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THE TERM STRUCTURE OF UNCERTAINTY IMPLIED BY OPTION PREMIA

John A. Shaffer and Bruce J. Sherrick*

Abstract

Information describing future asset price distributions is fundamental to nearly all risk management activities. Futures markets are often relied upon to provide information about the mean of future price distributions, and option markets are often used to recover measures of volatility. In cases where multiple maturities of an underlying asset trade, techniques for inferring implied forward price levels have been generally found to perform well under empirical scrutiny. Although implied forward price levels are well-understood, less information is readily accessible about other moments of a future asset price distribution or about the future time path of uncertainty. There is strong and widely-accepted empirical evidence that asset prices do not follow constant volatility models but instead contain intervals of relatively increased and subdued information events (Heynen, et al.). Techniques for recovering implied forward volatility estimates are conceptually equivalent to recovering implied forward price levels, yet little empirical work doing so currently exists. This paper demonstrates a simple technique for inferring implied forward volatilities of asset prices from options data. Using hog futures options market data, the implied term structure of uncertainty is recovered and used to examine the constancy of forecasted volatility, and the expected significance of USDA Hogs and Pigs Report releases. The results indicate a non-constant term structure of uncertainty, yet demonstrate little impact on forecasted volatilities in periods that contain USDA Hogs and Pigs Reports.

Introduction

Commodity futures markets have been generally found to provide accurate information about future price levels, with the accepted convention that the current futures price is the market's estimate of the mean of an expected future price distribution. Option markets contain additional information about the distribution of the underlying asset future prices, and have been frequently used to infer information about the volatility of an expected future price distribution.

The most common application of this concept involves the use of Black-Scholes style option models inverted to recover an implied volatility (IV) or implied standard deviation (ISD) that results in the market price equaling the model price. This model parameter is taken to be the collective market expectation about the underlying asset's future price variance. In conjunction with the estimate of the mean and an assumed parametric form, the complete future price distribution can often be described.

Generalizations to the B-S approach have been numerous. Cox and Ross and others have noted that under fairly innocuous no-arbitrage assumptions, an option value can be expressed as

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the discounted expected value of its payoffs. The expectation is taken over a risk-neutralized valuation measure (the RNVM is the distribution describing both the underlying asset and the probabilities of the option's payoffs), and requires no particular assumptions about the time path of the underlying asset's price. Although the identification of a unique RNVM remains somewhat controversial, the implied distributions recovered from these general approaches also seem to describe market expectations reasonably well.

At any given time, multiple futures contracts trade on an underlying asset, differing only by their expiration dates. And, for each expiration contract, associated options also trade which contain information about future price distributions at each of the expiration dates. This market structure of trading assets whose prices contain assessments of information in partially overlapping intervals of time permits the recovery of a term structure of volatility. From this term structure of volatility, implied forward measures of volatility can be computed, and the forward volatility decomposed into potentially nonconstant intervals.

The term structure of uncertainty described by a complex of implied volatilities, is analogous to the term structure of interest rates as commonly understood.¹ As the term structure of interest rates is often used to derive implied forward rates -- or rates between distinct future points in time -- a term structure of volatility allows future volatility to be separated into different time intervals. This approach provides a unique view of the constancy of volatility and allows the impact of events that are expected to have disproportionate effects on the resolution of future uncertainty to be examined directly in terms of market expectations.

This study first uses a variety of common option-pricing models to recover implied distributions of underlying asset prices. The impact of differing modeling assumptions is examined briefly in the choice of the model used to describe future price distributions at a given point in time. These estimates of future price distributions are used to construct implied forward volatility "buckets" and the expected rates of resolution of uncertainty examined. Simple tests of the constancy of the implied forward volatilities are conducted. Then, in light of the considerable literature examining the informational content of USDA Report releases, the implied forward volatility intervals containing these reports are considered for evidence of the impact of known report releases on the market's expectation of volatility. A brief summary then concludes the paper.

¹ Heuristically, if the (average) two period interest rate is 10% and the one period interest rate is 9%, then the implied one period forward one period rate is 11% ignoring compounding. More generally, from a set of today's spot interest rates lasting i periods into the future, $r_{0,i}$, $i = 1, 2, \dots, n$, implied forward rates between time period k and n can be inferred as:

$$r_{k,n-k} = \left[\frac{(1 + r_{0,n})^n}{(1 + r_{0,k})^k} \right]^{(n-k)}$$

No-Arbitrage Option Pricing Model

Cox and Ross present the implications of no-arbitrage conditions and derive a general description of option prices as the discounted expected future payoffs against a risk neutral valuation measure (RNVM) that governs the outcomes of the underlying asset price. Adopting those conventions, equations (1) and (2) below describe the values of European call and put options respectively under these assumptions:

$$V_c(x) = b(T) \int_0^{\infty} \max(0, F_T - x) g(F_T) dF_T \quad (1)$$

$$V_p(x) = b(T) \int_0^{\infty} \max(0, x - F_T) g(F_T) dF_T \quad (2)$$

where V_c and V_p are the prices of call and put options, x is the strike price of the call or put option, F_T is the price of the underlying asset, T is the time to expiration, $b(T)$ is the discount factor, and $g(F_T)$ is the market expected probability density function of the asset price F_T for time T in the future. These equations simply state the current option price as its discounted expected future payoff with the expectation taken over $g(F_T)$. As has been noted by numerous previous authors, if $g(F_T)$ is lognormal, the equations are equivalent to Black-Scholes option pricing models (Fackler and King, Garven, McNew and Espinosa).

Given a RNVM and the current discount rate, one can estimate the appropriate price of a call or put option, which can be compared to the market price. Or, as more commonly employed, given an option price and the discount rate, one can recover the implied RNVM, under the assumption that the market's price is determined by the model (Fackler and King, Sherrick, et al. McNew and Espinosa). With this RNVM one has the implied probability measure in equations (1) and (2).

This process of recovering complete probabilistic descriptions of future asset prices differs slightly from traditional B-S option pricing applications in two ways. First, the underlying mean is not restricted to equal the current asset price or a known function of current price, and there are no particular assumptions about the underlying price process. Secondly, this procedure permits simplified combination of information from multiple strike prices and/or options to recover the underlying price distribution. While previous research has investigated which strike price, or set of strike prices, should be used to provide the best implied volatility (Turvey, for example), this paper makes no specified assumptions about which strikes might provide the best "fit" or predictive ability.

Previous research has investigated numerous parameterizations of the RNVM including lognormal, stable Paretian, Burr XII, Burr III, gamma, Weibull, and exponential distributions. Herein, three different distributions are considered -- lognormal, Burr XII, and Burr III --- and both European and American approximations are estimated. This final choice of modeling

approaches is treated as a purely empirical issue in the context of recovering implied price distributions.

The following objective function is used to recover the implied distributional parameters for the European specifications (Fackler; Sherrick et al.; McNew and Espinosa):

$$\text{Min}_{\beta} \left[\sum_{j=1}^k \left(V_{p,j} - b(T) \int_0^{x_j} g(Y_T | \beta) (x_j - Y_T) dY_T \right)^2 \right] + \sum_{j=1}^l \left(V_{p,j} - b(T) \int_0^{x_j} g(Y_T | \beta) (x_j - Y_T) dY_T \right)^2 \quad (3)$$

where β is the parameter vector for the particular distribution, $V_{c,i}$ and $V_{p,j}$ are the observed option premia, x_i and x_j are the call or put strike prices, and k and l are the number of calls and puts for a particular day. This equation is solved for each day's data using nonlinear least squares methods in Mathematica.

In addition, a modified estimation procedure is incorporated to allow for early exercise (i.e. American options). This estimation uses the Barone-Adesi and Whaley model in a context which allows for the simultaneous estimation across multiple strikes. The procedure begins with the implied volatility estimate generated using the lognormal specification of (3). Because the early exercise premium cannot be negative, the American implied volatility must be less than or equal to the European implied volatility. The procedure searches iteratively for the volatility that provides the best fit to the actual premia across all strikes and option types.

Forward Implied Volatility

Previous work on the term structure of volatility can be roughly categorized into two general areas. First, there is a body of work that estimates IVs through time and then examines the time-series properties of these estimates. Among those, Stein develops a simple mean reverting model that separates S&P 500 Index volatility into a short- and long-term component. Rational expectations dictates that a longer maturing option's average ISD should move less on a percentage basis on a given day than a shorter maturing option's ISD due to the impact of the removal of a single day's uncertainty. The author concludes that longer term options "overreact" to changes in short term volatility as the implied change in the average does not adequately describe volatility behavior. Heynan, et al. extends Stein's framework by incorporating GARCH and EGARCH models of a term structure of volatility. They find an EGARCH specification to be superior to the simple mean reversion model suggested by Stein.

The other general area of research considers the term structure by comparing volatility for different time intervals. Resnick, et al. use the term structure of volatility to test the January effect of different returns for small and large firms. They utilize monthly estimates of volatility for different sized firms to compare differences by month and also by firm size. Their use of IV is somewhat indirect, but is important in that it confirms the existence of a volatility conditioned January effect for small firms. Xu and Taylor consider other methods for inferring forwards from the relationship among a complex of volatilities. Ederington and Lee likewise investigate

the implied forward uncertainty and employ the relationship between marginal and average rates in a series of intervals differing by one day's information to test for differences in the quantity of uncertainty resolved in a given day.

After volatility estimates for various future intervals are estimated at a point in time, implied volatilities for the non-overlapping time intervals can easily be calculated. Just as implied forward interest rates represent the rate of interest implied in a yield curve between two distant points in time, forward implied volatility can be extracted from the set of volatilities estimated at a point in time for assets with differing expirations. If variance is additive, it is a straightforward calculation to compute a forward implied volatility from a set of implied volatilities. In this case, the following is used to represent the forward-implied volatility (or standard deviation) between periods $t_{n-\alpha}$ and t_n (Taleb):

$$\sigma_{n-\alpha, n} = \sqrt{\frac{\sigma_{t_0, n}^2 (t_n - t_0) - \sigma_{t_0, n-\alpha}^2 (t_n - \alpha - t_0)}{t_n - t_n - \alpha}} \quad (4)$$

This expression provides a convenient means to assess market "forecasts" of volatility for future periods of time. Forward implied volatility also provides a different perspective on the impact of scheduled news releases on volatility, or market expectations. Specifically, expected public news releases occur within specific forward "buckets", or intervals, of volatility. Utilizing this approach, the importance of news releases can be viewed from their effect on expected volatility. Considerable prior research exists on the impact or value of news releases as evidenced by changes in futures prices levels around news release dates (Colling and Irwin, Carter and Galopin). And, there is also considerable research on the changes in implied volatility around news release dates (Patell and Wolfson, Ederington and Lee, McNew and Espinosa). These works all examine the effect on prices or volatility in the immediate days prior to and following important public news releases. The approach taken herein is somewhat different in that it examines the expected impact on volatility of information releases before the release dates. In this context any change in volatility that will occur around a release date should be reflected in the term structure of volatility. It is a simple matter to test the equivalence of volatility in an interval which contains a news release from an interval without a known anticipated report release. For example, in crops, one may expect heightened volatility around planting, pollination, and harvest intervals relative to intervening periods. The approach herein could be used to disaggregate total uncertainty into forward intervals that would be expected to display non constant evolution through time.

Data

The data used are all daily Chicago Mercantile Exchange options and futures closing prices of live hogs for 1995-1996.² For each day, every contract traded was analyzed across all option settlement prices reported. To examine issues related to implied forward volatility, the contract months of February, April, June, July, and October were examined as a complex of

² Other contracts examined include S&P Index futures, soybeans, and corn. Only results related to live hogs are reported herein for space considerations.

trades. The August contract is excluded because this contract is much more thinly traded and has been historically viewed as less reliable than others. The December contract is not traded concurrently with the February contract, and thus, both contracts cannot be simultaneously included. The first twelve trading days of the calendar year are used to derive the implied volatility samples for comparison. The five contract months listed above are trading concurrently in that interval, and the two week interval before expiration of the Feb. contract is avoided to prevent major expiry effects in the sample period. The years 1995 and 1996 are tested separately.

The use of the five different contract months allows for separate time “buckets” or intervals of implied volatility to be compared. The set of shortest forward intervals initially compared includes January - February (JF), February – April (FA), April – June (AJ), June – July (JJ), and July – October (JO). Other longer intervals (e.g., June – October) can likewise be constructed and are more fully documented in Shaffer, 1999. The first interval contains the current, or spot level of implied volatility. The four other intervals are represented by forward-implied volatilities as defined in (4). The intervals FA, JJ, and JO contain USDA *Hogs and Pigs* report releases, while the JF and AJ intervals do not.

Table 1 shows the average number of puts and calls for the contracts used. The April contract contains the most inputs (or different option prices) to the no-arbitrage pricing model.

Table 1. Average Number of Puts and Calls per day

Contract	Calls	Puts	Total
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As expected, the October contract does not contain as many sample points, but there is no obvious reason to suspect that this reduced number of sample points will have any particular effect on the results. There is no obvious pattern between the number of calls and the number of puts over for the sample period across the different contract months.

Estimated RNVM Results

The results of this section are used to select the form of model used for the remainder of the paper and the construction of implied forward volatilities. First, the ability to fit to observed option prices is used under the assumption that observed prices are correct, and that the goal of

the option model is to permit accurate summaries of market expectations about future price distributions. Figure 1 shows the resulting estimated RNVs at four different points in time for the Dec. 1996 contract in PDF form. Figure 2 is simply the CDF representation of the same estimated distributions. These are available for every option trade-day and contract combination, and under three different European specifications as well as the modified American model.

The selection of a parameterization of the option models was treated as a purely empirical issue. Table 2 shows the minimum average absolute pricing error for the three different European and American (Whaley) models. The minimum average absolute pricing error is computed using the distributional parameters recovered during the estimation process. These distributional parameters are then used to obtain a “fitted” premium which is compared to the actual premium for each input premia in equation (3) above. The average absolute pricing error is the absolute value of the average across all inputs of these fitted premiums for a particular day and contract. In this case, the Burr 12 distribution most often provides the best fit.

Table 2. Number of Occurrences of Minimum Average Absolute Pricing Error

Contract	<i>European</i>			<i>American</i>
	Lognormal	Burr 3	Burr 12	Whaley
Feb-95	4	3	4	1
Apr-95	2	0	6	4
Jun-95	0	2	10	0
Jul-95	0	2	8	2
Oct-95	2	2	8	0
Feb-96	2	1	7	2
Apr-96	4	0	1	7
Jun-96	0	2	8	2
Jul-96	2		9	0
Oct-96	5	4	3	0
	2			

Table 3 confirms the performance of the Burr 12 by comparing the average absolute pricing errors for the 12 trading days. This table is constructed by simply averaging the average absolute pricing errors across the 12-day sample period.

Table 3. 12 Day Average of the Average Absolute Pricing Errors

Contract	<i>European</i>		<i>American</i>	
	Lognormal	Burr 3	Burr 12	Whaley
Feb-95			0.00437	0.00808
Apr-95			0.00424	0.00494
Jun-95			0.00196	0.00804
Jul-95			0.00182	0.00492
Oct-95			0.00037	0.00389
Feb-96			0.00291	0.01895
Apr-96			0.00738	0.00429
Jun-96			0.00313	0.00953
Jul-96			0.00450	0.00786
Oct-96			0.00079	0.00519
				0.00757

Because of these results, the statistical comparisons in the next section are provided only for the Burr 12 results. The results for the other distributions are virtually identical and do not provide additional insight into the central research questions.

Empirical Testing

Figure 3 shows annualized volatility time plots for each contract estimated daily from August 1994 through the end of 1996. As can be seen in the graph, there appear to be periods of varied volatility that are not commonly reflected across contracts, but instead affect individual contracts more significantly – especially nearer expiration. If there are periods of nonconstant volatility, then the options complex prior to maturity of the earliest dated contract can be used to infer the forward volatilities of intervening periods. To make the relationships more obvious, four individual series are separately treated in the following figures. Figure 4 shows the volatility per unit time (remaining average volatility) for Feb. and July 1995 and 1996 contracts. Figure 5 simply graphs these in terms of remaining total volatility against time to maturity to make the relationship to time more obvious.

While much could be done to next analyze the time series nature of the implied variances and/or other characteristics, the focus of this paper is instead only on the informational nature of the implied forward volatility measures that can be recovered from differences in volatility of options traded concurrently with differing expirations. Thus, to first test the constancy of volatility and the significance of news releases on the term structure of volatility, one-way ANOVA tests are first conducted on the five implied forward intervals described earlier. Resnick, Sheikh, and Song employ this method to test whether there are monthly differences in stock option volatility (a description of these intervals is given in the next section). The null hypothesis is that each of the interval's IV is equal to that over the entire sample. Similarly, the

ex ante volatility in intervals containing USDA HPR releases can be tested for equivalence to that in intervals not containing releases.

Table 4 presents the results of the hypothesis of equivalent volatility (for the JF Period) and implied-forward volatilities (for the other five time periods). The *F*-Statistics reported for each year allows an easy rejection of equal volatilities across the sample groups. The July-October has an implied volatility that is much lower than the other intervals. While this result could be due simply to data effects, it indicates expectation of less volatility resolution in that period given acceptance of market prices. If this time period is excluded, the hypothesis of equal volatilities across the other four time periods is still rejected at the 0.01 significance level.

Table 4. Average Volatilities

Year	JF	FA	AJ	JJ	JO	<i>F</i>
1996	9.97	8.95	8.27	7.89	5.55	60.54

1. *F*-Statistics are significant at 0.01 level.

The results of the two way *t* tests to compare the two sample groups, one which contains periods with a USDA *Hogs and Pigs* report and the other that does not, are shown in Table 5. Because the JO time period yielded such dramatically different results, this time period is not used to test the impact of the news release on forward volatility. The test is conducted with 30 observations in each set. For 1995 the average volatility is lower for the group that does not contain a news release. However, the p-value for the reported statistic is only 0.069. For 1996 the average volatility for the group containing no release is actually significantly higher than the group which contains a release.

Table 5. Average Volatilities and Reported *t* Statistics

Year	HP Report	No HP Report	<i>T</i>
		9.51	
1996	8.42	9.12	3.36

1. *t*-statistic significant at the 0.01 critical level.

Other factors which cause changes in perceived volatilities in addition to USDA *Hogs and Pigs* reports also influence relative volatility, and it may simply be that the effect is diluted enough to be difficult to detect. The other main USDA publications, the ERS *Hog Outlook* and USDA *Cold Storage*, could have an impact. *Hog Outlook* is also released quarterly, but in somewhat different time periods than the *Hogs and Pigs* report. The *Cold Storage* report is monthly so this would not have an impact in this context. Other seasonal factors could also be

important, such as grain planting and harvest (where supplies to market can be disrupted) and seasonal demand changes for pork products.

Summary

Daily option data on a complex of live hog futures were used to estimate market-based expected price distributions. These estimated distributions were recovered under various model specifications for use in examining the implied forward volatility of prices. The term structure of volatility was demonstrated selected contracts from 1995 – 1996. Simple tests of equivalence indicate that the market forecast of volatility (implicit in the term structure of volatility) is not constant. With volatility segmented into different time periods using the forward volatilities, significantly different levels of volatility are obtained. In addition, time periods which contain significant public news releases do not appear to contain higher levels of forecasted volatility.

Future work develops simple trading strategies based on the relationship between average and marginal volatilities and uses these as indirect measures of market performance. The lack of storability of the underlying, in this case hogs, prevents implications of arbitrage, but similar results appear to hold in the S&P and within market years for corn and soybeans. In the latter two contracts, these methods provide a simple mechanism to deconstruct volatility into intervals covering potentially different types of risks within the growing season.

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Figure 1. Implied PDFs - Dec. 1996 contract

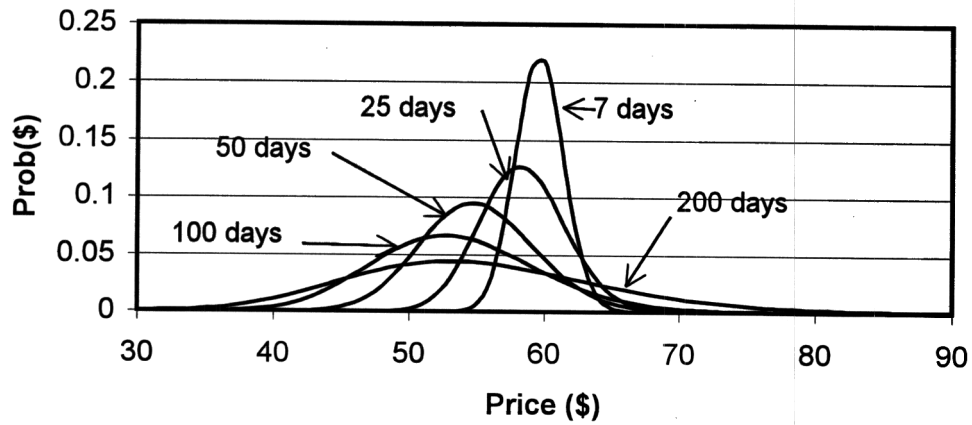


Figure 2. Implied CDFs by TTM - Dec. 1996 contract

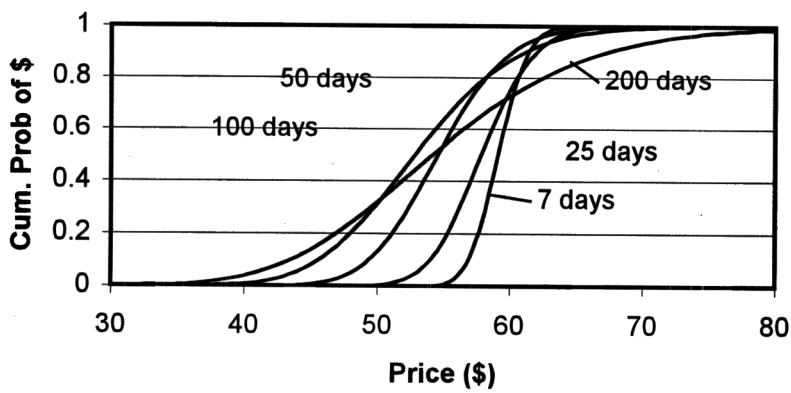


Figure 3. Annualized Volatility

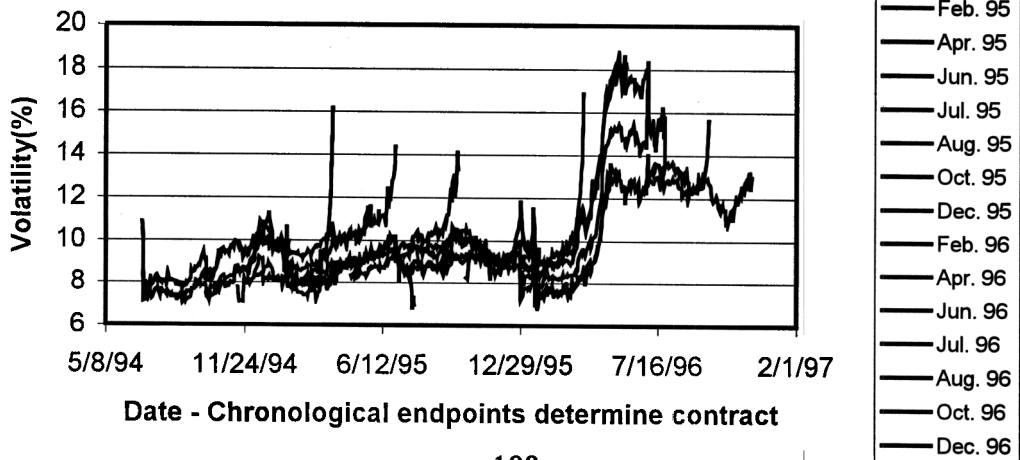


Figure 4. Volatility per unit time

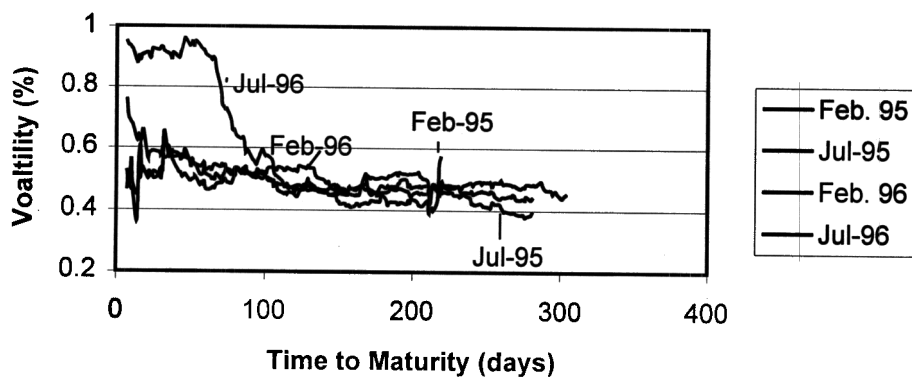


Figure 5. Volatility by TTM

