Heterogeneous Subjective Moments and Price Dynamics

by

Darren L. Frechette and Robert D. Weaver

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Differential expectations have long been presumed necessary for the existence of speculative markets. At an empirical level, considerable evidence further suggests that agents may not hold rational expectations. The representative agent hypothesis is disputable on theoretical grounds because it is not consistent with observed trading behavior and the existence of markets. It has been favored in the past due to intractability of aggregation associated with heterogeneity. Now, due to improved computing technology, explicit aggregation problems are becoming tractable. Heterogeneous expectations must be considered seriously in price analysis because they bring our models one step closer to reality.

INTRODUCTION

Managed futures trading by commodity pools using positive feedback rules has grown dramatically in the last decade, demonstrating that some noise traders cannot be driven out of the market (Irwin and Yoshimaru). The coincident persistence of noise and rational traders within commodity markets calls into question the representative agent hypothesis. These two groups of traders form expectations differently but neither can be driven out of the market in the short run (Black).

Differential expectations have long been presumed necessary for the existence of speculative markets (Keynes, Grossman, etc.). At an empirical level, considerable evidence suggests that agents may not hold a uniform rational expectation (Irwin and Thraen; Frankel and Froot). The result has been considerable interest in developing new models to account for heterogeneous expectations (Chavas; Frechette; Markson). This paper contributes by developing a powerful theoretical framework for analyzing heterogeneous expectations and using it to estimate the time path of the distribution of agent-level expectations.

THEORETICAL FRAMEWORK

We adopt a very general specification for the market:

(1) Current supply $S_t = S(p^e_{t-1}, Z^s_t, \beta) + v_t$ $\quad v_t \sim V(\mu^v_t, \sigma^v_t)$

(2) Current demand $D_t = D(p_t, Z^d_t, \alpha) + w_t$ $\quad w_t \sim W(\mu^w_t, \sigma^w_t)$

(3) Current balance $S_t = D_t$

where p_t is current price, p^e_{t-1} is last period's expectation of current price, Z^s_t and Z^d_t, are exogenous shifters, α and β are parameters, and v_t and w_t are stochastic shocks. If a cash market is being considered, then inventories and carryout are included within the supply and demand functions.

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Equations (1) - (3) imply a stochastic excess demand:

(4) $$D(p_t, Z^d_t, \alpha) - S(p^e_t, Z^s_t, \beta) + u_t = 0 \qquad u_t \sim U(\mu_t, \sigma^u_t)$$

where $u_t = w_t - v_t$.

To proceed, we need to specify the relationship between current price and expected price. Where expectations are exogenous to the market, this relationship is defined by (4) and intertemporal arbitrage equilibrium. Rational expectations assumes:

(5) $$p^e_{t-1} = E_{t-1}p_t$$

(6) $$p_t = p^e_{t-1} + \omega_t \qquad \omega_t \sim \Omega(0, \sigma^e_t)$$

Where the market structure is linear, we have from (4) and (5):

(7) $$E_{t-1}p_t = (\beta_{0t} - \alpha_{0t} - \mu_t)/(\alpha_p - \beta_p).$$

The time subscripts remind us that these parameters may vary over time with the exogenous determinants of demand and supply, Z^d_t and Z^s_t, respectively.

A series of cases may now be generated depending on the specification of the stochastic properties of u_t. Where u_t is i.i.d. and zero mean, the conventional rational expectations result follows; (6) and (7) imply that expectations and current price will evolve depending on the properties of the underlying exogenous variables. Alternatively, assume u_t is covariance stationary and representable by a linear filter of i.i.d. disturbances: $u_t = h(B)\varepsilon_t$. It follows from (6) and (7) that p_t is representable using a related filter of the same disturbances. Dropping β_{0t} and α_{0t} to simplify, (6) and (7) imply p_t and p^e_{t-1} are representable with linear filters of i.i.d. disturbances:

(8) $$p_t = H(B)\varepsilon_t \text{ where } E\varepsilon_t = 0, E\varepsilon_t^2 = \sigma^2.$$

(9) $$p^e_{t-1} = H(B)\varepsilon_{t-1}$$

That is, under rational expectations the evolution of both p_t and p^e_{t-1} is representable in terms of the stochastic processes impacting the market. Importantly, the filter H(B) is defined by (4) - (6) and depends on the filter h(B), as well as the structure of the demand and supply functions. Nonstationary shocks may be introduced as affecting u_t and (8) and (9) generalized accordingly.

Apparently, rational expectations are attractive because a tractable approach to measuring such expectations is suggested, e.g. by (7) or (9). Further, the intuitive appeal of the underlying assumptions ((5) and (6)) appears powerful for many economists. However, heterogeneity in expectations is observable and evident by both the existence of markets as well as by the continuity of trade in those markets. October 1987 reminds us that homogeneous expectations can lead to one-sided markets, when just as predicted by (6), arbitrage incentives exist for only a buy or a sell, but not both.

Consider the possibility of heterogeneous expectations, and suppose that for each k^{th} agent the expectation can be thought of as a linear combination of a covariance stationary component representable by a linear filter of i.i.d. disturbances and a stochastic component that is not necessarily stationary, i.e.

(10) $$p^e_{kt} = H_k(B)\varepsilon_t + \varepsilon_{kt}$$

To allow consideration of processes of convergence in expectations, we define the filter as follows:

(11) $$H_k(B)\varepsilon_{t-1} = H'\boldsymbol{B}\Lambda_k\varepsilon_{t-1} = \Sigma_i H_{t-i}\lambda^k_{t-i}\varepsilon_{t-i},$$

where the weights H_{t-i} are elements of H(B) in (8), $\boldsymbol{B}$ is a diagonal matrix of the backward shift operators B contained in H(B), H' is a vector of parameters H_{t-i} involved in H(B), Λ^k is a vector of the parameters λ^k_{t-i} involved in an agent-specific linear filter $\lambda^k(B)$, and ε^k_t is similarly related to the nonstationary shock affecting p_t. This notation conveniently allows specification of particular expectations structures. For example, the k^{th} agent is said to hold rational expectations if $\lambda^k_{t-i} = 1 \; \forall \; i = 1, \ldots, \infty$. Alternatively, the agent may be said to be myopic within horizon [0, t] if $\lambda^k_{t-i} > 1 \; \forall \; i = 1, \ldots, \tau$.

To proceed, (4), (10), and (11) no longer provide a complete system for determination of p_t and its relationship with price expectations. In contrast to rational expectations, the arbitrage equilibrium conditions (e.g. (6)) no longer play a role in relating current price to agent specific expected prices. Instead, individualized supply functions exist, each a function of an agent-specific expectation. Aggregate supply and, therefore, current price would no longer be determined by a single market-wide expectation. The implications of this specification can now be assessed and its empirical relevance established by estimation of the parameters involved. The relationship among agent expectations and the current price are established by arbitrage entry equilibrium conditions.

Consider the storage problem. To start, suppose the agent is risk neutral with expected profits determined as:

(12) $$\pi^e_{kt-1} = (p^e_{kt} - p_t)I_{tk} - C(I_{kt}) ,$$

where $C(I_{kt})$ represents the cost of holding inventories I_{kt}. Agent equilibrium occurs when further expansion of inventories beyond some I^*_{kt} is no longer marginally profitable, i.e.

(13) $$p^e_{kt} - p_t - C'(I_{kt}) = 0 \text{ for } I_{kt} > 0.$$

However, if we assume current price and, possibly, agent expectations are somehow conditional on market-wide storage, then the traditional zero profits entry equilibrium condition would be achieved, though it would be complicated by the presence of heterogeneity. Agent entry would occur until:

(14) $$p^e_{kt} - p_t - AC(I_{kt}) = 0 \text{ for all agents } k = 1, \ldots, K .$$

Summing over K agents, entry occurs until:

(15) $$\Sigma_k p^e_{kt} - Kp_t - \Sigma_k AC(I_{kt}) = 0 ,$$

or using bold to indicate market-wide means:

(16) $$\mathbf{p}^e_t - p_t - \mathbf{AC}_t = 0.$$

Thus, while agent-specific decisions establish heterogeneous arbitrage equilibrium conditions, competitive entry lead to a relationship between the market-wide expectation $\mathbf{p}^e_t$ and current price, p_t. Equations (14) and (15) define an equilibrium relationship between agent and market-wide expectations: $p^e_{kt} = \mathbf{p}^e_t + AC(I_{kt}) - \mathbf{AC}_t$. If we define the distribution of agent expectations as $f(p; \Gamma)$, then (1) may be rewritten as the integral of agent supplies across the span of expectations. In analogy to (4), equilibrium price would be determined by the characterizing parameters Γ of f(.). For the case considered by (11), Γ would contain both the agent-specific parameters λ^k_{t-i} as well as the market rational weights H_{t-i}. Importantly, current price is no longer recoverable from knowledge of the expectation only. Instead, the agent's average net benefit (or average cost above) must also be known. This implies the relationship among the current price and expectations are defined by an analogue of (4).

EMPIRICAL APPLICATION

The theory developed in the previous section leads naturally to an empirical application. Assume that there exists a continuum of heterogeneous traders, indexed by their expectations. Trader k, at time t, expects next period's futures price to be $p^e_t(k)$. The distribution of expectations across agents at time t is denoted $f_t(p)$, where $f_t(p)$ is the portion of agents who expect the price to be p at time t+1. $f_t(p)$ is bounded between zero and one for all p and t. Time is measured in discrete units (days).

From the last section, equation (14) implies $p_{t+1} = p_t + AC_{kt} + e_{kt+1}$, where $AC_{kt} = AC(I_{kt})$ and e_{kt+1} is an idiosyncratic shock at time t+1. Assume e_{kt+1} has a zero conditional mean with respect to time, t, but may be correlated across agents, k. Averaging over k yields $p_{t+1} = p_t + \mathbf{AC}_t + \mathbf{e}_t$, where bold type represents market-wide averages as in the previous section. Substituting from equation (16) yields $p_{t+1} = \mathbf{p}^e_t + \mathbf{e}_t$, and $\mathbf{e}_t$ has a conditional mean of zero.

That means, on average, the mean trader's expectation is correct. In other words, $\mathbf{p}^e_t$, which equals $\int_0^\infty pf_t(p)dp$ when agents are represented as a continuum, is an unbiased estimator of p_{t+1}. In that case, $p_{t+1} = \int_0^\infty pf_t(p)dp + \mathbf{e}_{t+1}$, with mean-zero stochastic shock $\mathbf{e}_{t+1}$. A distributional assumption can be made about $\mathbf{e}_{t+1}$, e. g. $N(0, \gamma^2)$, and a flexible semiparametric structure can be imposed on $f_t(.)$ such that the integral can be calculated numerically and f(.) can be estimated by maximum likelihood.

Specifically, let $f_t(p)$ be a mixture of two Normal distributions:

$$f_t(p) = \alpha \exp(-\frac{p-\mu_{1t}}{2\sigma_{1t}^2})/\sigma_{1t}\sqrt{2\pi} + (1-\alpha)\exp(-\frac{p-\mu_{2t}}{2\sigma_{2t}^2})/\sigma_{2t}\sqrt{2\pi},$$

where μ_{it} and σ_{it} are means and standard deviations of the individual distributions, and α is a weighting parameter between zero and one. This distributional assumption allows for, but does not require, a bimodal shape, corresponding to "bulls" and "bears" in the market. The approach is similar to one used by Cukierman and Wachtel to study inflation expectations.

We would like to estimate the distributions of price expectations for each time t. To do so, we must impose a time series structure onto the parameters of f(.). For example,

$$\mu_{it} = a_{i0} + a_{i1}p_{t-1} + a_{i2}t$$
$$\sigma_{it} = b_{i0} + b_{i2}t\,;\ \mathrm{i} = 1, 2.$$

The reduced form for price comes from embedding these time series specifications into the distribution and integrating numerically. The time series parameters can then be estimated using a nonlinear estimator, such as maximum likelihood. This procedure allows the distributions to be estimated for each time t and convergence properties to be tested.

There are a total of 12 parameters to be estimated: six a_{ij}s, four b_{ij}s, α, and γ. The procedure for estimating them is as follows. First, choose reasonable starting values, such as $a_{i0} = 0$, $a_{i1} = 1$, $a_{i2} = 0$, $b_{i1} = 0$, and $\alpha = 0.5$. Second, embed a numerical integrator inside a numerical likelihood maximizer. Every time the 12-by-1 parameter vector is iterated, the numerical integrator runs once per data point, for a total of 269 integrations per iteration in this case. The log-likelihood function is:

$$L = \sum_{t=1}^{269} \log\{\phi[p_{t+1} - \int_0^\infty pf_t(p \mid a_{ij}, b_{ij}, \alpha)dp \mid \gamma]\},$$

where $\phi(.|\gamma)$ is the distribution function represented by $N(0, \gamma^2)$.

The importance of oilseeds to both domestic and international food supply motivates the study of soybeans futures. The following analysis uses daily closing futures prices (nominal) for the September 1997 futures contract, traded from August 27, 1996 through September 19, 1997. The data was purchased from Prophet Information Services, Inc. There are a total of 270 observations, and the mean price over the sample was 710.2 cents. Parameter estimates are presented in Table 1.

In an idealized rational market, the expectation of tomorrow's futures price is today's futures price, so $a_{i0} = 0$, $a_{i1} = 1$, and $a_{i2} = 0$. The estimates for a_{i0} and a_{i2} are all close to zero, but the estimates for a_{i1} differ by 2.6%, an economically but not statistically significant difference. If every participant's expectations were identical, then $b_{ij}=0$ for all i and j. However, the estimates for b_{i0} are roughly 10.2 and 6.9 cents per bushel, indicating that the bullish side of the market is more heterogeneous than the bearish side. However, the 6.9 cents is statistically significant at the 1% level, but the 10.2 cents is not statistically significant.

Correspondingly, the estimates for b_{i1} also differ; they are -0.006 and -0.012. They are negative, implying that agents' price expectations converge toward a common value as expiration approaches, but they are not significantly different from zero in the statistical sense. The estimate of α is 0.35, indicating that the bullish side of the market is weighted less heavily than the bearish side. That is not to say that the bears outnumber the bulls, but rather to say that the bullish "hump" of the distribution is shorter and fatter than the bearish "hump" in the distribution, as in Figure 1. Also, the asymptotic standard error of the estimate is 1.62, indicating that the bull-bear distinction is not a fine one.

The distribution changes shape and position every day, based on the previous day's data. Thus, a moving picture is the best method of describing the evolution of the expectations distribution over time. Such a "movie" is available from the authors, using MatLab. Figures 1 - 3 display three frames from the movie, for day 2 (the second day of trading), day 135 (mid-sample), and day 270 (the last day of trading).

Overall, the results are economically meaningful but provide only weak statistical evidence that heterogeneous expectations play a role in futures price dynamics. Previous studies have sometimes supported the rational representative agent model, and this study's results do not refute it. Instead, they provide an example of the universal truth that many different hypotheses can be supported by the same data.

The representative agent hypothesis is disputable on theoretical grounds because it is not consistent with market trading and the existence of the market place. It has been favored in the past due to intractable aggregation problems associated with heterogeneity. Now, due to improved computing technology, those aggregation problems are becoming tractable. Heterogeneous expectations must be considered seriously now because they bring our models one step closer to reality.

CONCLUSION

This paper has relaxed the representative agent hypothesis, which is inconsistent with market existence. The major findings of the paper are (1) one unbiased estimator for future price is the average expectation over all agents in the market; (2) an unbiased estimator exists, but efficiency remains an empirical question; and (3) the distribution of price expectations is estimable and forecastable over time.

One weakness in the empirical analysis is in the distributional assumption. Future improvements may allow for a more flexible distribution of expectations and more driving terms in the distribution's parameters. These improvements would allow the distribution to change shape based on information not captured by lagged prices and a time trend.

Another weakness is the link between the theoretical model and the empirical specification. Right now, the necessary restrictions appear to include homogeneous risk preferences, constant absolute risk aversion, and no liquidity constraints. These restrictions and the role they play in the empirical specification must be considered more carefully in future work.

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Parameter	Estimate	Asy. Std. Error	Asy. t-statistic
		0.01472	-0.15
		12.94	0.00
		0.255	3.98
		0.076	13.01
		0.00025	11.15
		0.01025	0.47
		66.52	0.15
		0.93	7.37
		0.02796	-0.19
		0.07551	-0.16
		1.6162	0.22

Table 1
Parameter Estimates

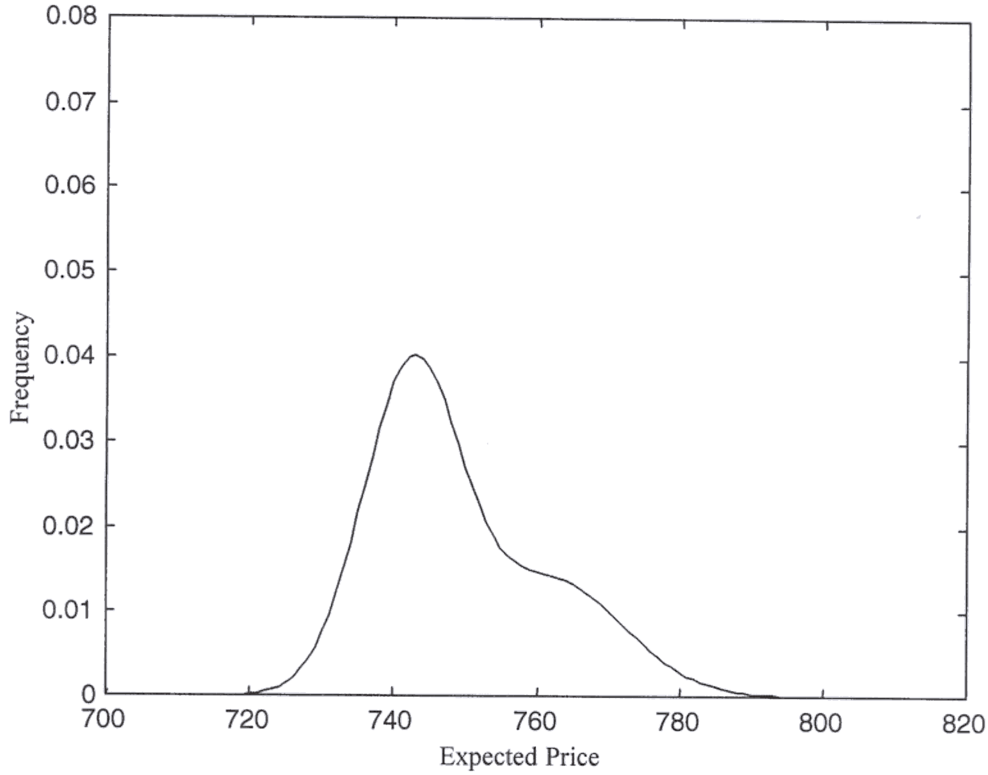

Figure 1
Estimated Expectations Distribution
Day 2

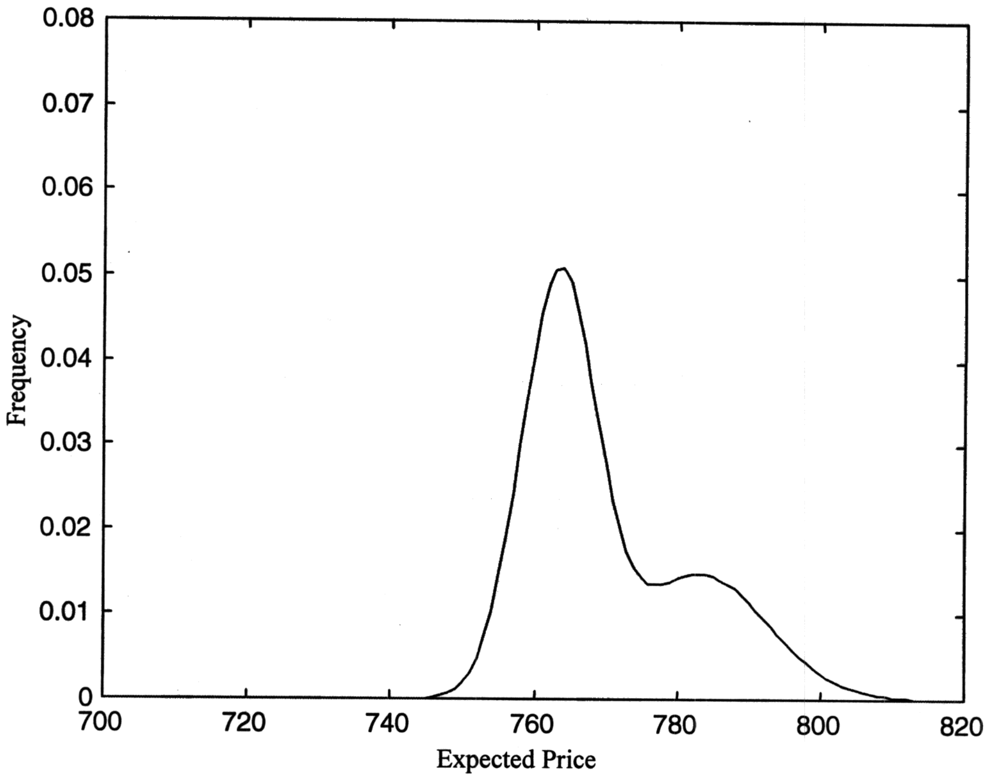

Figure 2
Estimated Expectations Distribution
Day 135

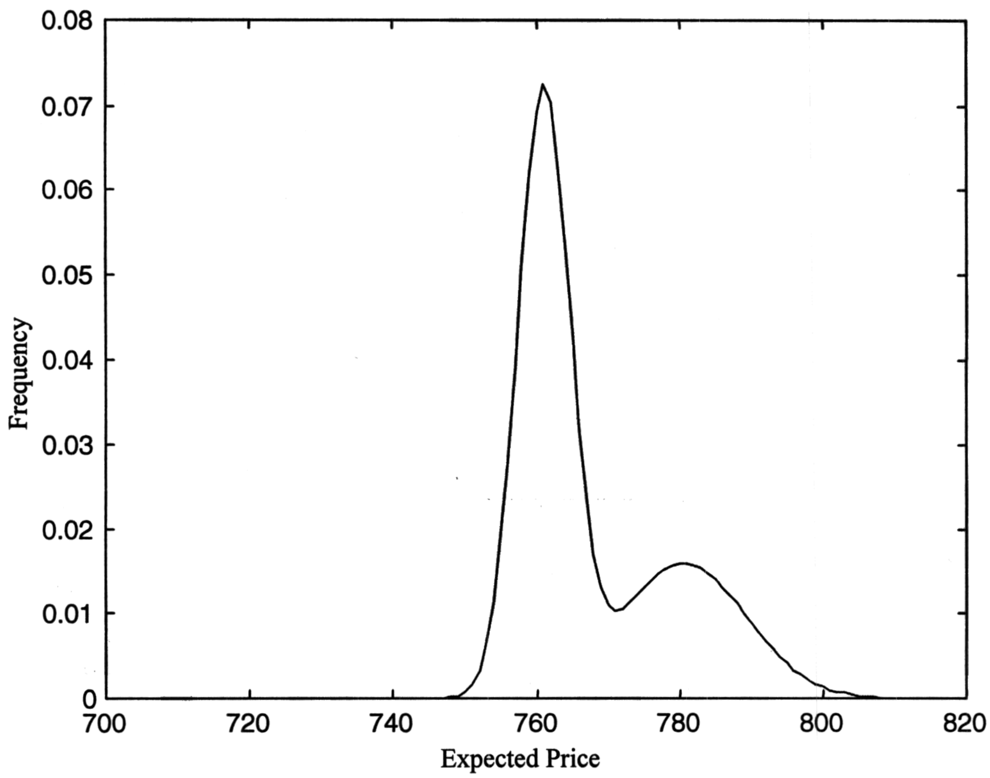

Figure 3
Estimated Expectations Distribution
Day 270

As noted by most authors, there may be several reasons for observed asymmetries in commodity prices. Unfortunately, the econometric methods commonly used in the price asymmetry studies do not allow us to refine the set of plausible explanations. The purpose of this paper is to consider an alternate approach to the asymmetry issue based on frequency domain time series methods. Although the frequency-based methods cannot conclusively determine the cause of all observed price asymmetries, we may be able to use this approach to further refine our research agenda.

Theories of Asymmetric Price Transmission

Borenstein *et al.* and Balke *et al.* discuss the competing theories of asymmetric price transmission. In general, the explanations offered in the literature may be roughly grouped in three categories: theories of local market power and search costs, inventory management costs, and tacit or explicit collusion among firms in an oligopoly.

Local market power and costly search

Grocery stores, meat processors, gasoline stations, and other wholesale and retail firms may enjoy local market power due to the lack of similar firms in a given neighborhood or region. Although customers of these firms face a finite number of choices, they may not be able to gather full information about prices offered by other firms due to the costs of search. In particular, a consumer may be unsure if a recent price increase at their local retail outlet was matched by the other retail firms. At the time, the consumer may simply pay the higher price to avoid making a costly search for a better price. Thus, retail firms can temporarily widen their profit margins by taking advantage of search costs.

Asymmetric costs of inventory management

Reagan and Weitzman and others have shown that firms may asymmetrically adjust prices due to the unequal costs of maintaining relatively high or low inventory levels. In general, the costs of experiencing a stockout are greater than the cost of carrying excess stocks. If upstream prices fall several periods in a row, the increase in quantity demanded may prevent firms from maintaining or replenishing inventories. To protect against a stockout, the firms may lower their prices at a slower rate than the decline in upstream prices Consequently, asymmetric price transmission may reflect the difference in inventory costs.

Cooperative Oligopolies

Suppose a few dominant firms explicitly or tacitly cooperate to maintain an effective cartel in an industry. The traditional kinked-demand oligopoly predicts sticky prices, and other features of the market may provide plausible reasons for asymmetric price adjustment. For example, even if market conditions are ideal for collusive behavior (highly inelastic demand and concentrated supply), cartels are difficult to maintain due to the incentive for one firm to cheat on the group. To maintain market power, the collusive firms can use trigger prices, defined market areas or shares, industry associations, or other means to identify cheating behavior. Suppose the firms set a trigger price that serves as a minimum for the cartel. If a firm attracts excess market share by setting its price too low, the other firms will punish the cheating firm in some way. As upstream prices rise, firms can quickly raise their prices to maintain profit margins without fear of punishment. However, as upstream prices fall, firms may hesitate to lower prices too quickly in order to avoid punishment.

At this point, we emphasize that other plausible explanations have been offered in the literature. In most cases, one or more of the possibilities may be eliminated by referring to our knowledge of the relevant markets or institutions. For example, the inventory theory is not applicable to livestock or other non-storable commodities. However, we are often left with two or more competing theories that cannot be empirically distinguished with the methods commonly used in practice. In the following section, we argue that some of the alternatives may be distinguished if we consider firm responses to quickly or slowly evolving cycles in upstream prices. The proposed method of analysis focuses on the frequency of price cycles and is commonly known as spectral analysis.

Price Cycles and Firm Behavior

In time series analysis, prices or other observed outcomes of a stochastic process may be viewed in terms of their frequency of occurrence as well as their order of occurrence. Although the time series methods commonly used in economic research focus on events in the time domain, a large literature on frequency domain methods has evolved in statistics and econometrics. Before discussing the competing theories asymmetric price adjustment from a frequency domain perspective, we first review the concepts of periodic behavior and the frequency of the stochastic cycles.

Under Cramer's spectral representation theorem, a mean-zero covariance stationary random process Y_t may be composed as the sum of periodic or cyclic components of different frequencies, $\omega \in [-\pi, \pi]$. In the context of prices, high frequency (ω near π or $-\pi$) price cycles occur rapidly and are associated with transitory shocks to the market or economic system. In contrast, low frequency (ω near zero) price cycles represent slow moving changes in the system. Some prominent economic phenomena have been analyzed in terms of the frequency of related events. For example, Engle (1974) noted that the permanent income hypothesis implies that the marginal propensity to consume is different for permanent (low frequency) and transitory (high frequency) changes in income. To examine empirical support for the claim, Engle estimated the marginal propensity to consume for high and low frequency income data. Interestingly, the estimated marginal propensity parameter was not significantly different across the frequency regimes, which contradicts the permanent income hypothesis.

Other contributions to the economics literature suggest that the competing theories of asymmetric price transmission may be distinguished in the frequency domain. First, consider firm behavior in locally imperfect markets with search costs. For high frequency price cycles, firms can quickly raise prices as upstream prices rise but slowly reduce prices as the upstream price declines. Given the transitory nature of the change, consumers cannot afford to search for a better price and are caught paying higher prices in the short-run. Alternatively, firms cannot widen their margins if prices fall slowly and consumers have time to search for a better price. Thus, the presence of search costs in locally imperfect markets implies that asymmetric price transmission may occur for high (but not low) frequency price changes.

Regarding the inventory management theory, Blinder finds that firms allow inventories to build or decline for short periods of time as prices change in transitory fashion. However, firms alter their behavior in response to sustained price movements that cause inventories to increase or decline over subsequent periods. Thus, firms adjust their inventory management practices in response to low (but not high) frequency price cycles. The principle reason firms do not respond to transitory price changes are menu costs, or the costs of adopting new price or quantity strategies. A large number of studies in the macroeconomics literature find that small menu costs can result in wages or other prices that are "sticky" in the short-run. Further, Reagan and Weitzman find that the low frequency responses are asymmetric due to the higher cost assigned to low inventory situations. Consequently, asymmetric price transmission occurs for low (but not high) frequency price cycles under the inventory theory, and the expected pattern of asymmetric adjustment is exactly opposite the search cost case.

Price Transmission in the U. S. Pork Market

The U.S. markets for pork, beef, and other meat products have received considerable attention from researchers over the past three decades. The efficiency of price transmission in the markets is a common theme, perhaps due to the concentration of the meat processing and retail food marketing sectors. Recent reports indicate that the domestic pork-packing industry has a four-firm concentration ratio of 53%, and Ward finds that local four-firm concentration ratios can exceed 70%. As well, livestock are non-storable commodities subject to biological production lags, and the short-run supply curves for meat are highly inelastic. Consequently, livestock producers are unable to adjust production in response to transitory price changes, and some have expressed concern that they are at the mercy of downstream firms in the short-run.

Data for this study are weekly observations of pork prices at the farm, wholesale, and retail levels for 1981-95.[1] The farm-level data are interior Iowa-southern Minnesota live hog prices. The wholesale and retail series are composite prices formed as weighted averages from the major meat cuts. To conduct the time domain and spectral analyses, we must first

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The data were generously provided by Professor Ted Schroeder, Kansas State University remove significant trend and seasonal components from the data. Graphically, the pork price series exhibit slight linear trends, and a linear trend variable was statistically significant in least squares regressions for each price series. To remove the linear trend, we use the first-differences of the pork prices. We also conducted time domain tests for the presence of seasonal effects by regressing the prices on quarterly, monthly, and weekly dummy variables, which were not statistically significant.

Next, we test the stationarity of each price series using the augmented Dickey-Fuller (ADF) and the Enders-Granger tests. The latter hypothesis test is designed to overcome power problems with ADF and other stationarity tests in the presence of asymmetric responses. The adjusted critical values for the ADF tests appear in Table A of Enders, and the T-Max* and F critical values are provided in Tables 2a and 2b by Enders and Granger. The test results are presented in Table 1 and provide strong evidence that the price series are stationary.

We use a vector autoregressive (VAR) process to model the farm, wholesale, and retail pork prices. In general, a VAR model for an m-vector of observations may be stated as

(1) $$\mathbf{y}_t = \Phi_0 + \Phi_1 \mathbf{y}_{t-1} + \quad + \Phi_k \mathbf{y}_{t-k} + \varepsilon$$

For the present study, the vector $\mathbf{y}_t$ is $[\Delta R_t, \Delta W_t, \Delta F_t]'$, where R_t, W_t, and F_t are the retail, wholesale, and farm pork prices, respectively. Equation (1) may be viewed as a reduced-form model in which all lagged dependent variables on the righthand side are predetermined, which simplifies the estimation task. The model may be derived from a structural representation of the VAR in which the contemporaneous observations may appear on the righthand side of (1).[2] To determine the order (number of lags) for the pork VAR model, we choose the common number of lags k to minimize the Akaike Information Criterion (AIC) for the system of VAR equations. We identified $k = 4$ as the optimal order for the VAR model. Although other authors have reported longer lags, especially in the wholesale-retail relationship, the following results were not significantly affected if we include higher order lag terms.

To determine the causal structure of the model, we conduct Granger causality and block exogeneity tests. The results of the tests are presented in Table 2. We find that farm prices strongly cause wholesale and retail prices, wholesale prices weakly cause retail prices, and there is no significant feedback from the downstream prices to the upstream markets. Although ordinary least squares is typically used to estimate VAR models, the estimator is inefficient if we impose the causality restrictions (i.e., the set of explanatory variables in each equation are not identical). We use the seemingly unrelated regressions (SUR) estimator to

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[2] If the price series are non-stationary and cointegrated, the long-run restriction on the VAR model may be imposed with an error-correction mechanism.

compute the SUR estimates of the VAR model parameters. The estimated model parameters are reported in Table A. The model exhibits reasonably good fit to the pork price series. Although the results are not presented to save space, the diagnostic statistics and hypothesis test results provide further evidence in support of the stated VAR model.

Time Domain Tests for Asymmetry

To evaluate empirical support for the symmetry of price response among levels in the pork marketing channel, we restate the VAR model (1) as

$$(2) \qquad \mathbf{y}_t = \Phi_0 + \Phi_1^+\mathbf{y}_{t-1}^+ + \Phi_1^-\mathbf{y}_{t-1}^- + \ldots + \Phi_k^+\mathbf{y}_{t-k}^+ + \Phi_k^-\mathbf{y}_{t-k}^- + \varepsilon_t$$

Equation (2) is a special case of a threshold autoregressive process that allows for asymmetric response to increases or decreases in $\mathbf{y}_t$ for preceding periods. The lagged dependent variables are defined as $\mathbf{y}_{t-j}^+ = \max(\mathbf{y}_{t-j}, \mathbf{0})$ and as $\mathbf{y}_{t-j}^- = \min(\mathbf{y}_{t-j}, \mathbf{0})$, and Φ_j^+ and Φ_j^- are conformable (m x m) parameter matrices with the causality restrictions imposed. The pork VAR model is based on first-differences of prices, and the $\mathbf{y}_{t-j}^+$ and $\mathbf{y}_{t-j}^-$ variables represent positive and negative changes (respectively) in retail, wholesale, and farm pork prices.

Classical hypothesis tests of price transmission symmetry are conducted by comparing the response parameters for price increases and decreases. In terms of the VAR model stated in Equation (2), we consider joint null hypotheses of the form

$$(3) \qquad \mathbf{H}_0\text{: } \Phi_k^+(m, m+1) = \Phi_k^-(m, m+1) \text{ for m of interest and for each k}$$

The hypotheses may be imposed on the asymmetric VAR model as a set of k = 4 linear restrictions $\mathbf{c}\Phi = \mathbf{0}$, and the associated Wald test statistic

$$(4) \qquad \text{Wald} = \hat{\Phi}'\mathbf{c}'\left[\mathbf{c}\text{Var}\left(\hat{\beta}\right)\mathbf{c}\right]^{-1}\mathbf{c}\hat{\Phi}$$

is asymptotically $\chi^2(k)$ under the null assumption (3).

We report the Wald test results in Table 3 in the row labeled "Full sample". In the retail equation, the response of retail pork prices to changes in the wholesale price is not significantly asymmetric (the observed p-value is 0.53). Conversely, we reject the symmetry hypothesis in the farm-wholesale margin for tests conducted at levels greater than 0.055. In summary, we find no evidence of asymmetries in the wholesale-retail margin and marginally significant asymmetries in the farm-wholesale margin.

Band Spectrum Regression

To examine the symmetry of price transmission for high and low frequency price cycles, we use an estimation method known as band spectrum regression. The method was formally introduced and developed by Engle (1974, 1980), and a comprehensive discussion of the method is provided by Hylleberg (Section 4.3). Although spectral regression methods are not widely used, the required steps are relatively straightforward: 1) convert the price data to the frequency domain by a Fourier transformation, 2) select the band of high or low frequency observations of interest, and 3) use the observations for this band to estimate the model parameters. By analogy to time domain regression, note that a simple way to model time-varying behavior is to estimate separate regression models for subsets of the sample period. In band spectrum regression, the same idea is applied by estimating the model parameters for subsets (bands) of the frequency domain rather than subsets of the time domain.

For present purposes, we rewrite the VAR model (2) as a linear regression model of the form $\mathbf{y} = \mathbf{X}\Phi + \varepsilon$. The time domain observations $\mathbf{y}$ and $\mathbf{X}$ may be converted to frequency domain observations (indexed by $\omega_j = j\pi / n$ for $j = 1,\ldots,n$) with a discrete Fourier transform, $\mathbf{Wy}$ and $\mathbf{WX}$. The transformation matrix $\mathbf{W}$ (known as the Fourier matrix) is orthogonal, and the transpose (inverse) of the matrix $\mathbf{W}'$ can be used to convert frequency domain observations back to the time domain. To identify the frequency band of interest, let $\mathbf{A}$ be an (n x n) diagonal matrix with elements $a_{ii} = 1$ for $i = [\omega/2\pi]$ (i.e, integer portion of the ratio) defined with respect to the frequencies of interest, $\omega_1 \le \omega \le \omega_2$. Then, the band spectrum regression estimate is based on the transformed data, $\mathbf{y}^* = \mathbf{AZy}$ and $\mathbf{X}^* = \mathbf{AZX}$.[3]

The band spectrum regression estimator is simply $\tilde{\Phi} = (\mathbf{X}'\mathbf{Z}'\mathbf{AZX})^{-1}\mathbf{X}'\mathbf{Z}'\mathbf{AZy}$, and the estimates are used to compute the Wald test statistics (4). Under appropriate regularity conditions, the estimator satisfies the Gauss-Markov Theorem in small (finite) samples and is consistent and asymptotically normal and efficient in large samples. Due to the orthogonality property of the Fourier matrix $\mathbf{W}'\mathbf{W} = \mathbf{I}_n$, the estimator reduces to the ordinary least squares regressor if we select the full set of frequencies (i.e., $\mathbf{A} = \mathbf{I}_n$).

Band Spectral Tests of Asymmetry

To test the symmetry of response at various frequencies, we first divide the frequency domain in four overlapping subsets, [0, 0.2], [0.1, 0.3], [0.2, 0.4], and [0.3, 0.5]. The frequency bands are associated with weekly cycles lasting at least 5 weeks (lowest frequency), 3.33 to 10 weeks, 2.5 to 5 weeks, and 2 to 3.33 weeks (highest frequency). We identify the bands by the mid-points of the frequency intervals (0.1, 0.2, 0.3, and 0.4), which correspond to 2.5, 3.33,

[3] The frequency-based data may be complex-valued under some Fourier transformations. Engle (1974) and Harvey (1980) discuss alternate means of forming real-valued data that may be used in most regression packages. We use the transformation recommended by Harvey.

5, and 10 week cycles. The hypothesis test results are presented in Table 3. For the retail pork price equation, we find that the response to changes in the wholesale price is significantly asymmetric for frequency bands with midpoints $\omega = 0.1$ and $\omega = 0.2$. Recall that the time domain Wald test could not reject the symmetry hypothesis for the wholesale-retail margin. Thus, the asymmetric response at low frequencies is masked by the symmetric response at high frequencies in the time domain test.

Given our discussion of the competing theories of asymmetric behavior, the observed pattern is not consistent with the presence of search costs in locally imperfect markets but may be explained by the other theories. As previously noted, the inventory management theory is not directly applicable to livestock and other non-storable commodities, but much of the pork sold at the retail level is processed (e.g., bacon, sausage, cured hams) and may be stored for several weeks. Second, the observed symmetry in price transmission for high frequency changes may be consistent with a cooperative retail oligopoly in the presence of significant menu costs. For typical retail foodstores with electronic inventory (scanner) systems, the costs of changing prices are not small and only about 16% of the food prices change in the average week (Levy, *et al.*). Consequently, retailers may not respond to high frequency (transitory) changes in wholesale prices, but they may exhibit asymmetric response to longer-lived price cycles. Although the spectral methods do not allow us to further distinguish between the inventory or cooperative oligopoly theories, we can firmly reject the search cost theory.

Regarding the farm-wholesale margin, the band spectral tests confirm the asymmetries observed in the time domain test. From Table 3, we find that the p-values for each of the Wald test statistics are much lower than the Type I error rates commonly used in practice. Consequently, the farm-wholesale margin is asymmetric at all frequencies, which conflicts with the search cost and inventory management theories of asymmetric price response. Although we emphasize that our rejection of two theories does not automatically imply that we should "accept" the remaining theory, we note that other studies (e.g., Koontz, Garcia, and Hudson) present evidence of cooperative behavior among beef packers. The test results suggest that a similar study of the wholesale pork sector is warranted.

Concluding Remarks

In this paper, we use band spectrum regression to estimate the symmetry of farm-wholesale-retail price transmission for high and low frequency changes in pork prices. The band spectral test results indicate that traditional time domain methods can mask underlying asymmetries that can occur in subsets of the frequency domain. In particular, we are unable to reject the null hypothesis of symmetric price transmission in the wholesale-retail price margin based on the time domain (full sample) results. The band spectral tests also indicate that retail price changes are significantly asymmetric for low frequency cycles in wholesale prices.

Further, the spectral evidence presented in this paper indicates that the observed asymmetries in the wholesale-retail margin are not consistent with search costs or other theories that imply asymmetries at high frequencies. Conversely, the farm-wholesale margin is asymmetric at all frequencies, which is not consistent with search costs, inventory management, or other theories that imply asymmetries at high or low frequencies (but not both). Again, we strongly emphasize that our rejection of two of the three competing theories does not imply that we should automatically accept the remaining alternative. The advantage provided by the spectral methods is the ability to eliminate explanations that may be plausible but are not empirically supported in the frequency domain. As such, the search for reasons underlying asymmetric price transmission may continue with a refined set of objectives.

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Table 1. Stationarity Test Results

	Retail	Wholesale	Farm
Augmented Dickey-Fuller	-3.65	-3.85	-4.88

(Reject unit root with drift or trend at 5% level if t-ratio < -2.68)

	Retail	Wholesale	Farm
Enders-Granger T-Max$_T$*	-21.252	-20.159	-19.595

(Reject unit root with drift or trend if the largest t-ratio in absolute value falls outside the interval (-3.60, -0.76) for a 5% level test)

Table 2. Causality Test Results (asymptotic p-values in parentheses)

Granger Causality (row variable Granger-causes the column variable)

	Retail	Wholesale	Farm
Retail		2.527	0.385
		(0.0386)	(0.819)
Wholesale	2.695		0.411
	(0.0299)		(0.801)
Farm	3.412	36.886	
	(0.0089)	(0.0000)	

Block exogeneity (significance of the variable in the other equations)

Retail	Wholesale	Farm
13.564	12.812	192.516
(0.938)	(0.1185)	(0.0000)

Table 3. Wald test statistics for symmetry of the band spectrum regression models

Midpoint of the Frequency Band	Symmetry of the Retail-Wholesale Equation	Symmetry of the Wholesale-Farm Equation
10 weeks (ω=0.10)	10.153	28.884
	(0.0379)	(0.0000)
5 weeks (ω=0.20)	12.591	11.739
	(0.0135)	(0.0194)
3.33 weeks (ω=0.30)	1.290	16.212
	(0.8631)	(0.0027)
2.50 weeks (ω=0.40)	1.576	34.416
	(0.8131)	(0.0000)
Full sample	2.727	9.341
	(0.605)	(0.0531)

(Asymptotic p-values are in parentheses below the observed Wald test statistics)

Table A Estimated VAR, symmetry and causality imposed (t-ratios in parentheses)

		ΔW_t	ΔF_t
Constant	0.0331	-0.0027	0.0022
	(0.129)	(0.058)	(0.053)
ΔR_{t-1}	-0.720		
	(20.18)		
ΔR_{t-2}	-0.513		
	(12.00)		
ΔR_{t-3}	-0.340		
	(7.92)		
ΔR_{t-4}	-0.147		
	(4.11)		
ΔW_{t-}	0.176	-0.278	
	(0.757)	(7.73)	
ΔW_{t-2}	0.716	-0.176	
	(2.98)	(4.71)	
ΔW_{t-3}	0.103	-0.098	
	(0.43)	(2.68)	
ΔW_{t-4}	0.371	-0.0031	
	(1.62)	(0.087)	
ΔF_{t-1}	0.393	0.420	0.418
	(1.49)	(9.10)	(11.61)
ΔF_{t-2}	-0.960	0.318	0.116
	(3.39)	(6.44)	(2.98)
ΔF_{t-3}	0.393	0.0320	-0.050
	(1.35)	(0.63)	(1.28)
ΔF_{t-4}	0.185	-0.007	-0.055
	(0.67)	(0.15)	(1.53)
System R^2		0.554	
Equation R^2	0.368	0.167	0.216