key respect: the model can generate autocorrelations as high as 0.80, which is comparable to what is observed in practice. In other respects the convenience yield model is similar in implications to the constant returns to storage model: lower harvest volatility implies lower price autocorrelation, lower cost of storage implies higher autocorrelation, and iso-elastic demand displays greater positive skewness.

TABLE 2. Distributions of Simulated Prices: Constant Returns to Storage Model

Supply Shocks	$P(x) = a - b\,x$ y is N(1,s=0.1), r=0.05				$P(x) = x^{-r}$ ln y is N(0,s=0.1), r=0.5			
Parameters	d=0.05 a=2.00 b=1.00	d=0.00 a=2.00 b=1.00	d=0.05 a=6.00 b=5.00	d=0.00 a=6.00 b=5.00	d=0.05 r=1.00	d=0.00 r=1.00	d=0.05 r=5.00	d=0.05 r=5.00
Coeff of variation	0.09	0.08	0.28	0.24	0.09	0.08	0.36	0.30
First-order a.c.[(1)]	0.08	0.20	0.34	0.47	0.10	0.19	0.29	0.40
Second-order a.c.[(1)]	0.01	0.06	0.18	0.30	0.00	0.06	0.14	0.25
Skewness γ_1[(2)]	0.47	0.86	1.63	2.01	0.67	1.00	3.08	3.64

Notes: (1) Autocorrelation of prices.
(2) $\gamma_1 = \mu_3/(\mu_2)^{1.5}$, where μ_γ is the γ-th central moment

TABLE 3. Distributions of Simulated Prices: Cost of Storage with "Convenience Yield" Model

Supply Shocks	$P(x) = a - b\,x$ y is N(1,σ=0.1), r=0.05 $c(x)=\alpha+0.1\ln(x)$				$P(x) = x^{-\rho}$ ln y is N(0,σ=0.1), r=0.05 $c(x)=\alpha+0.1\ln(x)$			
Parameters	α=0.30 a=2.00 b=1.00	α=0.05 a=2.00 b=1.00	α=0.30 a=6.00 b=5.00	α=0.05 a=6.00 b=5.00	α=0.30 ρ=1.00	α=0.05 ρ=1.00	α=0.30 ρ=5.00	α=0.05 ρ=5.00
Coeff of variation	0.08	0.05	0.30	0.16	0.08	0.05	0.36	0.17
First-order a.c.[(1)]	0.20	0.60	0.41	0.80	0.19	0.60	0.33	0.80
Second-order a.c.[(1)]	0.04	0.36	0.20	0.64	0.04	0.37	0.14	0.63
Skewness γ_1[(2)]	0.27	0.16	0.98	0.37	0.42	0.15	2.60	0.84

Notes: (1) Autocorrelation of prices.
(2) $\gamma_1 = \mu_3/(\mu_2)^{1.5}$, where μ_γ is the γ-th central moment

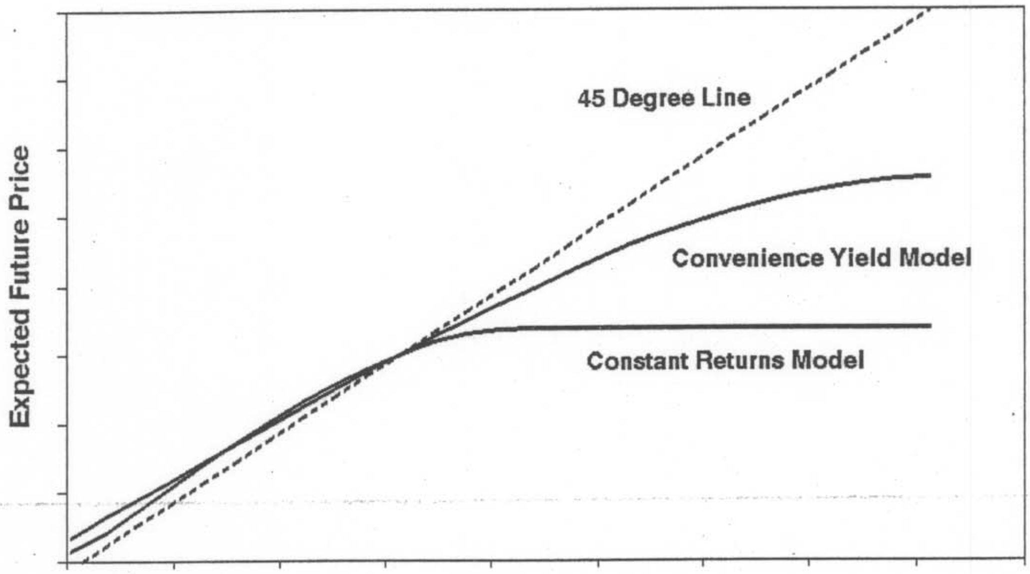

Figure 2. Commodity Price Dynamics

The source of differences in price autocorrelation between the two models is illustrated in figure 2. This figure shows the expected price in period t+1 conditional on the price in period t. The relationship was computed by deriving the equilibrium price function for the two storage models and taking expectations over the harvest in equations (12) and (13). As can be seen in figure 2, low current prices imply abundant supplies, substantial stockholding, and a strong link between the current and successive period's prices. As price rises, stocks fall and expected price rises in tandem. This is true for both the constant returns to storage model and the convenience yield model.

The main difference between the constant returns to storage model and the convenience yield model arises when prices are high. Given sufficiently high current prices, there will be no stocks held in the constant returns to storage model; once the threshold price for stockouts is reached, higher prices have no effect whatsoever on stockholding or on the expected future price. In the convenience yield model, however, stocks never fall to zero. Thus, a higher current price will be associated with lower stockholding and a higher expected future price. Unlike the constant returns to storage model, the convenience yield model does not allow the link between successive prices to be broken. It is not be surprising, therefore, that the convenience yield model generates the higher autocorrelation in prices.

Conclusion

In this paper we have continued Deaton and Laroque search for a variant of storage model that can explain the observed behavior of commodity prices, particularly the high autocorrelations.

We confirmed Deaton and Laroque's findings that the constant returns to storage model cannot satisfactorily explain high autocorrelations. We found, however, that a classical cost of storage function with "convenience yield" could generate the high levels of price autocorrelation observed empirically.

Our analysis was restricted to simple models with independent and identically distributed harvest shocks. Whether our conclusions will be strengthened or weakened by the presence of autocorrelation in supply, or by more complex structures remains to be seen. Our analysis was also restricted to illustrative simulations of models with artificial parameterizations. Our next goal in pursuit of a viable storage model is to estimate the convenience yield model from historical data using full information maximum likelihood methods. This technique has been demonstrated to be feasible in other studies involving the nonlinear rational expectations storage model (Miranda 1995). Until such an empirical tests are performed, the ability of the classical convenience yield storage model to explain commodity price behavior will not be fully established.

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