Implications of Deflating Commodity Prices for Time-Series Analysis

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Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management
Chicago, Illinois, April 17-18, 2000

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Practitioner's Abstract

The choice of deflators of commodity prices can change the time-series properties of the original series. This is a specific application of the general phenomenon that various kinds of data transformations can create spurious cycles that did not exist in the original data. Different empirical models of expectations result from nominal and various deflated series that have distinct time-series properties, and these models, in turn, produce varying estimates of supply response and measures of price risk. The foregoing is illustrated by annual grain prices, monthly milk prices, and a milk supply analysis. Annual prices of corn and soybeans, for example, appear to vary around a constant mean, but when deflated by general price indexes such as the CPI, the deflated prices are autocorrelated around a declining deterministic trend and/or have a stochastic trend. The quasi-rational expectations hypotheses assumes that farmers base expectations on forecasts from time-series models, but forecasts of real prices, that ultimately become negative, are not rational.

Keywords

deflating, time-series analysis, price expectations, price risk, supply analysis

Introduction

Much of economics is about the consequences of changes in relative prices. Sometimes, as in the specification of an Almost Ideal Demand System, theory provides a rather specific guide about deflators. At other times, theory and logic provide only general guides for empirical analyses that use relative prices, and price analysts have used a variety of different deflators in otherwise similar applications. This is particularly true in ad hoc, single equation models. In supply analysis, for example, the output price has been divided by a single major input price (the hog/corn price ratio in pork supply equations), by indexes of input prices (the index of prices paid by farmers), by the price of the main commodity competing for the same resources (the corn/soybean price ratio in corn supply equations), by indexes of a combination of such commodity prices, and by the Consumer Price Index (CPI).

The question of an appropriate deflator, if any, also occurs in specifications of price risk. The measurement of risk appears in portfolio problems, which involve the probability distributions of prices of items that are potential members of the portfolio. Risk variables may be used in supply equations for commodities. Risk is sometimes measured by the unconditional variance of nominal prices. If both output and input prices are risky, then the variances of two nominal prices or of the (single) price ratio might be used. Relevant prices or price indexes are problem dependent.

Theoretically, risk involves deviations from expected prices, but this requires a definition of expectations. Expectations can be viewed as forecasts from an underlying economic model, and models of commodity prices often use deflated series. In an influential paper, which is concerned with the theory and empirical behavior of commodity prices, Deaton and Laroque deflate annual prices of 13 commodities (including bananas, coffee, and palm oil) by the CPI of the United States. They apply a theory of storage — a structural model that incorporates non-negativity of inventories — to explain the skewness, autocorrelation, and occasional spikes that appear in the price series. If the prices had not been deflated, however, little or no autocorrelation would have existed to be explained. Further, even if interest centers on the probability distributions of real prices, it is not obvious that the real price should be defined by dividing nominal world prices by the CPI of the United States.

Assumptions regarding how expectations are formed alter empirical results. Antonovitz and Green, for example, compare several supply response models for fed beef that vary by definition of price expectations. Mean and variance of output prices were estimated from naïve, quasi-rational, adaptive, and rational expectations models and futures prices using CPI-deflated monthly prices. Their results show that the signs and elasticities of the supply response vary significantly across models. Most likely they would have reached the same conclusion—that alternative definitions of expectations result in various estimates of supply responses—using nominal prices or other deflators. But, the quantitative results would certainly have differed, and again, it is not clear why beef suppliers form expectations on prices deflated by the CPI.

The fact that data transformations change time-series properties of the original data is not new. Harvey discusses a phenomenon, called the Yule-Slutzky effect, that was discovered over half a century ago, where applying a set of summing operations induces a spurious cycle in data that had already been differenced. In an example he cites by Kuznets, a claimed 20-year cycle in an economic time series could have been spuriously induced by the two filtering operations: moving averages and differencing. Holbrook Working also notes that averaging monthly differences of prices into bimonthly, quarterly, or annual prices generates autocorrelation in the error term that does not exist in the original, first-differenced series (see also Nerlove, Grether, and Carvalho). Here, deflating is another operator on the original price series that could significantly impact empirical results.

The objective of this paper is to illustrate the potential effects of the choice of deflators on time-series properties of prices and hence to demonstrate the consequences of common deflating practices. In particular, we discuss the specification of quasi-rational price expectations of grain and dairy farmers, where it is assumed that producers form expectations as forecasts based on time-series (ARIMA) models. The analysis uses expectations based on nominal and deflated prices for multiple deflators.

The paper is organized as follows. The first section describes the data and methods. The data include annual prices of soybeans and corn, monthly prices of milk and corn, and two price indexes. The properties of nominal and deflated series are described by graphs, spectral analysis, and ARIMA models. The second section presents results for the time-series properties of annual and monthly prices. The effects on the definition of expected prices and supply analysis are illustrated in the third section. Alternative measures of expected prices are compared and used in a supply model for milk. Some conclusions are summarized in a final section.

Data and Methods

Annual observations for the U.S. average prices of corn and soybeans (USDA), for the Index of Prices Paid by Farmers (PPF) for production items (1992=100), and for the CPI (1982-84=100) were collected for a 25-year period, crop years 1973-74 through 1997-98. Starting with the 1973-74 year is justified by the probable structural break in commodity prices at this point in time, associated with a major increase in exports to the (former) Soviet Union. It is true that corn and soybean prices remained under the influence of government programs through the end of the 1980s, but both price series evidence considerable variability from 1973 onward.

Government programs were a major influence on milk prices through 1988, and the changes in program provisions created a structural break between 1988 and 1989. Thus, we analyze monthly milk prices for September 1989 through August 1998 (108 months). The series is the Minnesota-Wisconsin manufacturing grade milk price adjusted to 3.5 percent milkfat basis. The monthly price of corn and the CPI were obtained for the same time period.[1] The use of monthly prices allows us to examine the effects of seasonality. Averages and standard deviations for the series are reported in Table 1. Note, the average nominal prices of corn are almost identical for the 25-year period and for the more recent monthly period.

The methods of analysis involve a combination of descriptive charts, spectral analysis, and ARIMA models. The series are differenced when appropriate, as will be discussed. Essentially, a variety of empirical methods are used to describe the various time series with the objective of comparing the properties of nominal and deflated prices.

The spectral density function (or spectrum) of a time series uses the frequency domain, and hence describes how much contribution each frequency makes to the overall variance of the series. There are several ways to estimate the spectrum. Periodograms are obtained by plotting the Fourier-transformed data against the frequencies. While not statistically consistent estimators of the underlying spectra, the periodograms are a useful way to illustrate the effects of deflating choices. They are particularly relevant for monthly series to examine the seasonal component(s) of prices.

The literature suggests that maximum entropy estimators of spectra have some statistical advantages over the periodogram (Burg; Akaike; Woitek). In particular, maximum entropy methods are better suited for short time series. The procedure involves fitting an AR model of order p and computing the spectrum of the estimated model. The choice of p is based on the Akaike Information criterion (Woitek). Formulas used to calculate the maximum entropy spectrums are found in Appendix 1.

The time domain is estimated using ARIMA models. They were fitted to all of the series, using Akaike Information and Schwartz-Bayesian criteria as a basis for choosing the orders p and q of the AR and MA processes. The Box-Liung statistic was used as a tool to assure that the residuals were free of autocorrelation.

[1] The monthly PPF was not available for the entire sample period.

The final models are evaluated for their ability to simulate the sample period data; the monthly models are also used to make *ex post* forecasts for 1998-99. In addition, the time-series are simulated beyond the sample to determine their stability. Forecasts correspond to prices expected by agents who are aware of the stochastic processes underlying the observed prices, i.e., quasi-rational expectations (Feige and Pearce; Nerlove, Grether, and Carvalho; Chavas).

Time-Series Properties

Annual Corn and Soybean Prices

Visual inspection of annual corn and soybean prices, plotted in Figure 1, suggests no trend in the nominal series. In contrast, the CPI and the PPF trended upward over the sample period, though the PPF has more variability around the trend. Thus, deflating by either index tends to introduce a downward trend in corn and soybean prices and to dampen price variability in the later portion of the sample relative to the earlier period.

The periodograms and maximum entropy spectra are similar and suggest that the nominal commodity prices are white noise; there are no apparent systematic components.[2] Given the trends in the price indexes, they were first differenced, and the CPI and the PPF are estimated to have nine- and 11.8-year cycles, respectively. These may be attributable to macro-economic cycles, or alternatively, to multiple operators applied to an original set of observations, as in previously mentioned Harvey and Working. Here, we regard the published annual data as the original data series.

Corn prices were deflated by the two indexes, and a unit root was detected in both of the deflated series. Based on the estimated spectral densities, the first-differenced series are white noise, implying that deflating introduced the stochastic trend. The deflating did not impose any cyclical behavior on corn prices.

Soybean prices were also deflated by both indexes, but unlike corn, deflating did not introduce a unit root. An autoregressive process best represents the systematic component in real soybean prices. Again, the longer cycles in the price indexes do not appear in the spectral analyses of the deflated series.

The preferred ARIMA models are summarized in Appendix 2. In all models, residuals are statistically white noise. Consistent with the spectral analysis, nominal corn and soybean prices are best represented as white noise around a constant mean (thus not reported). The preferred model for the annual CPI is an AR(2) of the first differences. The annual PPF is represented by a MA(1) model of the first differences, although for the maximum entropy spectrum estimation, a second-order AR was used.

If real corn prices are random walk with drift, then the preferred model is simply $y_t = a + y_{t-1} + e_t$, and the fitted model does indeed have a slope coefficient very close to one. Also, while

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[2] To limit the length of the paper, periodograms and maximum entropy spectra are not included but are available upon request.

the Box-Liung test suggests that the error terms are white noise, alternative ARIMA models for the first differences of the deflated corn prices were estimated as a double check of the test, and for the forecasting applications, we used a MA(2) model for the CPI-deflated prices and a MA(1) model for the PPF-deflated prices. The slope coefficients of the MA(2) process are statistically insignificant, but the intercepts of the first-differenced real prices suggest a downward trend in real corn prices.

For soybeans, the preferred model is AR(2) for both deflated price series. We also report an AR(1) model using the first differences of real soybean prices (Appendix 2). In sum, deflating introduces autocorrelation into the series that did not exist in the nominal prices, and real corn prices seemingly have a unit root as a consequence of deflating.

In considering alternative variances to measure risk, one possibility is the unconditional variance of the nominal prices; as noted above, these prices appear to vary around a constant mean. Using real prices, the measure for corn would be the variance of the first differences, if corn prices were truly random walk with drift, and for soybeans, it would be a conditional variance based on the residuals of a second-order autoregression, if their price had indeed no unit root. Standard errors for alternative models are presented in Table 2. The unconditional standard errors of the nominal prices of corn and soybeans are the smallest of the models considered. Note, the standard errors of CPI- and PPF-deflated prices were higher than those of nominal series (1.334 and 1.080 for corn, 2.744 and 2.078 for soybeans, respectively), as were the means (2.849 and 3.191 for corn, 6.570 and 7.385 for soybeans, respectively). If an earlier year were selected for the base of the price indexes, and given that the indexes have been trending upward, the means of the deflated series would have been smaller than the nominal means and perhaps the standard errors as well.

Monthly Milk and Corn Prices

Monthly data are graphed in Figure 2. Nominal milk prices have no apparent trend. Though variable and difficult to generalize (the standard deviation of milk prices was about 10 percent of the mean, see Table 1), the prices tend to be higher in the fall and decrease in the winter. Monthly corn prices also have no trend, but an obvious spike in prices occurred, starting in late 1995 and persisting into the summer of 1996. Corn prices are likely to be higher in early spring than in the remaining months.

The ratio of milk to corn prices is also variable from month to month and has no apparent trend. Both milk and corn prices deflated by the CPI trend downward. For the deflated milk series, peaks in the summer of 1996 and 1998 are reduced relative to the high prices observed at the beginning of the sample period. Similarly, since the spike in corn prices appear later in the sample period, it is dampened as well. The May 1996 price was 2.5 times higher than the October 94 price in nominal terms — in the CPI-deflated terms, 2.3 times as high.

Turning to the spectral analysis, the variability of milk prices is concentrated at lower frequencies, implying that long-run factors contribute more to the overall price variability than do seasonal variations. Seasonality is illustrated by plotting the periodogram against period — months per cycle (Figure 3). The periodogram indicates three major seasonal components at 5.7, 9, and 12 months in the nominal series. The CPI-deflated milk prices preserve these cycles, except that the contribution of the 5.7-month cycle is reduced relative to the nine-month cycle, which may be caused by the six-month cycle observed in the CPI first differences.

Nominal corn prices also have seasonal components that are similar to milk prices, except corn prices do not have the 5.7-month component (Figure 3). By taking the ratio of milk to corn prices, the 5.7-month cycle is uniquely preserved, while the common seasonal factors seem to be dampened. In essence, the nature of seasonality in the deflators, if any, influences the seasonality of the real prices. Indeed, a common method of adjusting for seasonality is to divide by a seasonal index. This notion can be generalized — a divisor that has seasonality can impart or delete seasonality from the numerator. Nerlove, Grether, and Carvalho, in their discussion of an "optimal" method of seasonal adjustment of time-series, caution against inducing spurious fluctuations in the adjusted series (Chapter 8).

The nominal prices of milk and corn were fitted with third- and second-order AR models, respectively (Appendix 2). The ratio of milk to corn prices seems to be best fitted by an AR(2) model that has coefficients similar to the corn equation. The CPI is differenced, and the preferred model appears to be one with first, 12^{th}, 24^{th}, and 36^{th} MA terms. When milk prices are deflated by the CPI, however, the preferred model is a third-order AR, with coefficients similar to those for the undeflated series. That is, the nominal prices are autocorrelated, which is not surprising given the seasonality of the series, and this autocorrelation persists after deflating. The AR coefficients of nominal and real milk price models are similar.

For both nominal and CPI-deflated milk prices, a risk measure is a conditional variance based on the AR(3) residuals. In Table 2, the standard error for the deflated price is the smaller of the two. In contrast to the annual prices, the base year for the CPI is prior to the sample period, and the deflated price level is below the nominal level throughout the sample (see Figures 5a and 5b).

We discuss a few additional time-series models in the next section, in the context of measuring price expectations of grain and dairy farmers. It should be noted, however, that all of the models presented in this paper are linear in the mean and in the variance of the various random variables. Perhaps models that are non-linear, at least in the mean, are more appropriate in modeling farmers' expectations, but given the vast array of such models, they were not considered.

Supply Analysis and Quasi-Rational Expectations

In supply analysis, assuming optimizing firms that produce more than one product with a set of inputs, the conceptual model involves a system of product supply and input demand equations. The supply of a particular product depends on its own price relative to the other product prices and relative to the input prices. With lags in the production process, decisions are based on expected prices, and since expectations may not be realized, risk may be an important argument in the supply functions. Moreover, a variety of hypotheses about the formation of price expectations has been used, and their empirical implementation influences results (e.g., Antonovitz and Green; Tronstad and McNeill).

There are three issues. First, what are the relevant relative prices for the producers? Second, how do they form expectations about these prices? Third, how does the analyst actually implement the empirical model?

The answer to the first issue is problem specific. Researchers should rely on theory and logic as much as possible, yet many applications require the use of judgement. Regarding the second question, farmers are not homogeneous — their costs of obtaining and using information differ, and they form price expectations in different ways — and evidence supports heterogeneous expectations (e.g., Antonovitz and Green; Chavas). Some farmers may have naïve expectations, others rational or quasi-rational expectations, and still others perhaps adaptive expectations. In an empirical analysis of pork supply, Chavas estimated (p. 34) that 73.3% of pork production comes from farmers using quasi-rational expectations; the remainder are estimated to use rational expectations (19.5%) or naïve expectations (7.2%).

This paper focuses on the third issue. Even if the quasi-rational expectations hypothesis is appropriate, the analyst must make choices in developing the empirical model. In Chavas, quasi-rational expectations for pork prices are based on an AR(3) model with a linear trend term, and the analogous equation for corn prices is an AR(2) model also with a linear trend term. Both models were estimated using annual prices deflated by the CPI for the years 1960-96. Given our results (though for a different sample period), it is not surprising that the linear trend term is significantly negative in both equations. How would the results have changed, if any, if quasi-rationally expected prices were defined differently?

First in this section, models of annual corn prices are used to illustrate the behavior of different estimates of quasi-rationally expected prices. Then, we examine several measures of quasi-rational expectations from the perspective of a dairy farmer, contrasting simulated values ("forecasts") of monthly prices using the alternate measures (no deflating or different deflators). We cannot, of course, say which empirical implementation of quasi-rational expectations is best — actual expectations are unobservable — and we can only compare estimated expectations with historical data. The largest correlation with data or the smallest mean square error does not imply the best measure of expectations. But, the choice of deflators is shown to make important differences in the measures of expectations. Also, the choice of the particular time-series model influences forecast outcomes, hence the nature of the estimated expectations. In the latter subsection, the effect of various empirical definitions of expected prices is examined in a milk supply analysis.

Quasi-rational price expectations

Annual prices. For corn, the prior analysis suggests that, given our sample and the models we examined, annual nominal prices are best forecast by naïve expectations; i.e., the forecast equals the last sample observation. For the CPI-deflated corn prices, the within-sample performance of four models is examined. Model 1A treats the first differences as white noise, which is consistent with the spectral analysis. Allowing for the possibility that the test for white noise might not be correct, we searched for an alternate model of the first differences and selected one with MA(2) terms, though the coefficients are not statistically significant (Model

2A). The intercept in this equation is -0.137, implying an almost 14 cent per bushel downward trend in the deflated prices.

To explore the Chavas approach, Model 3A includes a deterministic trend, and the preferred alternative was AR(1) plus trend. (Chavas used an AR(2) plus trend for a different, though over-lapping sample period.) Finally, nominal corn prices and the CPI are forecast separately, and then the ratio of the separate forecasts is used (Model 4A). PPF-deflated corn prices are analyzed in a similar way.

The in-sample root mean square errors (RMSEs), in percentage terms, for nominal corn prices and the four models of CPI-deflated corn prices are reported in Table 3. If estimates of risk are based on deviations from expectations, the different values across the four CPI-deflated models could be interpreted as alternative measures of historical price risk, in terms of deflated prices. The RMSE for Model 3A is the smallest among them, but as noted above, the model with the smallest RMSE is not necessarily synonymous with the "best" measure of farmers' (unobservable) expectations. It should also be noted that trend-stationary and unit root specifications have different implications for out-of-sample standard errors of forecast, i.e., confidence intervals (Hamilton, p. 441).

The correlations of the simulated ("forecast") values over the sample are shown in Table 4. For the CPI-deflated corn prices, Model 3A with deterministic trend has the best fit to the sample data (0.899) by a small margin, but the specification of a continued downward linear trend outside the sample period quickly becomes untenable (Figure 4). The other three models have very similar fits with the data (about 0.87). They too have potential problems for post-sample forecasts. Model 2A with MA(2) terms has a negative intercept, and the first differences as white noise in Model 1A imply that the last observed first difference, a negative number, is a constant out-of-sample forecast of first differences, both resulting in a downward trend. Model 4A, using separate forecasts, seems to provide the most logical simulation for a 10-year, out-of-sample period, but the estimated ratio also trends downward.

The fundamental problem is that nominal corn prices have no trend, but the CPI trended upward over the sample period. Thus, any model that uses CPI-deflated corn prices must deal with a downward trend. Using the CPI deflator to measure farmers' expectations for corn appears to be a dubious idea, precisely because it is not rational to predict negative prices.

Since the PPF is also trending upward, one might expect analogous issues to arise. In order to limit the length of the paper, we do not discuss the analysis of the PPF-deflated series nor of soybean prices. Basically, the results of the PPF-deflated series were consistent with those of the CPI-deflated series, and the analyses of nominal and deflated soybean prices support the conclusions drawn from the analyses of corn prices.

<u>Monthly prices.</u> The nominal and CPI-deflated monthly milk prices are simulated using AR(3) models. Although the models simulate the sample period closely, neither one is particularly successful in forecasting the upturn in nominal and real prices in the immediate post-sample months (Figures 5a and 5b); the RMSEs of the ex post forecasts are 15 to 20 times as large as the in-sample RMSEs (Table 3).

The models were also simulated beyond the sample period through August 2001. Given that both equations (nominal and CPI-deflated) result in a convergent cycles, the nominal price forecast stabilizes at about $12.25 per cwt, while the forecast of real prices stabilizes at almost $8.50 (in 1982-84 dollars).

Farmers perhaps form expectations about the milk-corn price ratio. This ratio is "forecast" directly using AR(2) models, with and without monthly dummy variables (Models 2M and 1M, respectively). In addition, the separate components of the ratio are forecast from models of the two nominal prices (Model 3M) and from models of the two CPI-deflated prices (Model 4M). All of the models fit the sample period data about equally well (Table 4). For the simulations that include out-of-sample computations, the direct forecasts of the ratio stabilize at about 4.8, and a similar result is obtained when the ratio of the separate nominal forecasts is used (Figures 5c and 5e). The simulated values are naturally highly correlated. If one is working with nominal prices, the two approaches — direct estimate of the ratio or taking the ratio of nominal forecasts — are equivalent. Again, this does not mean that this is actually the way farmers form expectations.

The correlations among the various out-of-sample simulations of the milk-corn price ratio tend to be small and variable as compared with the correlations among the out-of-sample deflated annual price series simulations (Table 4). If the autoregression with monthly dummies is used, then the out-of-sample forecasts distinctly reflect the estimated monthly changes (Figure 5d). If the ratio of the forecasts of the separate CPI-deflated milk and prices is used, then the projected ratio rises to nearly double the sample mean by 36 months after the sample period (Figure 5f). This occurs because real corn prices are forecast to decline relative to real milk prices. This point is important, not because we would use separately forecast prices to forecast the ratio, but because an analyst might generate quasi-rationally expected prices individually and then use them in analyzing producers' decisions. (This is our understanding of the Chavas analysis of pork supply, where separate equations were fitted for the CPI-deflated prices of corn and pork.)

The definition of expected prices again makes a difference in estimates of risk. If farmers actually based expectations on a CPI-deflated price (which seems unlikely), then this measure could be smaller or larger than the nominal prices. In our sample, the CPI-deflated measure is slightly larger (Table 3). Of course, as previously discussed, neither the nominal nor CPI-deflated prices of milk forecast well outside of the sample — the prices which occurred just after the end of the sample period were not captured by the models. Both the in-sample and ex post forecasts of the milk-corn price ratio has the largest percentage RMSE among the choices compared, but this is perhaps a more plausible relative price to consider in milk supply analysis.

In sum, quasi-rational expectations assumes that producers use (implicitly) a time-series model of past prices as a basis for forming expectations, i.e., forecasting prices, and the foregoing analysis illustrates that an important prior issue is, what prices? Different deflators result in different empirical models of expectations. In addition, a model of deflated prices can sometimes produce a smaller RMSE than a model of nominal prices, but this does not necessarily mean that the deflated price series is relevant to farmers' decisions. Moreover, some deflated series have models that produce impossible or illogical outcomes (forecasts). Thus, analysts, who assume farmers' decisions are based on rational expectations, need to ask whether their models produce logical forecasts, both within- and out-of-sample.

A Milk Supply Model

Sun, Kaiser, and Forker (hereafter SKF) published a model of milk supply in 1995 that used quarterly observations for 1970 through 1992. The model consisted of three equations: retention rate of cows, recruitment rate of cows, and production per cow. The retention and recruitment equations determine the number of cows and heifers in the dairy herd. The equations use naïve expectations based on the ratio of milk to feed prices. SKF cite prior studies, including Chavas and Klemme, as justification for the naïve expectation specification. In SKF, expected price is a one-quarter lag of the price ratio, but the equations contain the respective dependent variables lagged (which could be interpreted as an adaptive expectations specification).

With Kaiser's assistance, the model was re-estimated for a different sample period, the first quarter of 1977 through the fourth quarter of 1994. Thus, results from the new sample are compared with the original analysis for selected coefficients (Table 5); specifically, the coefficients for the expected price ratio variable in two equations (retention rate and production per cow) are reported.[3] While the coefficients change considerably in the new sample, they are positive with t-ratios of 2.5 or larger.

Subsequently, we replaced the price of feed as the deflator with the CPI and the PPF in the entire model and re-estimated it using naïvely formed expectations of the deflated prices. In Table 5, the results are presented in the columns labeled *M/CPI* and *M/PPF* under the heading "Naïve." Statistical significance of the coefficients on the expected price variables worsened under the alternative deflators.

To explore an alternative definition of expectations, we developed time-series models of the three price ratios to specify quasi-rational expectations (Appendix 2). Using quarterly observations, the models were acquired using the empirical criteria discussed earlier. The "forecasts" are merely the fitted values from the estimated equations. That is, expected price is defined as the within-sample projection of the time-series equation, and of course is not a true forecast. Indeed, this approach likely exaggerates the potential forecast quality, as a true forecast would involve fitting an equation to historical data, making an out-of-sample, one-step ahead forecast, refitting the equation recursively adding a data point, making the next out-of-sample forecast, etc. Thus, our analysis should be biased toward the quasi-rational expectations approach, because the within-sample, best fit should out-perform the out-of-sample forecasts.

The results using the three quasi-rational expected prices are presented in the last three columns of Table 5. The coefficients of the expected milk-feed price ratio (*M/F*) have smaller t-ratios than those for naïve expectations, and one sign is negative. For the other expected price ratios, the coefficients have negative signs and/or small t-ratios. There are two points: naïve

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[3] In SKF, the expected price ratio appears only in these two equations. The recruitment rate is specified as a function of the past price ratios but not their expected values.

expectations appears to perform better than quasi-rational expectations specification in the SKF model, and different deflators give different results. In this example, the inferiority of the coefficient estimates of the alternatives to the naïve milk-feed price ratio is "obvious," but in other applications, different measures of expectations might perform better.

If price elasticity of aggregate milk production from the alternative specifications is considered, large differences exist (Table 5), but it is not obvious which is the preferred alternative.[4] Indeed, if the long-run is defined as a 10-year adjustment period, the elasticities range from 1.19 (based on the quasi-rational *M/CPI*]) to 6.90 (based on the naïve *M/PPF*). In SKF, the comparable estimate for earlier sample period was 2.28. The quasi-rational estimate of expected milk-feed ratio produced an elasticity estimate of 2.66 at 10 years. Thus, while the naïve milk-feed ratio produced the most logical estimates of the coefficients of the expected price variable, it results in seemingly large estimates of price elasticities of supply.

The bottom line is that the choice of deflators influences the choice of the time-series model used to represent expectations, and the alternative definitions of expectations can have a profound influence on empirical results. We are reminded that the definition of expectations is an integral part of model specification that impacts the model performance.

Conclusions

The empirical results reported in this paper illustrate that a deflated price series can have very different time-series properties than the original nominal series. Annual prices, that are essentially white noise, become autocorrelated with trends after deflating by general price indexes. On the one hand, it is surprising that nominal annual prices of agricultural commodities are not autocorrelated; theoretical models of prices suggest that they should be. On the other hand, if a nominal price series has no trend, then dividing by an upward trending index will result in a time series with a downward trend. A variety of models, which might involve stochastic or deterministic trends, may fit such data reasonably well within the sample period. Nonetheless, it is difficult to find a model that performs well beyond the sample.

In the case of two monthly (or quarterly) series which both have seasonal components, we should not be surprised that their ratio has a modified seasonal pattern. This is analogous to using a seasonal adjuster, and "seasonal effects" can be modeled in several ways.

It follows that deflating influences the definitions of expected prices. Since quasi-rational expectations assume that farmers base expectations on time-series forecasts of prices and since the time-series properties will vary with the deflator used, it must be the case that different deflators will result in different empirical models of expectations. These alternative definitions of expectations can, in turn, result in varying estimates of supply elasticities. The different measures of expectations also imply diverse estimates of price risk. Given that actual

[4] SKF calculate the supply elasticity as the ratio of percentage change of total production to a (permanent) 10 percent increase in the milk price. Following their procedure, the estimated model is dynamically simulated from the first quarter of 1977 with historical data, and then, with 10 percent increased milk prices.

expectations are unobservable, most desirable models should produce logical results for various definitions of expectations. Sensitivity analysis with respect to alternate measures of expectations might be appropriate.

Obviously, the more fundamental question is, what is the relevant set of relative prices for the research problem? To the extent that theory and logic cannot answer this question, the researcher is dependent on empirical criteria. Naïve expectations performed well in our examples. Yet, perhaps we should not be surprised that it did for the grains, since "...futures [contract] prices reflect essentially no prophecy that is not reflected in the cash price...(Tomek and Gray, p. 273);" i.e., futures and cash prices are expected to be highly correlated.

The decision whether or not to deflate and by what deflator requires thoughtful justification. It is important to ask, can the deflator used in the analysis be justified theoretically for the analyst's particular research problem? Also, does the model of the deflated series produce logical results? In particular, models that involve trending series can quickly produce illogical out-of-sample forecasts; the so-called rational forecast is not rational. These points are perhaps obvious, but are too often ignored.

Table 1. Descriptive Statistics for Prices

Variable	Mean	Std. Dev.
Annual (1973-1997)		
Corn, \$/bu	2.670	0.474
Soybean, \$/bu	6.216	0.795
CPI, 1982-84=100	107.921	36.270
PPF, 1992=100	88.712	19.405
Monthly (1989.9-1998.8)		
Milk, \$/cwt	12.187	1.177
Corn, \$/bu	2.686	0.565
CPI, 1982-84=100	146.652	10.963

Table 2. Standard Error of Residuals, Selected Models

Model	Std. Error
Corn prices, annual	
Nominal price level	0.474
CPI-deflated, first differences	0.659
CPI-deflated, first differences, MA(2)	0.631
CPI-deflated, AR(1) with trend	0.519
PPF-deflated, first differences	0.688
PPF-deflated, first differences, MA(1)	0.677
PPF-deflated, AR(1) with trend	0.575
Soybean prices, annual	
Nominal price level	0.795
CPI-deflated, AR(2)	1.284
CPI-deflated, first differences, AR(1)	1.144
CPI-deflated, AR(1) with trend	1.179
PPF-deflated, AR(2)	1.335
PPF-deflated, first differences, AR(1)	1.283
PPF-deflated, AR(1) with trend	1.202
Milk prices, monthly	
Nominal price level, AR(3)	0.523
CPI-deflated, AR(3)	0.363
/Corn price, AR(2)	0.313
/Corn price, AR(2) with monthly dummy variables	0.298

Table 3. Root Mean Square Errors of Forecasts

	Type of Forecast[a]	
	In sample	Ex post
	- - - - - percent - - - - -	
Corn, nominal (annual)	4.59	
Corn, CPI-deflated (annual)		
Model 1A[b]	11.69	
Model 2A	4.42	
Model 3A	3.93	
Model 4A	4.48	
Milk, nominal (monthly)	0.18	2.89
Milk, CPI-deflated (monthly)	0.19	3.86
Milk/Corn (monthly)		
Model 1M	0.42	3.89
Model 2M	0.95	3.80
Model 3M	0.40	3.83
Model 4M	0.41	11.61

a/ "Ex post" forecasts are for 1998.9 through 1999.8. See text for additional information.

b/ Model codes: 1A = first differences as white noise; 2A = first differences as MA(2); 3A = level as AR(1) + deterministic trend; 4A = ratio of separate estimates of the two series; 1M = ratio as AR(2); 2M = ratio as AR(2) + monthly shifts; 3M = ratio of separate estimates of nominal prices; 4M = ratio of separate estimates of CPI-deflated prices.

Table 4. Correlation Coefficients among Alternative Simulations[a]

A. CPI-deflated corn price, 1973-97					
	Data	1A	2A	3A	4A
Data	1.0	0.861	0.887	0.899	0.866
1A		1.0	0.877	0.890	0.908
2A			1.0	0.993	0.974
3A				1.0	0.988
4A					1.0
B. Milk-corn price ratio, 1989.9-98.8					
	Data	1M	2M	3M	4M
Data	1.0	0.917	0.865	0.919	0.921
1M		1.0	0.933	0.989	0.989
2M			1.0	0.921	0.919
3M				1.0	0.999
4M					1.0
C. Milk-corn price ratio, 1989.9-2001.8					
		1M	2M	3M	4M
1M		1.0	0.647	0.990	0.523
2M			1.0	0.634	0.658
3M				1.0	0.501
4M					1.0

a/ Model codes: 1A = first differences as white noise; 2A = first differences as MA(2); 3A = level as AR(1) + deterministic trend; 4A = ratio of separate estimates of the two series; 1M = ratio as AR(2); 2M = ratio as AR(2) + monthly shifts; 3M = ratio of separate estimates of nominal prices; 4M = ratio of separate estimates of CPI-deflated prices.

Table 5. Selected Price Coefficients and Price Elasticities of Supply for Milk

	Original[a]	Replication					
	Naïve				Quasi-Rational		
	M/F	*M/F*	*M/CPI*	*M/PPF*	*M/F*	*M/CPI*	*M/PPF*
Coefficient A1[b]	6.15	28.11	13.26	13.99	13.82	-5.56	-0.02
	(5.40)[c]	(2.75)	(1.64)	(1.74)	(1.40)	(0.61)	(0.00)
Coefficient C1	2526.6	2191.8	99.05	1582.2	-137.5	-1522.6	242.6
	(3.12)	(2.51)	(0.16)	(2.02)	(0.12)	(2.80)	(0.46)
Price elasticities							
Short-run (1 quarter)	0.09	0.99	1.03	1.23	0.94	-0.09	0.02
Intermediate-run (1 year)	0.22	0.55	0.30	0.85	0.36	0.03	0.34
Long-run (10 years)	2.28	5.30	1.32	6.90	2.66	1.19	2.41

a/ SKF results for 1972(4th quarter)-1992(4th quarter). Replication results for 1977(1st quarter)-1994(4th quarter), using different deflators and price expectations.

b/ A1 and C1 are the price coefficients in the retention rate of cows and production per cow equations in SKF, respectively.

c/ t-ratio.

Figure 1. Annual Prices, 1973/74-1997/98

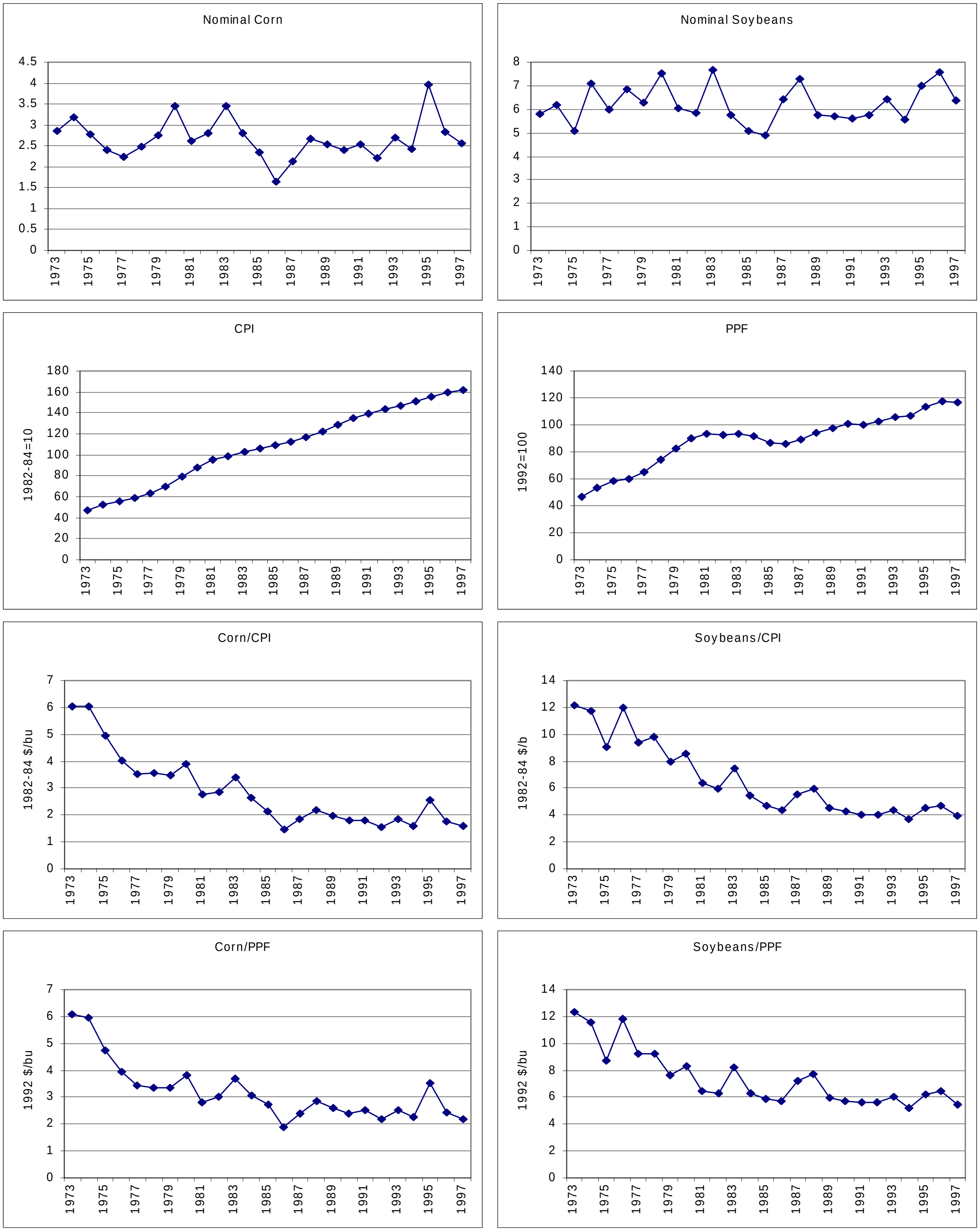


Figure 2. Monthly Prices, 1989.9-1998.8

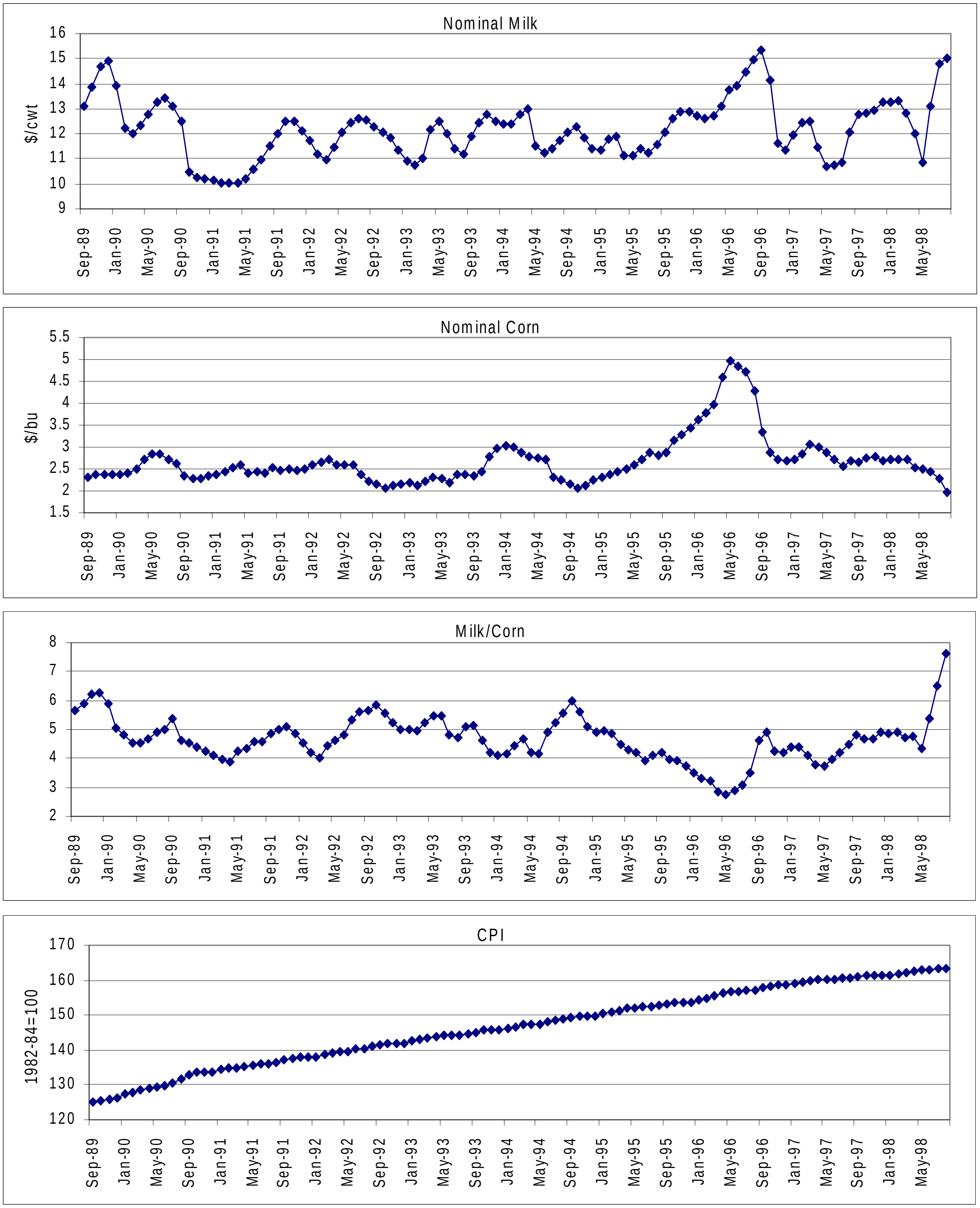


Figure 2. Continued.

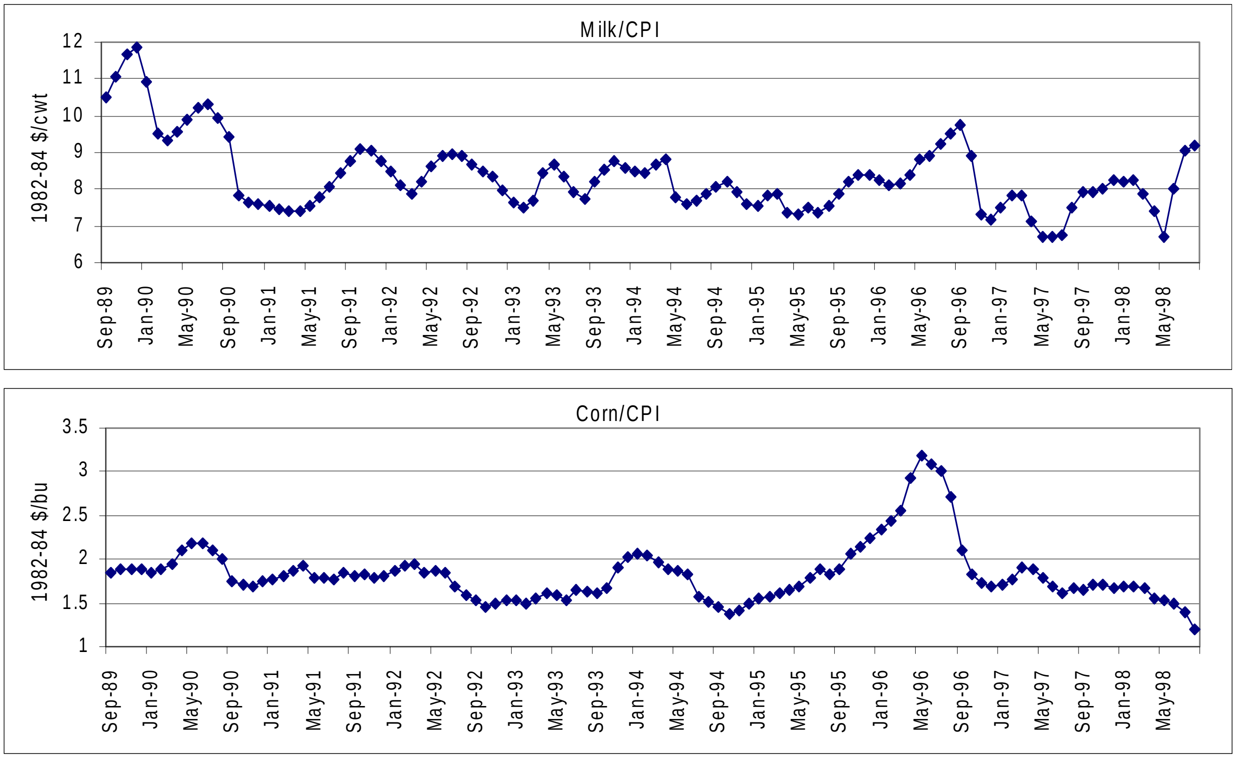


Figure 3. Periodograms for Selected Monthly Series

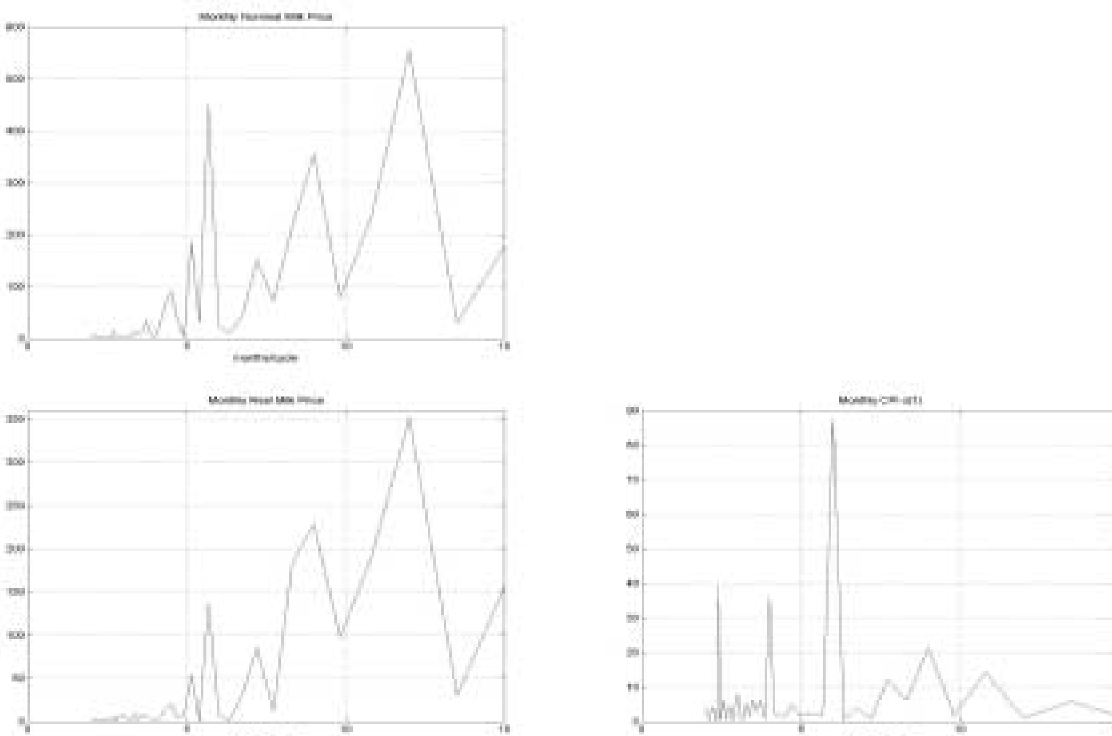


Figure 3. Continued.

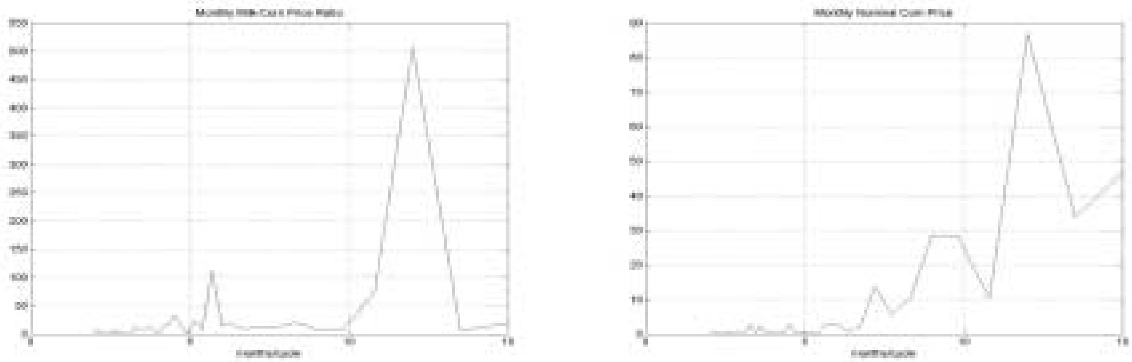
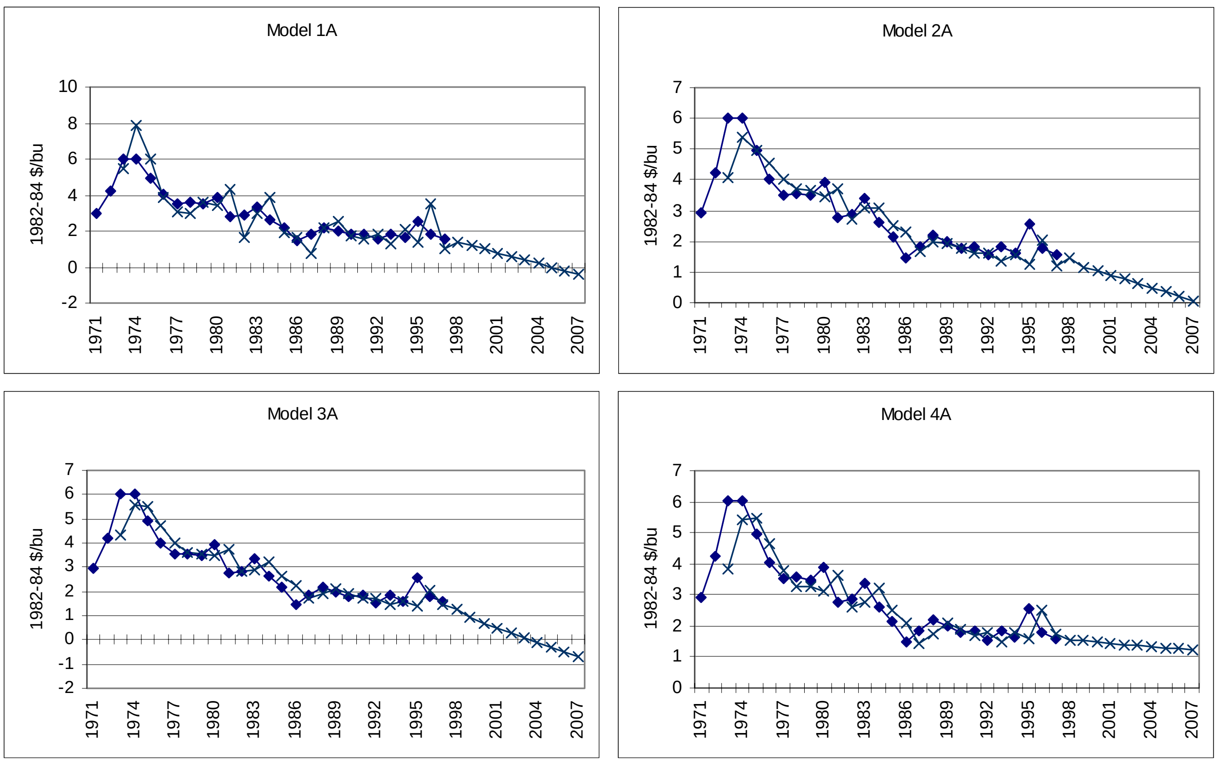
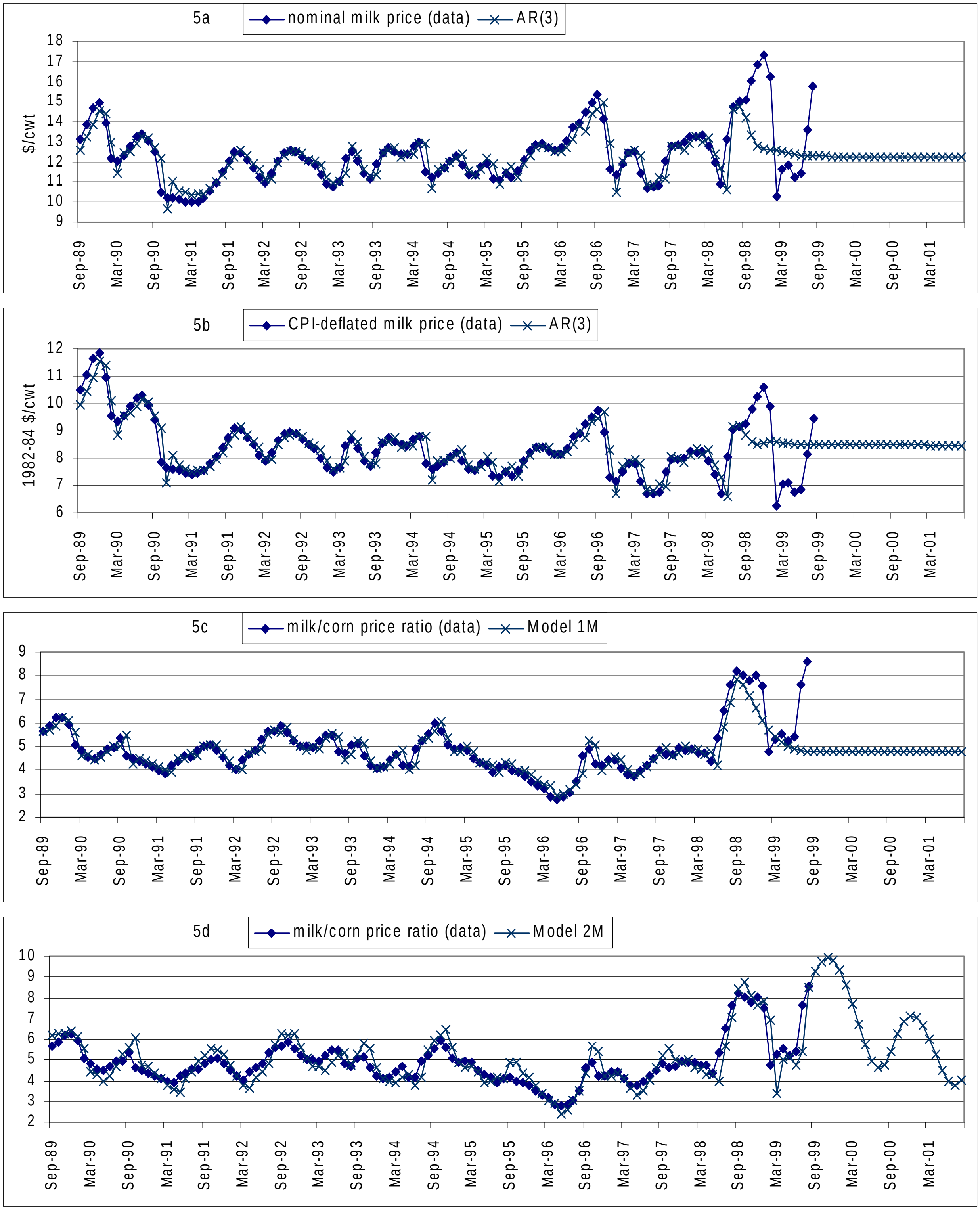


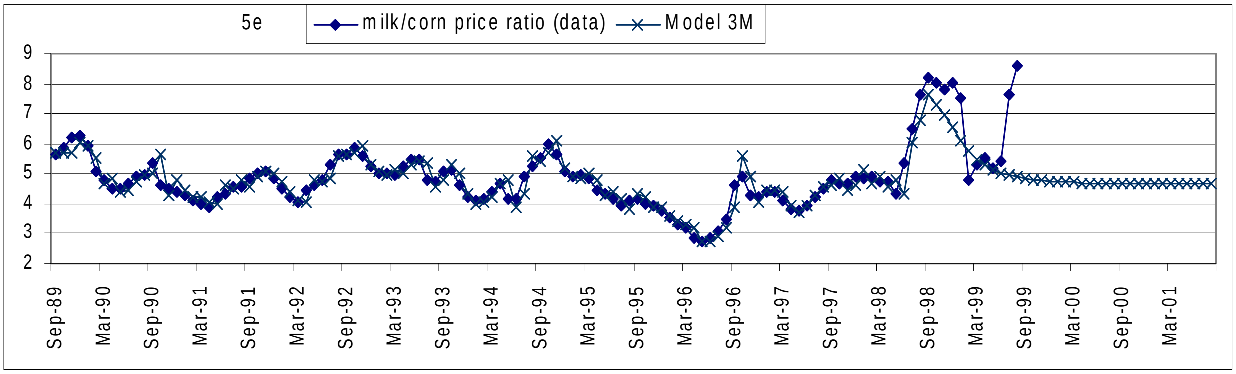
Figure 4. ARIMA Forecasts, Annual Corn Prices Deflated by CPI[a]



a/ Legend: ♦ = data, × = ARIMA forecasts. Model codes: 1A = first differences as white noise; 2A = first differences as MA(2); 3A = level as AR(1) + deterministic trend; 4A = ratio of separate estimates of the two series.

Figure 5. ARIMA Forecasts, Monthly Milk Prices





5e
milk/corn price ratio (data)
Model 3M
9
8
7
6
5
4
3
2
Sep-89
Mar-90
Sep-90
Mar-91
Sep-91
Mar-92
Sep-92
Mar-93
Sep-93
Mar-94
Sep-94
Mar-95
Sep-95
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Mar-97
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Mar-98
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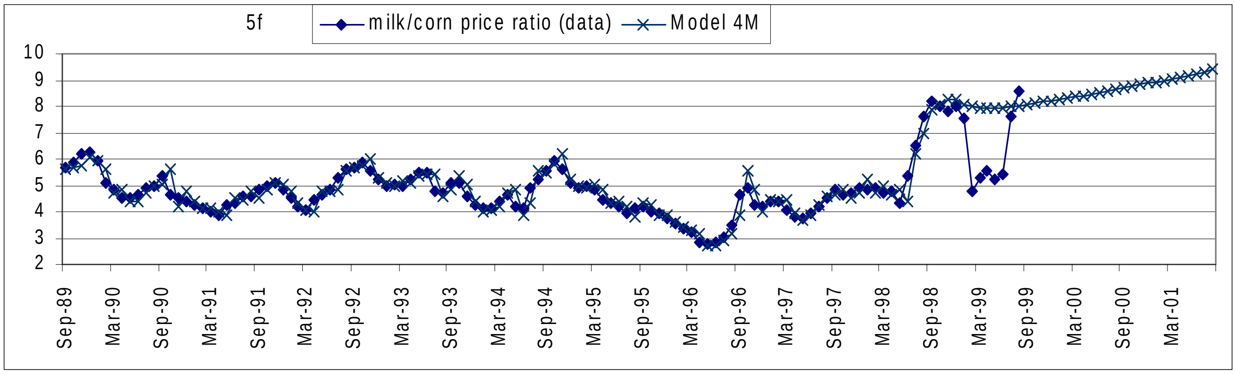
Sep-99
Mar-00
Sep-00
Mar-01



5f
milk/corn price ratio (data)
Model 4M
10
9
8
7
6
5
4
3
2
Sep-89
Mar-90
Sep-90
Mar-91
Sep-91
Mar-92
Sep-92
Mar-93
Sep-93
Mar-94
Sep-94
Mar-95
Sep-95
Mar-96
Sep-96
Mar-97
Sep-97
Mar-98
Sep-98
Mar-99
Sep-99
Mar-00
Sep-00
Mar-01

Appendix 1. Formulas Used to Calculate Maximum Entropy Spectrum

It is shown elsewhere that the maximum entropy spectrum takes an identical form to the spectrum of AR processes (Burg, Woitek).

The spectrum of the AR(p) process, $\left(1-\sum_{j=1}^{p}\phi_j \,\mathrm{L}^j\right) y_t = \varepsilon_t$ (white noise), is given by (Harvey):

$$f(\lambda)=\left(\frac{\sigma^2}{2\pi}\right)\left(\frac{1}{\left|1-\sum_{j=1}^{p}\phi_j \,\mathrm{e}^{-i\lambda j}\right|^2}\right), \quad 0<\lambda<\pi .$$

Thus, for an AR(1) process,

$$f(\lambda)=\left(\frac{\sigma^2}{2\pi}\right)\left(\frac{1}{1+\phi^2-2\phi\cos\lambda}\right), \quad 0<\lambda<\pi .$$

For an AR(2) process, the spectrum is

$$f(\lambda)=\left(\frac{\sigma^2}{2\pi}\right)\left(\frac{1}{1+\phi_1^2+\phi_2^2-2\phi_1(1-\phi_2)\cos\lambda-2\phi_2\cos 2\lambda}\right), \quad 0<\lambda<\pi ,$$

and for an AR(3) process,

$$f(\lambda)=\left(\frac{\sigma^2}{2\pi}\right)\left(\frac{1}{1+\phi_1^2+\phi_2^2+\phi_3^2-2(\phi_1-\phi_1\phi_2-\phi_2\phi_3)\cos\lambda-2(\phi_2-\phi_1\phi_3)\cos 2\lambda-2\phi_3\cos 3\lambda}\right),$$

$0<\lambda<\pi$.

Appendix 2. Estimated ARIMA Models[a]

Annual, 1973/74-1997/98	
Corn	
CPI-deflated [3A]	$(1 - 0.716\,\mathrm{L}^1)\, y_t = 1.478 - 0.169\, T_t$
	(0.15) (0.60) (0.04)
CPI-deflated, first differences [2A]	$\Delta y_t = -0.137 - 0.271\,\Delta e_{t-1} - 0.398\,\Delta e_{t-2}$
	(0.05) (0.24) (0.22)
PPF-deflated, first differences	$\Delta y_t = -0.112 - 0.483\,\Delta e_{t-1}$
	(0.07) (0.19)
Soybeans	
CPI-deflated	$y_t = 0.321 + 0.535\, y_{t-1} + 0.424\, y_{t-2}$
	(3.80) (0.18) (0.19)
CPI-deflated, first differences	$\Delta y_t = -0.548 - 0.525\,\Delta y_{t-1}$
	(0.15) (0.17)

Soybeans (Continued)	
PPF-deflated	$y_t = 0.714 + 0.525\, y_{t-1} + 0.388\, y_{t-2}$
	(2.19) (0.19) (0.20)
PPF-deflated, first differences	$\Delta y_t = -0.465 - 0.456\, \Delta y_{t-1}$
	(0.18) (0.18)
CPI, first differences	$\Delta y_t = 2.318 + 0.945\, \Delta y_{t-1} - 0.442\, \Delta y_{t-2}$
	(0.49) (0.19) (0.19)
PPF, first differences[b]	$\Delta y_t = 2.985 + 0.785\, \Delta e_{t-1}$
	(0.88) (0.14)
	$\Delta y_t = 1.401 + 0.833\, \Delta y_{t-1} - 0.319\, \Delta y_{t-2}$
	(1.06) (0.21) (0.21)
<u>Monthly, 1989.9-1998.8</u>	
Milk	
Nominal	$y_t = 2.113 + 1.444\, y_{t-1} - 0.891\, y_{t-2} + 0.275\, y_{t-3}$
	(0.28) (0.09) (0.15) (0.10)
CPI-deflated	$y_t = 0.854 + 1.518\, y_{t-1} - 0.929\, y_{t-2} + 0.310\, y_{t-3}$
	(0.33) (0.09) (0.15) (0.09)
/Corn price [1M]	$y_t = 0.578 + 1.394\, y_{t-1} - 0.514\, y_{t-2}$
	(0.25) (0.09) (0.10)
[2M]	$y_t = 0.394 + 1.301\, y_{t-1} - 0.382\, y_{t-2} - 0.175\, Feb_t - 0.324\, Mar_t$
	(0.38) (0.10) (0.10) (0.10) (0.16)
	$- 0.395\, Apr_t - 0.461\, May_t - 0.296\, Jun_t - 0.040\, Jul_t + 0.242\, Aug_t$
	(0.21) (0.23) (0.25) (0.26) (0.25)
	$+ 0.617\, Sep_t + 0.612\, Oct_t + 0.402\, Nov_t + 0.193\, Dec_t$
	(0.24) (0.21) (0.16) (0.10)
Corn	
nominal	$y_t = 0.200 + 1.550\, y_{t-1} - 0.626\, y_{t-2}$
	(0.17) (0.08) (0.08)
CPI-deflated, first differences	$\Delta y_t = -0.014 + 0.476\, \Delta y_{t-1} - 0.251\, \Delta y_{t-3}$
	(0.01) (0.05) (0.05)
CPI, first differences	$\Delta y_t = 0.364 + 0.280\, \Delta e_{t-1} + 0.272\, \Delta e_{t-12} + 0.320\, \Delta e_{t-24} + 0.304\, \Delta e_{t-36}$
	(0.04) (0.09) (0.09) (0.11) (0.12)
<u>Used in Supply Analysis (quarterly, 1970-1994)</u>	
M/F	$(1 - 0.489\, L^1 - 0.314\, L^4 + 0.338\, L^5)\, y_t = 0.039 + 0.437\, e_{t-1}$
	(0.11) (0.10) (0.10) (0.00) (0.12)
M/CPI	$(1 - 0.362\, L^1 + 0.186\, L^5 - 0.270\, L^8)(1 + 0.630\, L^2)\, y_t = -0.057$
	(0.09) (0.09) (0.09) (0.09) (0.05)
M/PPF	$(1 + 0.921\, L^2)(1 + 0.619\, L^4)\, y_t = -0.165 - 0.387\, e_{t-3} - 0.422\, e_{t-5}$
	(0.04) (0.09) (0.00) (0.09) (0.09)

[a/] Figures in ( ) are approximate standard errors. Labels in [ ] are models defined in text. *T* is annual trend (1973=1); *Feb*, …, *Dec* are monthly dummies variables that equals one in respective months and 0 otherwise. L is the lag operator.

[b/] The second equation used to calculate maximum entropy spectrum

References

Akaike, H. "Power Spectrum Estimation through Autoregressive Model Fitting." *Annals of the Institute of Statistical Mathematics* 21(1969):407-419.

Antonovitz, F., and R. Green. "Alternative Estimates of Fed Beef Supply Response to Risk." *Amer. J. Agr. Econ.* 72(1990): 475-487.

Burg, J. "Maximum Entropy Spectral Analysis." *Proceedings of the 37th Meeting of the Society of Exploration Geophysicists*, 1967. Reprinted in Childers, D. *Modern Spectrum Analysis*. New York, NY: IEEE Press, 1978.

Chavas, J-P. "On the Economic Rationality of Market Participants: The Case of Expectations in the U.S. Pork Market." *J. Agr. Res. Econ.* 24(1, 1999): 19-37.

Chavas, J-P., and R. M. Klemme. "Aggregate Milk Supply Response and Investment Behavior on U.S. Dairy Farms." *Amer. J. Agr. Econ.* 68(February 1987): 55-66.

Deaton, A., and G. Laroque. "On the Behavior of Commodity Prices." *Rev. Econ. Studies* 59(1992): 1-23.

Feige, E. L., and D. K. Pearce. "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Monetary and Fiscal Policy?" *J. Polit. Econ.* 84(1976):499-522.

Hamilton, J. D. *Time Series Analysis*. Princeton, NJ: Princeton University Press, 1994.

Harvey, A. C. *Time Series Models*, 2nd Edition. Cambridge, MA: The MIT Press, 1993.

Kuznets, S. S. *Capital and the American Economy: Its Formation and Financing*. New York, NY: National Bureau of Economic Research, 1961.

Nerlove, M., D. M. Grether, and J. L. Carvalho. *Analysis of Economic Time Series: A Synthesis*. New York, NY: Academic Press, 1979.

Sun, C-H., H. M. Kaiser, and O. D. Forker. "Analysis of Seasonal Milk Price Incentive Plans." *Rev. of Agr. Econ.* 17 (1995): 383-393.

Tomek, W. G., and R. W. Gray. "Temporal Relationships Among Prices on Commodity Futures Markets: Their Allocative and Stabilizing Roles." *Amer. J. Agr. Econ.* 52(August 1970):372-380.

Tronstand, R., and T. J. McNeill. "Asymmetric Price Risk: An Econometric Analysis of Aggregate Sow Farrowings, 1973-87." *Amer. J. Agr. Econ.* 71(1989):630-637.

Woitek, U. *Business Cycles: An International Comparison of Stylized Facts in a Historical Perspective.* Germany: Physica-Verlag, 1997.

Working, H. "Note on the Correlation of First Differences of Averages in a Random Chain." *Econometrica* 28(March 1960):916-918.