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Suggested citation format:

Shepherd, T. L., and J. H. Dorfman. 1996. "Optimal Storage Decisions under Estimation and Prediction Risk." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

OPTIMAL STORAGE DECISIONS UNDER ESTIMATION AND PREDICTION RISK

Tommie L. Shepherd and Jeffrey H. Dorfman*

Estimation and prediction risk are shown to influence the optimal storage decision of a dominant firm facing a competitive fringe. The presence of risk with respect to demand estimation and supply prediction results in increased storage by a dominant firm exercising market power in a two period profit maximization scenario. Bayesian numerical integration is employed to derive the optimal storage decision for a hypothetical cooperative of Georgia pecan growers facing demand estimation risk, supply prediction risk and a combination of the two.

Introduction

The application of optimization techniques in economic decision making almost invariably involves the use of estimated parameters. Estimated parameters are commonly accepted as the certainty equivalent of the true parameters they represent in the decision process, ignoring any possible biases or sampling errors (Bawa, Brown, and Klein, 1979). Typical examples in the area of market risk management include the estimation of market supply and demand functions of various agricultural commodities for the purpose of determining optimal pricing and storage policies. Implicit in the use of such estimates is the presence of estimation risk, the probability that the true parameter differs significantly from the estimate (Lence and Hayes, 1994). Demand risk stems primarily from the variance associated with parameter estimates of the level and elasticity of demand for a commodity and from errors in model specification. When future conditions are relevant to some decision process, the decision maker may face prediction risk as well. Prediction risk may be viewed as the risk which arises from forecasting a future economic variable, since even an ideal model, correctly specified with perfectly estimated coefficients would still have associated risk due to the stochastic nature of the variable being forecast.

One important example of optimizing behavior in economic theory is the derivation of optimal strategies for competing firms exercising market power. Competition among oligopoly firms has been addressed for a variety of market structures by such noted economists as Bertrand (1883), Cournot (1897), Bowley (1924) and Stackelberg (1952). One common thread which runs throughout much of the oligopoly literature is the implicit assumption that all firms possess perfect knowledge of market supply and demand parameters. It is the failure of the classical theory to acknowledge the risk embodied in such informational assumptions and the importance of considering this risk in the derivation of optimal competitive strategies which motivates the following discussion.

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The objectives of this paper are (I) to review briefly, the theory of optimal decision making under risk, (II) to discuss the sources of risk in oligopoly theory and the implications of incorporating risk into the decision process, (III) to apply this methodology to a dominant firm/competitive fringe representation of the U.S. pecan industry and (IV) to summarize the results of this application and evaluate the profit potential of developing and employing storage facilities as a means of exercising market power.

Optimal Decisions Under Risk

Consider a profit maximizing firm which receives utility from profits via $U[\pi(d,y)]$ where the argument $\pi(d,y)$ describes profits as a function of a vector of decision variables d and a vector of random variables y which denotes future states of nature relevant to the decision making process. The joint probability density function (pdf), $p(y|\theta)$ summarizes the probability of observing states of nature y conditional on what is known about their distribution through the parameters θ. If $p(y|\theta)$ is known with certainty, the firm's problem is simply to find the decision d among the set of all feasible decisions D such that

$$\begin{aligned} d &= \operatorname{argmax}_{d\in D} E_{y|\theta}\{U[\pi(d,y)]\} \\ &= \operatorname{argmax}_{d\in D} \int_Y U[\pi(d,y)]\, p(y|\theta)dy , \end{aligned} \tag{1}$$

where E(•) is the expectations operator and Y is the domain of y. The above scenario describes the firm's decision process, in somewhat generic terms, when it possesses perfect knowledge regarding states of nature y which are relevant to the decision making process. In reality however, one might suspect that firms do not possess perfect information concerning future states of nature such as market supply and demand parameters. In fact, considerable amounts of time and effort are regularly expended by firms in estimating such parameters for decision making purposes. Inherent in these estimates is some degree of estimation risk which may be interpreted as incomplete or imperfect knowledge about $p(y|\theta)$. Estimation risk may be attributed to two sources, (1) lack of knowledge about the functional form of $p(y|\theta)$ or (2) lack of knowledge about the correct values of θ, given that the functional form is known (Bawa, Brown, and Klein, 1979). The remainder of this discussion will assume that estimation risk arises from scenario 2, as suggested by Lence and Hayes (1994).

If this assumption of perfect information is violated, i.e. $p(y|\theta)$ is unknown, $E_{y|\theta}(U)$ is unknown as well and therefore (1) cannot be solved. The "classical" solution to this problem of parameter uncertainty is to substitute a point estimate $\hat{\theta} = \hat{\theta}(X)$ for the unknown parameters θ, where X represents a matrix of explanatory variables related to past observations of the states of nature y in question. Substituting these point estimates into (1) yields,

$$\begin{aligned} d &= \operatorname{argmax}_{d\in D} E_{y|\theta=\hat{\theta}} \{U[\pi(d,y)]\} \\ &= \operatorname{argmax}_{d\in D} \int_Y U[\pi(d,y)]\, p(y|\hat{\theta}(X))dy. \end{aligned} \tag{2}$$

This approach is quite appropriately named the parameter certainty equivalent (PCE) method (Bawa, Brown, and Klein, 1979) due to its assumption of certainty regarding the parameters in $\hat{\theta}$. The obvious shortcoming of this method is that it fails to take into account the risk embodied in the estimate $\hat{\theta}$. Uncertainty or estimation risk about θ may be taken into account in a logical and systematic way by employing Bayes' decision criterion (Winkler, 1972; Anderson, Dillon, and Hardaker, 1977; Aitchison and Dunsmore, 1975). In its simplest form, Bayes theorem states that a posterior distribution $p(\theta|X)$ is proportional to the product of a prior distribution $p(\theta)$ which reflects one's subjective parameter estimates independent of any observed data and $p(X|\theta)$, a likelihood function entirely dependent on observed data:

$$p(\theta|X) \propto p(\theta)p(X|\theta).$$

Thus the posterior pdf $p(\theta|X)$ combines the sample information contained in X with the prior or nonsample information. Bayes theorem then allows us to integrate out the uncertainty embodied in θ by employing the posterior pdf $p(\theta|X)$, allowing for a solution for expected utility which is not conditional on a single value (estimate) of θ. Thus, the decision becomes

$$\begin{aligned} d &= \operatorname{argmax}_{d\in D} E_{\theta}\{E_{y|\theta}[U(\pi(d,y))]\} \\ &= \operatorname{argmax}_{d\in D} \int_{\Theta}\{\int_{Y} U[\pi(d,y)]\, p(y|\theta)dy\}\, p(\theta|X)d\theta \\ &= \operatorname{argmax}_{d\in D} \int_{Y} U[\pi(d,y)][\int_{\Theta} p(y|\theta)p(\theta|X)d\theta]dy \\ &= \operatorname{argmax}_{d\in D} \int_{Y} U[\pi(d,y)]p(y|X)dy, \end{aligned} \tag{3}$$

where Θ is the domain of θ and $p(y|X)$ is the predictive pdf of y. The difference between this and the PCE method is that the Bayesian decision is a function of all available information, both sample and nonsample and does not depend on any unknown parameters.

Uncertainty in Oligopoly Theory

The simplest "classical" models of market structure begin with the assumption of a known (inverse) demand curve of the form

$$P = \alpha - \beta(q_1 + q_2 + \ldots + q_n), \tag{4}$$

where price P is dependent on the total quantity ($Q = q_1 + q_2 + \ldots + q_n$), of a good produced by n firms through the parameters α and β. Firms face the problem of maximizing profits subject to the market structure within which they operate. Market structure may dictate that strategic decisions be made in terms of quantities (Cournot models) or prices (Bertrand models) for undifferentiated and differentiated products, respectively. It may also determine the degree of market power which firms are able to exercise due to comparative advantages in technology, experience, location and reputation.

The extension of oligopoly theory to multiperiod decision making with an objective of long run profit maximization was introduced by Shubik (1959) and further developed by Friedman (1968). Cyert and DeGroot (1987) approach the oligopolist's risk problem by means of Bayesian decision analysis for a variety of conjectural permutations including multiperiod decision making with simultaneous and alternating choice. The shortcoming of this approach is the problem of infinite regress and the need for additional simplifying assumptions which results from assigning risk to the conjectures of rival firms. Cyert and DeGroot summarize these difficulties by referring to this as the "I think that you think that I think . . ." model. Contrary to intuition, this amounts to assuming that a firm expects its rivals to know the values of market parameters with certainty, but is unsure how they will respond to any particular observed marketing strategy, when in fact, optimal competitive strategies may be derived with certainty given demand and supply schedules, cost information and the basic economic assumptions of profit maximizing behavior. Arguably then, a firm's risk problem arises not from uncertainty surrounding its competitors behavior, but from uncertainty about market demand and/or supply parameters.

A Bayesian approach to the estimation risk problem has two advantages. First, a joint pdf of α and β conditional on historical information X and subjective information makes use of all information relevant to the decision making process. The inclusion of subjective information may be used to incorporate expectations of future demand or supply which are not reflected in historical data or economic theory. The ability to accomplish this blending of historical information with future expectations is crucial to optimal decision making and is of particular relevance in markets characterized by highly variable conditions such as agricultural production. Second, a Bayesian approach allows the estimation risk of α and β to be integrated out by solving over the joint pdf $p(\alpha,\beta|X)$ as shown in (3).

In the application which follows, firms are assumed to behave as rational, risk averse profit maximizers who receive utility from profits via

$$U(\pi) = -\exp(-\phi\pi), \tag{5}$$

where ϕ is the Arrow-Pratt constant absolute risk aversion coefficient. The assumption of risk aversion is a necessary and sufficient basis for the inclusion of risk in the decision process since the treatment of estimated demand and supply parameters ($\hat{\theta}$) as certainty equivalents of the true parameters renders the utility function irrelevant, resulting in identical solutions for profit and utility of profit maximization.

Estimation risk is introduced into the decision process by employing the variances and covariances which result as a byproduct of estimating demand and supply parameters. The variances and covariances of parameter estimates may be viewed as a logical measure of the risk associated with using the accompanying parameter estimates for decision making purposes. Intuitively, parameter estimates which are characterized by relatively large variances may be viewed as somewhat unreliable or "risky" approximations of the true parameters, while relatively small variances suggest a greater degree of confidence that the estimates are reasonable approximations of the true parameters in question.

In the section which follows, this concept is demonstrated by application to a dominant firm/competitive fringe representation of the U.S. pecan industry. In this example, the dominant firm generates maximum likelihood estimates of demand and supply parameters and uses importance sampling to integrate over the accompanying risk as shown in (3).

Application To Pecan Storage

U.S. commercial pecan production began in a few Southeastern states during the late 1800's and had expanded substantially by the 1930's. By 1993, Georgia growers led the nation in pecan production, accounting for approximately 40% of total U.S. production and more than 75% of Southeastern production. Commercial growers range from small, part-time operators with a few acres of trees to pecan plantations consisting of well over a thousand acres. Pecans are marketed by a network of accumulators and shellers who perform the tasks of assembling, processing, grading and storing nuts which may then be sold through retail outlets in their raw form or to food processors for use in baked goods, candies and other confectioneries.

Georgia's pecan industry benefits from a significant comparative advantage in producing and marketing its crop relative to other states primarily due to the large number of mature orchards which were planted earlier in this century (Allison and Epperson, 1978). Orchard establishment and renewal is a long term investment, with improved (commercial) tree varieties requiring about five years to produce their first, small crop and ten years or more to reach maximum bearing potential (Hubbard, Purcell, and Crocker, 1988). An additional advantage of Georgia's early entry into the pecan industry is the accompanying development of marketing and processing infrastructure which are now firmly established. Georgia's pecan industry has the potential to maintain this comparative advantage well into the future as existing orchards are renewed and expanded with new, improved tree varieties (Ikerd, 1985).

Despite it's comparative advantages in production and marketing, Georgia's pecan industry faces a number of potentially serious problems. Chief among these are the erratic yields and large year to year price swings which result from adverse weather conditions as well as from the alternate bearing characteristics of the trees themselves. A second marketing problem faced by growers is a serious lack of adequate storage facilities suitable for pecans. Unrefrigerated, in-shell pecans may be stored for only a short period of time before they begin to deteriorate. Refrigerated storage facilities extend the period of time which in-shell pecans may be stored to approximately one year, but are prohibitively expensive for most if not all growers at the present time. Hubbard, Purcell and Ott (1987) report no refrigerated storage among growers surveyed. A third major threat to Georgia's pecan industry is that of increasing competition from growers in the Southwestern United States, primarily those in Arizona, Arkansas, Louisiana, New Mexico, Oklahoma and Texas (Texas A&M University, 1986) and from Mexico. As extensive plantings in the Southwest during the late 1980's approach bearing age, a potentially burgeoning supply of pecans threatens to depress prices and eliminate many small growers.

This combination of problems threatening Georgia's pecan industry results in the inability of growers to consistently produce profitable crops. Widely fluctuating year-to-year profitability is of primary concern to pecan growers, particularly those for whom pecan sales comprise a large percentage of farm income. The Southeast Georgia Branch Experiment Station's experimental farm management unit at Midville, Georgia reported that its pecan crops were profitable in only 11 out of 24 years from 1963 to 1986 (Perry and Saunders, 1987). Of the 13 years in which crops were unprofitable, 11 are included in the data set used here for estimation purposes. Of these 11 unprofitable growing seasons, 4 are associated with low alternate bearing years and 7 are associated with the resultant low prices which accompany high alternate bearing years.

Due to the prohibitive expense of refrigerated on farm storage facilities and competition from peanut growers for commercial storage space, growers are currently restricted to short-term, non-refrigerated storage of their crops. The feasibility of developing refrigerated storage facilities for pecans as a means of achieving consistent profitability by smoothing out year-to-year supply fluctuations is currently being investigated. A methodology for evaluating the economic feasibility of such facilities in the presence of estimation and prediction risk is presented in the following section.

Model Selection And Specification

The large percentage of total U.S. pecan production attributed to Georgia growers leads to the selection of a dominant firm/competitive fringe representation of the U.S. pecan industry. The primary assumption of this market structure is the ability of the dominant firm to exercise market power, in contrast to the firms comprising the competitive fringe, who assume the role of perfect competitors or price takers.

In light of the perfectly inelastic short run supply of pecans and the long lead time required to expand production, as well as the relatively homogeneous nature of the product, it is assumed that the dominant producer may exercise market power by controlling the quantity of nuts offered for sale through storage from high production to low production years. Georgia producers are thus modeled as a (hypothetical) risk averse marketing order (grower cooperative) with seasonal storage capabilities which behaves as a dominant firm facing a competitive fringe comprised of all other U.S. pecan growers.

This proposed grower cooperative (dominant firm) will solve a two-period utility of profit maximization problem in which its pecans may be stored for one year. Fringe firms are noncollusive, individually producing small percentages of total production and thus lacking market power. The dominant firm is assumed to be risk averse, receiving utility from profits via a negative exponential utility function as shown in (5). The Arrow-Pratt constant absolute risk aversion coefficient, ϕ, is expanded by Babcock, Choi and Feinerman (1993) to derive

$$\phi = \phi(\rho,h) = \ln[(1 + 2\rho)/(1 - 2\rho)] /h. \tag{6}$$

Equation (6) shows ϕ is a function of a probability premium (ρ) and of the gamble size (h). The gamble (h) size is taken to be the mean difference in profits which could have been realized *ex post* between optimal storage and no storage scenarios over the data set used. The dollar amount assigned to h was $70,377,000. When the chance of winning or losing risky wealth h is a fair gamble, the probability premium ρ measures the increase in probability above 1/2 that a risk averse agent requires to maintain a constant level of utility equal to some certain amount of wealth obtainable if the gamble is not taken. The probability premium ρ was set to .2, which leads to a risk premium Φ of .38 where

$$\Phi(\phi,h) = \ln[.5(e^{-\phi h} + e^{\phi h})/\phi h. \tag{7}$$

The risk premium associated with ρ is thus approximately 38% of the gamble size h, indicating a moderately though not excessively high level of risk aversion. The derived CARA coefficient ϕ, defined above in equation (6) equals .000012, which is well within the bounds of those cited in previous literature reviewed by Babcock et al. (1993).

In order to solve its storage problem, the dominant firm requires three key pieces of information which must first be estimated. These are the demand curve for pecans and the supply curve for its own production and that of its collective competition. Demand estimation for pecans has been investigated by a number of researchers. Fowler (1960) estimated the U.S. farm level price of pecans as a function of the quantity of pecans produced, per capita disposable income and time for the years 1922-1956. Epperson and Allison (1980) estimated a similar model for the years 1960-1976, with the exceptions that a population term was included and a double log specification was used. Shafer and Hertel (1981) employed stocks of nuts other than peanuts as a proxy for pecan stocks to estimate price as a function of production, per capita disposable income and the quantity of nuts in storage. Blake and Clevenger (1982) estimate pecan prices as a function of production, net change in the stock of all nuts, per capita income, exports and per capita consumption. In 1971, a time series of regional cold storage holdings became available (USDA, Regional Cold Storage-Annual Summary 1979-1983) which Wells et al. (1986) uses to demonstrate the impact of seasonal storage on pecan prices.

The desire for a concise and parsimonious model of pecan demand suitable for the methodological application performed here leads to the selection of

$$USP = \beta_0 + \beta_1 MKTQTY + \beta_2 DISPINC + \epsilon \tag{8}$$

where the β_i's are parameters, USP is the deflated U.S. season average price of pecans, MKTQTY is total marketable quantity; the sum of total U.S. production and total U.S. cold storage holdings of pecans, DISPINC is inflation adjusted per capita disposable income and ϵ is an error term. The data covers the time period from 1973-1993. U.S. price and production data is from Agricultural Statistics (1972-1995). Cold storage estimates are from Regional Cold Storage-Annual Summary (USDA) and population and income figures are from the Statistical Abstract of the United States (1972-1995).

The pecan supply model for the dominant firm (Georgia growers) is specified as a fir order integrated AR(1) process;

$$\Delta GAQTY_t = \gamma_0 + \gamma_1 \Delta GAQTY_{t-1} + u_1 \tag{9}$$

where the γ_i's are parameters, Δ is the difference operator, GAQTY is total Georgia productio: and u_1 is an error term. Dickey-Fuller unit root tests suggest the need for first orde differencing to achieve stationarity. Georgia price and production data is from Georgi; Agricultural Facts (1995). Competitive fringe production is obtained by subtracting Georgi: production from total U.S. production for each observation in the data set. The fring production equation estimated is

$$FRNGQTY_t = \Gamma_0 + \Gamma_1 FRNGQTY_{t-1} + \Gamma_2 LINTRND + u_2 \tag{10}$$

where the Γ_i's are parameters, FRNGQTY is competitive fringe production, LINTRND represents a positive linear trend and u_2 is an error term.

Estimation results are summarized in table 1. These parameter estimates provide an overview of the type and degree of estimation risk faced by Georgia pecan growers seeking to exercise market power through storage. It is interesting to note that the Georgia supply model, though clearly superior to alternative models tested, fails to capture the alternate bearing nature of pecan trees suggested earlier. This may be due to the prevalence of adverse, as well as favorable growing conditions over several consecutive seasons, which serves to interrupt the alternate bearing cycle. It may also be attributed in part to maturing orchards which are continuously realizing increased bearing potential. In either case, this result highlights the difficulties involved in forecasting Georgia's pecan production and the risk inherent in employing these methods to forecast future production. The competitive fringe supply model is found to exhibit the alternate bearing pattern, evidenced by the negative coefficient on lagged production.

Bayesian Integration Procedures and Results

With the above estimates in hand, we can solve for the dominant firm's optimal storage strategy in the presence of estimation risk. This decision is made during the annual pecan harvest, which begins about October and continues through the following January. At this time current production is known, if not with certainty, at least with a very high degree of accuracy. Using the 1993-1994 period as an example, a forecast of the 1994 crop is made for the dominant and fringe firms and each potential storage scenario from no storage to total storage (in increments of 100,000 lbs.) is evaluated for its expected impact on 1993 and 1994 prices and resulting gross profits. Costs are ignored as fixed so that the gross profit differential between optimal storage and zero storage provides an upper bound on the cost at which storage facilities are economically feasible. Gross profits are discounted at an annual rate of 5%.

Importance sampling is employed to derive the optimal storage strategy under estimation and prediction risk. Antithetic replication is used to improve numerical approximation accuracy

Table 1. Parameter Estimates

	Demand	Georgia Production	Fringe Production
Intercept	-31.66 (27.5)	1847.42 (7565.0)	187057.5 (18042.7)
MKTQTY	-0.00029 (.000063)		
DISPINC	0.0165 (.0028)		
$\Delta GAQTY_{t-1}$		-0.813 (.214)	
LINTRND			6022.88 (921.4)
$FRNGQTY_{t-1}$			-0.7786 (.124)
R^2	.640	.432	.754
obs.	21	21	21

Standard errors in parentheses.

(Geweke, 1988). Twenty thousand random draws are generated from the estimated joint distribution of the demand curve parameters conditional on the estimated covariance matrix of the parameters. This allows the draws to be generated from a normal distribution instead of a multivariate t-distribution. The proper likelihood function value is calculated and saved for use as the importance weight to correct the resultant undersampling of the tail areas. Twenty thousand draws are also generated from the forecast distributions of production for Georgia and the states representing the competitive fringe. The competitive fringe production forecast draws are taken from a normal distribution which has a variance incorporating both the sampling error of the parameters and the additional error variance associated with a prediction; that is,

$$Q^{F(i)}_{t+1} \sim N(QF_{t+1}, X_{t+1}\Sigma_{\Gamma}X'_{t+1} + \sigma^2_F),$$

where X = [1 FRNGQTY LINTRND], Σ_{Γ} is the estimated covariance of the fringe production equation coefficients, and σ^2_F is the estimated error variance.

Draws for the distribution of the Georgia production forecast are made in a manner analogous to the method used for the fringe with one exception. An informative prior is placed on the Georgia production model in the form of a truncated uniform providing support only for values above 20 million pounds. This restriction seems reasonable in light of the near failure of the 1992 crop which still amounted to 30 million pounds, the smallest in over 20 years. Thus, any draw below 20 million pounds is discarded, and a new draw is made to replace it.

For a candidate storage level, utility of gross profit is computed for each triplet of draws and expected utility is calculated as the weighted average of all draws for each potential storage decision. The weights are calculated using the likelihood functions of each distribution and the actual draws made to correct for the bias caused by drawing from normal rather than Student-f distributions. The storage scenario with the highest expected utility is then selected as the optimal quantity to store in the presence of estimation and prediction risk.

This procedure was repeated for three other scenarios in order to separate the effects of demand and supply uncertainty on the final solution. In one, only demand parameters were assigned uncertainty, while supply forecasts were taken as their parameter certainty equivalents. In another, demand parameters were accepted as certain while only supply forecasts were treated as risky. Finally, the traditional risk neutral (risk oblivious) solution was obtained by setting both demand and supply parameters to their certainty equivalents. Results are summarized in table 2 where PCE denotes the use of parameter certainty equivalent estimates and PER represents the use of parameter estimates which incorporate estimation/prediction risk.

A striking difference occurs in the optimal quantities of pecans stored between the certainty equivalent case (34,700,000 lbs.) and the estimation and prediction risk case (78,400,000 lbs.). A similar difference is found in the case of supply only uncertainty, while an almost negligible difference of only 200,000 lbs. is attributed to the demand only uncertainty case. The *ex post* gross profits which would have been realized from the above scenarios as well as from a zero storage situation are summarized in table 3. These results suggest an upper bound on economically feasible storage costs of about 38 cents per pound.

The objective of storage is to increase income by shifting sales from the present time period to some future time period. Storage is an effective means of accomplishing this if production falls in the inelastic portion of the demand curve during the first time period and in the elastic portion during subsequent time periods, thus increasing the probability that unit elasticity will be achieved in each period. The rather substantial increase in storage under estimation and prediction risk found in this example may be viewed as maximizing the probability that unit elasticity will be achieved in each period. Own price elasticities are calculated for each period for the scenarios of no storage, storage based on PCE estimates of supply and demand parameters and finally storage based on parameters with estimation and prediction risk. In the case of no storage, the large quantity of pecans marketed in period 1 results in an elasticity estimate of -.72, which falls well below the revenue/gross profit maximizing point of unit elasticity (-1.0). In the second period of the no storage case, the relatively small quantity of nuts marketed leads to an elasticity estimate of -2.03, this time falling substantially above the desired point of unit elasticity. Under a storage strategy based on PCE estimates (34,700,000 lbs. stored), the price elasticities for periods 1 and 2 are found to be -.90 and -1.59, respectively, indicating a move towards unit elasticity in each period. Finally, under a storage strategy based on parameters characterized by estimation and prediction risk (78,400,000 lbs. stored), price elasticities are nearly identical to unit elasticity at -1.19 and -1.20 for periods 1 and 2 respectively. The achievement of exact equality between these elasticities is prohibited by the discounting of second period utility.

Table 2. Optimal Storage

		Supply PCE	Supply PER
Demand	PCE	34,700,000 lbs.	76,900,000 lbs.
	PER	34,900,000 lbs.	78,400,000 lbs.

PCE = Parameter Certainty Equivalent Estimates
PER = Prediction and Estimation Risk Inclusive Estimates

Table 3. Ex Post Revenue.

	Quantity Stored	Ex Post Revenue
No Storage	0 LBS.	$180,917,460
PCE Storage	34,700,000 LBS.	$194,475,350
Storage with Demand Risk	34,900,000 LBS.	$194,514,130
Storage with Supply Risk	76,900,000 LBS.	$192,662,640
Supply and Demand Risk	78,400,000 LBS.	$192,228,550

Conclusions

A significant increase in expected gross profit is realized under all of the storage scenarios relative to that of no storage, however, only negligible differences are found to exist between the various storage scenarios themselves. The extremely flat nature of the underlying profit function, as indicated by small changes in expected gross profit over a wide range of potential storage decisions, suggests a rather large margin of error associated with determination of optimal storage decisions. In fact, the optimal storage decisions for the prediction risk only and prediction *and* estimation risk cases actually result in slightly lower expected profits than the parameter certainty equivalent case for the 1994 crop year due to the models tendency to under estimate production.

In conclusion, the substantial increase in expected gross profits attributable to storage of Georgia pecans from 1993 to 1994 offers support for further investigation into the development of refrigerated storage facilities as a means of exercising market power in the risky and increasingly competitive pecan industry. The inclusion of estimation risk in the decision process is shown to increase the optimal quantity of pecans stored only slightly, while the inclusion of prediction risk is shown to have a relatively large effect on the optimal storage decision. Expected gross profits, however, are found to vary little over the range of storage decisions considered, indicating a wide margin of error in selecting the appropriate storage strategy.

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