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### A Full Bayesian Analysis of Structural Changes with the AIDS Model: The Case of Meat Demand

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This study applies a Bayesian methodology to the oft-examined issue of whether a structural change has occurred in U.S. meat demand. The Bayesian approach allows us several advantages over earlier studies. First, we can estimate the true AIDS model instead of the commonly substituted linear-approximate version. Second, we derive the marginal posterior distribution of the time period in which the structural change occurred. This provides greater insight into whether the data is sharply or mildly informative as to such a structural change. Finally, we can derive posterior density results for the changes in parameters and elasticities across the two regimes. The results show the data to be highly informative with respect to a structural change in meat demand in the fourth quarter of 1975, but less evidence is found for changes in the majority of individual model parameters.

#### Introduction

Changes in consumption patterns for meat have been a major topic of study in the literature of food demand estimation. Those changes have been attributed to different factors. Some studies (Marion, 1986 p145) associated the changing pattern with a decrease in the relative price of poultry with respect to red meat. The main reason given for such a decline appears to be a higher rate of technical progress in the poultry industry than in the red meat industry. Others argue (Chavas, 1982) that health campaigns that encouraged consumers to avoid red meat because of its high content of fat and cholesterol, which could cause heart attacks, high blood pressure, and other health problems, have reduced the preference for red meat by the average American consumer. Still others believe that an increase in the opportunity cost of time of the consumer has moved consumption toward convenience food away from home, and that change has been detrimental to at-home meat consumption.

Many studies have used a point null hypothesis to test for structural changes in meat demand, including Nyankory and Miller (1982), Frank (1984), Cornell and Sorenson (1986), and Dahlgran (1986). Other papers have contested the use of parametric model for determining structural changes. Alston and Chalfant (1991) show that by using a parametric model one can detect structural changes in beef demand almost 100% of the time in a system in which, by construction, no such changes exist. According to Sakong and Hayes (1993), inferences drawn from parametric models are conditional on the assumption of functional form that might affect the outcome. To the contrary, inferences drawn in nonparametric methods

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involving testing the data with the weak axiom of revealed preference (WARP) avoid making assumptions about the distribution of the data. However, WARP is still conditional on the assumption of local nonsatiation of the utility function, which might not be true for some consumers. The question "What is the best or most powerful test?" is crucial to justifying the validity of any testing procedure. A major drawback of nonparametric inference is the lack of statistical power in hypothesis testing. Most studies that assumed parametric models for testing structural changes in meat demand have used the LAIDS model to avoid difficulties involved in the AIDS model evaluation. However since the LAIDS model is not equivalent to the AIDS model, using the LAIDS model to test changes in AIDS model parameter might be misleading.

We propose a full Bayesian parametric method to analyze structural changes in meat demand using the AIDS model. A Bayesian data analysis can be viewed as practical for making inferences from data using probability models for quantities we observe and for quantities about which we wish to learn. Unlike the frequentist inference that is based on the p-value which is not a true probability, the essential characteristic of Bayesians is their explicit use of probability for quantifying uncertainty based on statistical data analysis (Gelman et al. 1995, p3). Here, we propose a Bayesian methodology to derive posterior probabilities required for inferences and to generate statistics for the parameters. However, to alleviate some mathematical difficulties which often cannot be resolved using analytical or numerical calculation, as well as to avoid making unsatisfying compromises, we propose using the Gibbs sampler. The Gibbs sampling technique can generate a sample from desired posterior densities through a straightforward, iterative Monte Carlo method. Using the Bayesian framework to examine structural changes in meat demand, we are particularly interested in: (1) detecting if (and when) changes occurred in the consumption pattern for meat, (2) obtaining the parameter estimates before and after the changes, and 3) testing the hypothesis as to whether individual elasticity values are affected by structural change.

Many researchers have considered Bayesian approaches to detect structural changes, including Ferreira (1975), Holbert and Broemeling (1977), Chin Choy and Broemeling (1980), Holbert (1982), Booth and Smith (1982), and Diaz (1982). In order to estimate parameters and make inferences in the multivariate case with this approach, we have to consider a mixture of multivariate distributions that is usually difficult to evaluate using analytical or numerical methods. Broemeling and Tsurumi (1987) considered an approximation method that works for a particular case. In this paper, we propose a general method to deal with the mixture of distribution problem which avoids unsatisfying compromises with regard to the model specification.

#### The Model

We propose to investigate the structural change in meat consumption demand with the full AIDS model specification. Let  $W_1$  be an [m(n-1)]x1 vector of expenditure shares for n-1 meat categories during the period t = 1, ..., m, and  $W_2$  be an [(T-m)(n-1)]x1 vector of expenditure shares for the period t = m+1, ..., T. Let  $Z_{p1}$  be an m(n-1)xK matrix of row vector elements  $Z_{p1it}$  design such that

(1)  
$$Z_{phit}\theta_{h} = \alpha_{hi} + \sum_{j=1}^{n} \gamma_{hij} logp_{hjt} + \beta_{hi} log \frac{X_{ht}}{P_{ht}} \text{ with } \gamma_{hij} = \gamma_{hji}$$
$$logP_{ht} = \alpha_{h0} + \sum_{i=1}^{n} \alpha_{hi} logp_{hit} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{hij} logp_{hit} logp_{hjt} \text{ with } h = 1,2$$

where  $\theta_h$  is the AIDS model vector parameter at the h<sup>th</sup> period of the structural change without  $\alpha_{h0}$ . Thus if there is structural change at an unknown time t = m, the AIDS model can be specified as:

(2) 
$$\begin{array}{c} W_1 \sim N(Z_{PI}\theta_1, V_1) \quad for \quad t = 1, ..., m, and \\ W_2 \sim N(Z_{P2}\theta_2, V_2) \quad for \quad t = m+1, ..., T \end{array}$$

conditional on  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20} \Sigma_1$ ,  $\Sigma_2$  and m with  $V_1 = \Sigma_1 \otimes I_m$  and  $V_2 = \Sigma_2 \otimes I_{T-m}$ , where  $\Sigma_h = COV(W_{ht})$  for  $W_{ht} = (w_{h1t}, \dots w_{h(n-1)t})^t$  and h=1, 2. The term  $w_{hkt}$  denotes the expenditure share of the k<sup>th</sup> meat category at time t before (h=1) or after (h=2) the structural change.

Our objective is to find the posterior distribution of m in order to detect if/when a change does occur, and to find a posterior distribution for the elements of  $\theta_h$ , and  $\alpha_{h0}$  in order to make inferences about their components. The issue of whether there is a structural change at an unknown point will be decided by examining the marginal posterior distribution for m; if it is flat, there are no changes. A value(s) of m that is more probable is a likely change point, differentiating between two distinct levels of demand.

In order to determine the marginal posterior distribution for m, we consider the following procedure. The likelihood for  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20} \Sigma_1$ ,  $\Sigma_2$ , and m can be expressed as:

(3) 
$$L(\theta_1, \alpha_{10}, \Sigma_1, \theta_2, \alpha_{20}, \Sigma_2, m | W, Z) \propto |\Sigma_1|^{-\frac{m}{2}} |\Sigma_2|^{-\frac{T-m}{2}} \exp{-\frac{1}{2}(W_1 - Z_{PI}\theta_1)^t V_1^{-1}(W_1 - Z_{PI}\theta_1)} \times \exp{-\frac{1}{2}(W_2 - Z_{P2}\theta_2)^t V_2^{-1}(W_2 - Z_{P2}\theta_2)}$$

where  $W = (W_1^t, W_2^t)^t$  and  $Z = [Pr_1, Pr_2, X_1, X_2]$ , for  $Pr_h = [logp_{1h}, ...logp_{nh}]$ ,  $logp_{ih}$  is the vector prices for the i<sup>th</sup> meat category,  $X_h$  is vector of expenditures for meat before (at h=1) or after (at h=2) the break point. We assume that the change point is equally likely at any time between K+1 and T-K, where K is the number of elements in  $\theta_1$ . The prior probability mass function of m is given by

(4) 
$$P(m) = 1/(T-2K)$$
 m K+1, ,T-K.

Let's propose a joint multivariate prior, the Jeffrey's prior distribution for  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20} \Sigma_1$ , and  $\Sigma_2$ :

(5) 
$$P(\theta_1, \alpha_{01}, \Sigma_1, \theta_2, \alpha_{02}, \Sigma_2) \propto |\Sigma_1|^{-\frac{n}{2}} |\Sigma_2|^{-\frac{n}{2}}.$$

Then the joint prior for all the parameters is the product of equations (4) and (5). Using Bayes theorem, we can write the joint posterior density for  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20} \Sigma_1$ ,  $\Sigma_2$ , and m given  $Z_P$  and W is as follows:

(6) 
$$P(\theta_1, \alpha_{10}, \Sigma_1, \theta_2, \alpha_{20}, \Sigma_2, m | W, Z) \propto |\Sigma_1|^{-\frac{m+n}{2}} |\Sigma_2|^{-\frac{T+n-m}{2}} \exp{-\frac{1}{2}(W_1 - Z_{PI}\theta_1)^t V_1^{-1}(W_1 - Z_{PI}\theta_1)} \times \exp{-\frac{1}{2}(W_2 - Z_{P2}\theta_2)^t V_2^{-1}(W_2 - Z_{P2}\theta_2)}$$

We need to derive the marginal posterior probability of the break point m from (6). This distribution (6) is analytically intractable and requires finding high-dimensional numerical integral which might be unreliable. In order to overcome this problem we have used the Gibbs sampling technique, which is a very convenient tool for Monte Carlo integration. Given m, the model parameters  $\theta_1$ ,  $\alpha_{10}$ ,  $\Sigma_1$  before the structural change and the model parameters  $\theta_2$ ,  $\alpha_{20}$ ,  $\Sigma_2$  after the structural change are independent. That is,

(7) 
$$P(\theta_1, \alpha_{10}, \Sigma_1, \theta_2, \alpha_{20}, \Sigma_2 | mW, Z) = F_1(\theta_1, \alpha_{10}, \Sigma_1 | m, W_1, Z_{PI}) F_2(\theta_2, \alpha_{20}, \Sigma_2 | m, W_2, Z_{P2})$$

where the conditional densities  $F_1$  and  $F_2$  are defined as follows:

(8) 
$$F_1(\theta_1, \alpha_{10}, \Sigma_1/m, W_1, Z_1) \propto |\Sigma_1|^{-\frac{m+n}{2}} \exp{-\frac{1}{2}(W_1 - Z_{PI}\theta_1)^t V_1^{-1}(W_1 - Z_{PI}\theta_1)}$$

(9) 
$$F_2(\theta_2, \alpha_{20}, \Sigma_2/m, W_2, Z_2) \propto |\Sigma_2|^{-\frac{T+n-m}{2}} \exp{-\frac{1}{2}(W_2 - Z_{p2}\theta_2)^t V_2^{-1}(W_2 - Z_{p2}\theta_2)}$$

Let  $\lambda_{hit}$  be defined as

(10)  
$$\lambda_{hit} = w_{hit} - \alpha_{hi} - \sum_{i=1}^{n} \gamma_{hij} logp_{hjt} - \beta_{hi} logX_{ht} + \beta_{hi} \left[ \sum_{i=1}^{n} \alpha_{hi} logp_{hit} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{hij} logp_{hit} logp_{hjt} \right] where h=1,2.$$

Let 
$$\lambda_{ht} = (\lambda_{ht1}, \dots, \lambda_{ht(n-1)})$$
 and  $b_h^t = \{1/(n-1)\} [-1/\beta_{h1}, \dots, -1/\beta_{h(n-1)}]$ , and  $\lambda_{hM}$  defined as  
(11)  $\lambda_{hM} = \frac{1}{J} \sum_{t=1}^{J} \lambda_{ht}$  where  $J = m$  for  $h = 1$  and  $J = T - m$  for  $h = 2$ .

Consider the following hierarchical Bayes (HB) model:

- I. Conditional on  $\theta_h$ ,  $\Sigma_h$ , m, and  $Z_{Ph}$ ,  $W_h \sim N(Z_{Ph}\theta_h, V_h)$  for h = 1, 2.
- II. Conditional on  $\Sigma_h$ ,  $Z_{ph}$ , m, and  $W_h$ ,  $\theta_h \sim N(\theta_h^{hat}, \Omega_h)$ , where  $\theta_h^{hat} = (Z_{Ph}^{t}V_h^{-1}Z_{Ph})^{-1}Z_{Ph}^{t}V_h^{-1}W_h$ , and  $\Omega_h = (Z_{Ph}^{t}V_h^{-1}Z_{Ph})^{-1}$ .
- III. Conditional on  $\theta_h$ ,  $Z_{Ph}$ , m, and  $W_h$ ,  $\Sigma_h \sim Inv-Wishart_J(S_h^{-1})$  where J (= m for h=1 and T-m for h=2) is the degree of freedom and  $S_h$  is the (n-1)x(n-1) scale matrix with elements  $(W_{hi} Z_{ph}\theta_{hi})^t(W_{hj} Z_{ph}\theta_{hj})$ , i, j = 1, ... n-1.
- IV. Conditional on  $\theta_h$ ,  $W_h$ ,  $Pr_h$ ,  $X_h$ , m, and  $\Sigma_h$ ,  $\alpha_{h0} \sim N(b_h^{t} \lambda_{hM}, b_h^{t} \Sigma_h b_h/J)$  where  $Pr_h = [logp_{1h}, ...logp_{nh}]$ ,  $logp_{ih}$  is the vector prices for the i<sup>th</sup> meat category,  $X_h$  is vector of expenditures for meat before (at h=1) or after (at h=2) the beak point, and J = m for h=1, J = T-m for h=2.
- V. Conditional on  $\theta_h$ ,  $\alpha_{h0}$ , and  $Pr_h$ ,  $logP_h$  is constant.

In order to implement the Gibbs sampler for the marginal distribution of the break point m, we only need (i) to generate a Gibbs sequence  $\theta_1^{(0)}$ ,  $\theta_2^{(0)}$ ,  $\alpha_{10}^{(0)}$ ,  $\alpha_{20}^{(0)}$ ,  $\Sigma_1^{(0)}$ ,  $\Sigma_2^{(0)}$ ,  $P_1^{(0)}$ , and  $P_2^{(i)}$  from their full conditional distributions given above, (ii) generate the conditional distribution of m given  $\theta_1^{(0)}$ ,  $\theta_2^{(0)}$ ,  $\Sigma_1^{(0)}$ ,  $\Sigma_2^{(0)}$ ,  $Z_{P1}^{(0)}$ ,  $Z_{P2}^{(0)}$ ,  $W_1$ , and  $W_2$ , (j=1, 8000, we save the last 4000 draws) and (iii) estimate the marginal distribution of the breakpoint m, by averaging the conditional distributions of m from the Gibbs sequence using the saved draws (see Casella and George 1992). This can be done by using the following steps:

Start with some initial values for  $\Gamma = (\theta_1, \theta_2, \alpha_{10}, \alpha_{20}, \Sigma_1, \Sigma_2)$ , denoted by  $\Gamma^{(0)} = (\theta_1^{(0)}, \theta_2^{(0)}, \alpha_{10}^{(0)}, \alpha_{20}^{(0)}, \Sigma_1^{(0)}, \Sigma_2^{(0)})$ .

2. Generate sequential draws for  $\theta_h^{(i)}$ ,  $\Sigma_h^{(i)}$ ,  $\alpha_{h0}^{(i)}$  (for h=1,2) using their posterior conditional distributions:

$$\begin{split} \theta_{h}^{(i)} &\sim p(\theta_{h} | \Sigma_{h}^{(i-1)}, Z_{Ph}^{(i-1)}, W_{h}) \\ \Sigma_{h}^{(i)} &\sim p(\Sigma_{h} | \theta_{h}^{(i)}, Z_{Ph}^{(i-1)}, W_{h}) \\ \alpha_{h0}^{(i)} &\sim p(\alpha_{h0} | \theta_{h}^{(i)}, \Sigma_{h}^{(i)}, Pr_{h}, X_{h}) \end{split}$$

3. Construct  $logP_{h}^{(i)}$  vector of elements:

(12) 
$$logP_{ht}^{(i)} = \alpha_{h0}^{(i)} + \sum_{j=1}^{n} \alpha_{hj}^{(i)} logP_{hjt} + \sum_{j=1}^{n} \sum_{k=1}^{n} \gamma_{hjk}^{(i)} logP_{hjt} logP_{hkt}.$$

- 4 Construct  $Z_{Ph}^{(i)}$  from  $Z_{pht}^{(i)} = [Pr_{ht}, log(X_{ht} / P_{ht}^{(i)})].$
- 5. Compute  $P(m|\Gamma^{(i)}, Z, W)$ , where  $\Gamma^{(i)} = (\theta_1^{(i)}, \Sigma_1^{(i)}, \alpha_{10}^{(i)}, \theta_2^{(i)}, \Sigma_2^{(i)}, \alpha_{20}^{(i)})$  where  $Pr = (Pr_1, Pr_2) X = (X_1, X_2)$ ,  $W = (W_1, W_2)$ , and Z = [Pr, X].
- 6. Estimate the marginal posterior mass function P(m/Z, W)

(13) 
$$\hat{P}(m|Z,W) = \frac{1}{L} \sum_{i=L}^{2L} P(m|\Gamma^{(i)},Z,W).$$

Under reasonably general conditions,  $\lim_{L\to\infty} \hat{P}(m|Z,W) = P(m|Z,W)$ .

In implementing the Gibbs sampler we follow Gelman and Rubin (1992) and run 2 parallel sequences each for 2L iterations (L=4000). In each sequence, we repeat step 2 thru 5 2L times, and discard the first L iterations of each sequence to diminish the effects of starting points. The last L Gibbs iterates are used to monitor convergence using the potential scale reduction, to compute the marginal posterior mass probability P(m|Z,W), as well as to compute posterior means for each element of the parameter  $\Gamma$  denoted by  $\Gamma_m$ . This procedure is executed for m=K,...T-K. Note that  $\Gamma_m$  could be used as initial values when we have to simulate from mixture distributions as it is the case in this study.

The parameters of interest  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20}$ ,  $\Sigma_1$ ,  $\Sigma_2$ , can be simulated from the joint marginal posterior pdf of  $\theta_1$ ,  $\theta_2$ ,  $\alpha_{10}$ ,  $\alpha_{20}$ ,  $\Sigma_1$ , and  $\Sigma_2$ , which is given by

(14) 
$$P(\theta_1, \alpha_{10}, \Sigma_1, \theta_2, \alpha_{20}, \Sigma_2 | Z, W) = \sum_{m=K}^{T-K} P(\theta_1, \alpha_{10}, \Sigma_1, \theta_2, \alpha_{20}, \Sigma_2 | m, Z, W) P(m | Z, W).$$

The marginal posterior density  $P(\theta_1, \theta_2, \alpha_{10}, \alpha_{20}, \Sigma_1, \Sigma_2, Z, W)$ , is a mixture of normal and Inverse Wishart distributions, with the mixture distribution  $q = (q_K, ..., q_{T-K})$ , where  $q_i = P(m = i | Z, W)$ . In general, it is difficult to simulate from mixtures of distributions using analytical or numerical integration procedures. Some previous studies suggest that if the probability mass function of m is concentrated around  $m = m^*$ , the posterior distribution  $P(\theta_1, \theta_2, \alpha_{10}, \alpha_{20}, \Sigma_1, \Sigma_2 | m = m^*, Z, W)$  is a good approximation of the marginal probability density,  $P(\theta_1, \theta_2, \alpha_{10}, \alpha_{20}, \Sigma_1, \Sigma_2 | Z, W)$  (Broemeling and Tsurumi, 1987). That is, if  $q_m$  is large compared to the other  $q_i$ 's, we can set  $q_m = 1$  and all other  $q_i = 0$ . Unfortunately, this approximation method does not work in general.

We propose to analyze the mixture models as a hierarchical Bayesian (HB) model (Gelman and King, 1990). The basic principal for this procedure is to introduce unobserved indicators, random variables  $\zeta = (\zeta_{K}, ..., \zeta_{T-K})$ , that specify the mixture component from which each particular parameter is drawn. This idea of adding variables is also called data augmentation (see Tanner and Wong 1987). Consider the following HB model:

 $\zeta_i$  is defined as  $\zeta_i = 1$  if the parameter is drawn from the i<sup>th</sup> mixture and = 0 otherwise; given q, the distribution of  $\zeta = (\zeta_K, ..., \zeta_{T-K})$  is a multinomial Multin $(1, q_K, ..., q_{T-K})$ .

II Conditional on Z, W, and q, the joint distribution of  $\Gamma = (\theta_1, \theta_2, \alpha_{10}, \alpha_{20}, \Sigma_1, \Sigma_2)$ ,  $\zeta$  can be written as:

(15) 
$$P(\Gamma, \zeta | Z, W, q) = P(\zeta | q) P(\Gamma | \zeta, Z, W) = \prod_{m=K}^{T-K} \left( q_m P(\Gamma | Z, W, m) \right)^{\zeta_m}$$

III. The natural conjugate prior distribution for q is the Dirichlet distribution, q ~ Dirichlet( $\alpha_{K}$ , ...,  $\alpha_{T-k}$ ), where  $\alpha_i$  is estimated by the method of moments

Direct evaluation of this model involves high dimensional numerical integration and is not computationally efficient. Instead, we use the Gibbs sampler (cf. Geman and Geman 1984; Gelfand and Smith 1990). For this model, the Gibbs sampler alternates two major steps: A) Obtaining draws from the distribution of indicators given the model parameters. By using conjugate families as prior distributions, obtaining draws from the distribution of the indicators becomes straightforward. That is, these are simply multinomial draws in a finite mixture model. B) Obtaining draws from the model parameters, given the indicator. Note that, given the indicators, the mixture model reduces to a model which we can simulate  $\Gamma$ from using the steps 2) thru 4). We propose to use  $\Gamma_m$  for the initial value of each mixture to reach fast convergence to the target distribution.

To implement and monitor the convergence of the Gibbs sampler we follow Gelman and Rubin's (1992) approach. We run 4 parallel sequences for 4000 iterations each. In each sequence, we repeat A) and B) 4000 times, and discard the first 2000 iterations of the sequence to diminish the effects of starting points. The last Gibbs iterates are monitored for convergence using the potential scale reduction, used to estimate posterior means of the model parameters and the elasticities, and for computing measures of statistical precision.

#### **Data and Results**

To allow the most direct comparision with earlier work, we used Moschini and Meilke's (1989) data, which is quarterly data of U.S. meat demand and retail prices, from the first quarter of 1967 to the fourth quarter of 1988 (n = 88). For beef, pork, and chicken, quantities are per capita disappearance in retail weight as published by the U.S. Department of Agriculture (USDA). Quarterly consumption expenditures for fish and seafood data were obtained from the U.S. Department of Commerce (USDC). The potential scale reduction parmaeters for each estimated parameter before and after the structural change are very close to 1. Thus, we conclude that the Gibbs sequences have converged; the numerical approximations calculated here are sufficiently accurate.

The histogram (see fig. 1) representing the posterior probability of the break point is not flat. This supports the hypothesis that there is structural change in meat consumption between 1972 and 1989. More than 96 percent of the total mass posterior probability for the break point is concentrated between the first quarter of 1975 and the fourth quarter of 1976. The highest posterior probability for the break point is found at the fourth quarter of 1975. This indicates the most likely time of a structural change for meat consumption. The fact that the posterior probabilities of m from the first quarter of 1975 to the fourth quarter of 1976 are all statistically significantly different from the prior values is strong evidence that some significant shocks in meat demand occurred during these years.

In this study, we have not addressed the question, "how did the change occur? That is, was it a gradual change or a sudden change." Although, we expect gradual changes in consumer behavior, the hypothesis of sudden change in meat consumption patterns from the end of 1974 to the end of 1976 might be reasonable, because of sudden events, such as the severe drought of 1974 and the shock in grain prices caused by the high Soviet (USSR) demand for grain; both had profound effects on meat supply. This is consistent with Moschini and Meilke results which suggest that an abrupt change in meat consumption occurred between the fourth quarter of 1975 and the third quarter of 1976.

The issue of whether, after these peak points in figure 1, the consumption of meat adjusted back to the old pattern or to a new pattern can be addressed by examining how significant the changes in the parameters are from the first period to the second period. The parameters' posterior means before and after the structural change are given in table 1. The 95 percent posterior density regions of the parameter differences between the two periods indicate that three parameters have significantly changed after the structural change. That is, after the break points, meat consumption has not adjusted back to the old pattern, meaning that consumers have changed their behavior toward meat. Many more changes would be significant at, say, the 80 percent level.

The question "What might cause changes in meat consumption pattern?" can be partly investigated by examining how significant demand elasticities are affected by the structural change. Table 2 shows the Marshallian demand elasticities (with their 95 percent posterior

density regions) for beef, pork, chicken, and fish. As expected, before and after the structural change, the own price elasticity is negative for each category of meat. Before the structural change, only the own demand price elasticity for fish was not statistically significant, while the own price elasticities for beef, pork, chicken were all statistically significant. After the structural change, none of the own demand price elasticities for meat were statistically significant. That is, in the first period meat prices are a significant factor in the consumer decision to buy meat while in the second period a consumer decision to buy meat is not significantly affected by prices. The significance of these changes can be evaluated using Table 3.

In the first period, the own demand price elasticities for beef, pork, chicken, and fish are -0.717, -0.902, -0.314, and -0.104, respectively. In the second period, the own price elasticities for beef, pork, chicken, and fish are -0.460, -0.633, -1.273, and -3.758, respectively. In the first period, demand for red meat ( pork and beef) appears to be more elastic than demand for white meat (fish and chicken), while in the second period, although own price elasticities are not statistically significant, demand for white meat appears to be more elastic than demand for red meat. These results contrast with Moschini and Meilke's (1989), who found that in the first regime own price elasticities for beef, pork, chicken, and fish are -0.983, -1.015, -0.090, and -0.138, respectively. In the second regime, own price elasticities for beef, pork, chicken, and fish are -1.05, -0.839, -0.104, and -0196, respectively. They find red meat is more elastic than white meat in both periods; we find a reversal in the second period. Also, they found that in both regimes, none of the own price elasticities for meat were statistically significant.

In both regimes the cross price elasticities indicate complementarity. This is consistent with the Moschini and Meilke (1989) results.

In the first period a 10 percent increase in income is associated with a 11.65 percent increase in beef consumption, a 5.62 percent increase in pork consumption, a 22.86 percent in chicken consumption, and an 8.8 percent in fish consumption. In the second period, a 10 percent increase in income is associated with an 11.65 percent increase in beef demand, a 10.03 percent increase in pork consumption, a 5.65 percent increase in chicken consumption, and a 17.12 percent increase in fish consumption. Note that while in the first period only the income elasticity for fish is not statistically significant, in the second period, except for the income elasticity for beef, all other income elasticities are not statistically significant. These results indicate that beef appears to be a luxury good before and after the structural change whereas pork appears to a necessity good in both regimes. However, the chicken classification has changed from a luxury good before the structural change to a necessity good after the structural change. These results contrast with those of Moschini and Meilke who found that before the structural change a 10 percent increase in income caused a 12.20 percent increase in beef consumption, a 10.41 percent increase in pork consumption, a 2.38 percent increase in chicken demand, and a 4.32 percent increase in fish consumption. In the second period they found that a 10 percent increase in income was associated with a 13.94 percent increase in beef, and a 8.53 percent increase in pork consumption, a 2.11 percent increase in chicken demand, and a 4.32 percent increase in fish.

#### Conclusion

Moschini and Meilke investigated structural changes in meat demand using the LAIDS model. They found that there was sudden structural changes in meat demand in the mid-70's. More precisely, the change in meat demand occured between the fourth quarter of 1975 and the third quarter of 1976 (their model allowed for a transition period between two regimes; the two dates above bounded their maximum likelihood estimates of the transition period). Their results rejected the hypothesis that the model parameters were constant. They also found that beef was the only luxury good in both regimes. They estimated demand elasticities using the conventional LAIDS uncompensated elasticities. Their results showed that the cross price elasticities exhibited complementarity relationships. In order to avoid difficulties involved in computing the standard errors for the elasticities, Moschini and Meilke assumed that expenditure shares were constant (nonstochastic). They found that no elasticities were significantly different from zero in both periods, and that elasticity values were unaffected by structural change.

Our study used the full AIDS model to investigate structural changes in meat demand. We found structural changes in the meat consumption pattern in the mid-1970's. Our results revealed that change may have started at the first quarter of 1975, most likely occurred in the fourth quarter of 1975, and could have been as late as the fourth quarter of 1976. We rejected the hypothesis that the parameter vector before structural changes is the same as that afterward. Our result suggest that both beef and chicken were luxury goods in the first regime. However, only beef was classified as a luxury good in the second regime. We estimated demand elasticities using the AIDS elasticity formula. Our results indicate that there were more complementarity between meat products than expected. This could be explained by the fact that the income effect could outweigh substitution effect. However, the Hicksian demand elasticities in both periods indicate that beef, pork, chicken, and fish may be complements. This also might suggest that the use of aggregate data are not appropriate for meat demand analysis.

The 95 percent posterior density regions for elasticities indicate that in the first regime many demand elasticities were significantly different from zero, while in the second regime no elasticities was significantly different from zero. Furthermore, our results revealed that structural changes along with changes in relative prices and expenditure shares have significant effects on the demand elasticities.

The differences in the results of these two studies are mainly due to differences in methodology. The LAIDS model is not equivalent to the AIDS model. Thus, it is reasonable that our inferences, which are based on direct evaluation of AIDS model, be different from those of Moschini and Meilke which are based on the LAIDS model analysis. Also, the approach used by Moschini and Meilke to compute demand elasticities was theoretically incorrect (see Green and Alston, 1990), and computing the standard errors of elasticities by assuming that expenditure shares are constant might generate inconsistent estimators (see Buse, 1994). That is, inferences based on the Moschini and Meilke study might be misleading.

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F gure 1. Marginal Posterior D stribution of m



Parameters	Before Structural Change	After Structural Change	<u>95% PDR</u>
$\alpha_0$	-3.942	-4.163	(-3.254, 4.738)
$\alpha_1$	0.827	0.380	(-12.454, 13.114)
α <sub>2</sub>	0.662	0.272	(-1.274, 2.137)
α <sub>3</sub>	0.048	0.354	(-2.502, 1.439)
γ11	0.144	0.066	(-0.943, 0.982)
$\gamma_{12}$	-0.063	0.021	(-0.260, 0.069)
γ <sub>13</sub>	-0.059	0.012	(0.844, 0.145)
γ <sub>22</sub>	-0.015	0.060	(-0.222, -0.096)
γ <sub>23</sub>	0.057	-0.069	(0.035, 0.210)
γ <sub>33</sub>	0.030	0.039	(-0.099, 0.104)
$\beta_1$	0.021	0.031	(-0.219, 0.072)
$\beta_2$	-0.101	-0.007	(-0.145, 0.072)
$\beta_3$	0.120	-0.033	(-0.082, 0.279)
<b>α</b> <sub>11</sub>	0.0064	0.0107	(-0.310, 0.324)
α <sub>12</sub>	0.0101	0.0121	(-0.237, 0.240)
α <sub>13</sub>	0.0150	0.0135	(-0.233, 0.250)
α <sub>21</sub>	-0.0120	-0.0129	(-0.0419, 0.0452)
α <sub>22</sub>	-0.0295	-0.0229	(-0.0337, 0.0259)
α <sub>23</sub>	-0.0299	-0.0232	(-0.0615, 0.0496)
α <sub>31</sub>	0.0039	0.0021	(-0.0189, 0.0195)
α <sub>32</sub>	0.0164	0.0075	(-0.0098, 0.0224)
α <sub>33</sub>	0.0111	0.0089	(-0.0126, 0.0162)

Table 1. Parameters Posterior Means Before and After Structural Change

	<u>Before Structural Change</u>		After Structural Change		
	Estimated	95%	Estimated	<b>95</b> %	
Elasticity	Mean	PDR	Mean	PDR	
Price					
ebeef, beef	-0.717	(-0.897, -0.551)	-0.460	(-0.641, 2.561)	
e <sub>beef, pork</sub>	-0.051	(-0.184, 0.029)	0.358	(-0.441, 1.934)	
ebeef, chicken	-0.147	(-0.217, -0.059)	-0.365	(-0.974, 1.151)	
e <sub>beef, fish</sub>	-0.102	(-0.182, 0037)	-0.692	(-5.404, 0.171)	
e <sub>pork, beef</sub>	0.146	(-0.143, 0.463)	0.728	(-0.664, 3.395)	
e <sub>pork, pork</sub>	-0.902	(-1.312, -0.176)	-0.633	(-0.852, 4.515)	
e <sub>pork, chicken</sub>	0.164	(-0.308, 0.488)	0.359	(-3.713, 1.343)	
e <sub>pork, fish</sub>	0.036	(-0.202, 0.215)	-1.450	(-5.015, 0.705)	
e <sub>chicken, beef</sub>	-1.365	(-2.199, -0.619)	-0.802	(-3.406, 6.102)	
e <sub>chicken, pork</sub>	-0.035	(-1.647, 0.965)	1.308	(-3.015, 3.635)	
echicken, chicken	-0.314	(-1.762, -0.104)	-1.273	(-2.341, 0.092)	
e <sub>chicken, fish</sub>	-0.146	(-1.639, 1.760)	-1.642	(-4.092, 0.851)	
e <sub>fish, beef</sub>	-0.530	(-0.226, 0.119)	-3.190	(-6.954, 0.229)	
e <sub>fish, pork</sub>	-0.015	(-0685, 0.532)	- 3.479	(-7.009, 1.405)	
$e_{\mathrm{fish, chicken}}$	-0.492	(-0.843, 0.112)	1.192	(-3.135, 2.077)	
$\mathbf{e}_{\mathrm{fish, fish}}$	-0.104	(-0.503, 0.684)	- 3.758	(-4.836, 5.334)	
Income					
Ibeef	.012	(0.861, 1.241)	1.165	(-0.479, 1.410)	
I <sub>pork</sub>	0.562	(0.081, 0.925)	1.003	(-0.262, 1.870)	
$\mathbf{I}_{chicken}$	2.286	(1.219, 3.269)	0.565	(-2.686, 1.016)	
$\mathbf{I}_{fish}$	0.886	(-0.207, 2.095)	1.712	(-1.964, 7.293)	

Table 2. Marshallian Demand Elasticity Posterior Means

Elasticities Price	<u>95% H</u>	PDI
$\Delta e_{head}$	957,	-0 208
$\Delta e_{\text{heef.}}$	06	.3)
$\Delta e_{herf.}$	5 6,	-0.93
$\Delta e_{hasf}$ fish	(-0.267	i7)
$\Delta e_{nork.}$	<b>53</b> °	00.)
$\Delta e_{\text{nork}}$	(-5 799	-0 493)
$\Delta e_{nork, chicken}$	464	54)
$\Delta e_{ m pork.\ fish}$	.5	93
$\Delta e_{ m chicken}$	(· 343	899
$\Delta e_{\rm chicken}$	(-4	6.657)
$\Delta e_{ m chicken, chicken}$	048	02
$\Delta e_{ m chicken. fish}$	98	.240)
$\Delta e_{\rm fish.}$	-0.966	04.)
$\Delta e_{\rm fish, pork}$	-2.40	595)
$\Delta e_{\rm fich}$	<b>(-6</b> .009	232)
$\Delta e_{\text{fish. fish}}$	<b>(-</b> 8. 67	48)
Income		
	(-0.482	, .7:7)
	<b>(</b> 78	0.086)
$\Delta i_{chicken}$	(0.207	.328
	(-6.927	,2.602

Table 3 95	% PDRs	for Ch	anges in	Elasticities
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