NCCC-134

Modeling Contract Form:

An Examination of Cash Settled Futures

by

Dwight R. Sanders and Mark R. Manfredo

Modeling Contract Form: An Examination of Cash Settled Futures

Dwight R. Sanders

and

Mark R. Manfredo[*]

[*]Dwight R. Sanders (DwightS@siu.edu) is an Assistant Professor of Agribusiness Economics at
Southern Illinois University, Carbondale, Illinois. Mark R. Manfredo is an Assistant Professor at
Arizona State University.

Modeling Contract Form: An Examination of Cash Settled Futures

Abstract

This research presents an intuitive interpretation and expression for pricing cash settled futures contracts. In particular, the choice of the averaging period for the underlying cash index is evaluated. For example, the averaging period for the Lean Hog futures contract is two days, whereas it is thirty days for the Fed funds contract. Does the choice of the averaging period make a difference? Under certain assumptions, the behavior of the futures price prior to entering the expiration or averaging interval is independent of the length of the interval for storable commodities, but it is not for non-storable commodities.

Keywords: cash settlement, contract form, new futures contracts

Introduction

The Chicago Mercantile Exchange's lean hog futures contract cash settles to a two-day average of the underlying cash market; whereas, the Chicago Board of Trade's Fed funds futures contract settles to the monthly average of the underlying cash market. Clearly, forms of settlement can vary widely across cash settled futures contracts. The choice of the expiration interval for cash settled futures contracts (e.g., two days for lean hogs versus 30 calendar days for Fed funds) impacts how the futures price behaves in terms of volatility, pricing, and basis convergence. When an exchange develops a cash settled futures contract, they are faced with this problem: what should be the form of final settlement? The objective of this research is to provide insight into price behavior based on the length of the expiration interval. Specifically, the research develops a simple analytical expression for the pricing of cash settled futures contracts. Thereby, it develops an intuitive framework in which to examine alternative forms of cash settlement.

Optimal futures contract design involves two distinct steps (Tashjian). First, an underlying asset must be identified (corn or crude oil). Second, the settlement mechanism of the futures contract must be specified (physical delivery or cash settlement). Researchers have expended considerable effort to determine characteristics in the underlying asset market that are important for successful futures contracts (Black). Numerous case studies have identified instances where these conditions have not been met and contracts fail (Garcia, Thompson, and Wildman). However, far less research effort has explored the form of the futures contract in determining its success or failure, and that line of research has primarily focused on embedded options in delivery-settled futures (Martinez-Garmendia and Anderson). Other researchers have focused on alternative ways to construct underlying cash indices. This research assumes that a cash settled contract is optimal, and then examines alternative statistical methods to construct the underlying cash index (e.g., Cita and Lien). This line of research is important because it addresses a topic that academic researchers often take as a given; yet, it is a real problem for practitioners and industry researchers (personal interviews).

Cash settlement relies on "good" underlying cash prices, which are aggregated across time to create a "settlement index" (Jones; Garbade and Silber). The contract designer must select an expiration interval: the amount of time over which to calculate the final settlement index. This research examines how the choice of the expiration interval impacts the pricing of cash settled

futures contracts for storable and non-storable commodities. We develop an analytical expression for the futures price as a function of the expiration interval. The result is not based on an agent's optimization problem. Rather, it assumes that cash settlement is optimal and then derives a pricing expression. Examples and discussion build the intuition behind the analytics. The results of this research are important for two reasons. First, it is a research area that has not previously received a great deal of attention (Tashjian). Second, it provides a guide for examining cash settled futures prices. The Commodity Futures Trading Commission requires that the calculation procedure for cash settled futures contracts provides safeguards against manipulation. Time diversification is one such safeguard. Thus, the choice of expiration intervals is important from a performance and regulatory standpoint.

Methodology

Cash Settlement over an Expiration Interval
In this section the problem and terminology is presented. First, a *settlement time*, T, is defined as the date at which the futures contract ceases to trade and the price is set equal to the average spot price over the proceeding T-day *expiration interval*.   This settlement procedure is clarified in the following timeline where the time subscripts represent days into the T-length expiration interval. So, the spot price the day prior to entering the expiration interval is S_0. The first day into the expiration interval is S_1, and S_T is the final day in the expiration interval or the settlement day.
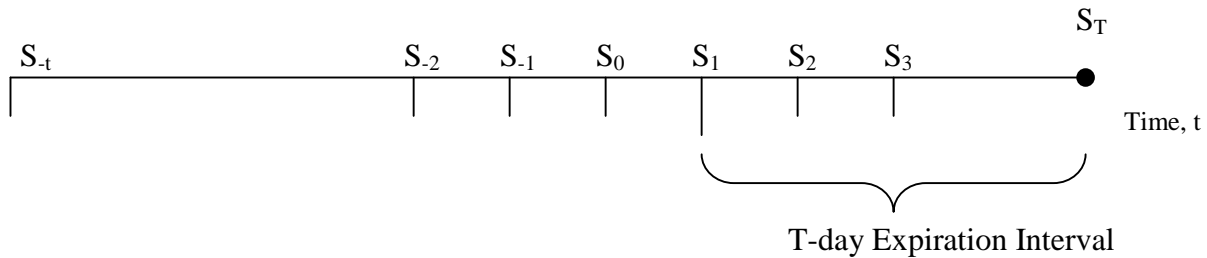
Next, define a futures contract that cash settles at time T to the average spot price over the proceeding T-day expiration interval,

$$F_{t,T} = F_{T,T} = \frac{1}{T}\sum_{i=1}^{T} S_i \quad \textit{for } t = T.$$

Equation 1

The futures price at any time t for settlement date T is denoted as $F_{t,T}$. On the settlement day, t=T, the futures price, $F_{T,T}$, by definition equals the average spot price, S, over the prior T trading days.

For example, say that T equals the 20th and last business day of December (T=20). Then the futures contract settles to the average spot price over the proceeding 20 business days—that is, the average spot price for the month:

$$F_{20,20} = \frac{1}{20}\sum_{i=1}^{20} S_i \quad \textit{for } t = T = 20.$$

3

The Case of Storable Commodities

How will the futures contract trade prior to and during the expiration interval? Analytically, this is answered by taking expectations of Equation 1 above. In an efficient and rational market, the futures price at time t for settlement time T is equal to the expected settlement price. So in the case of the cash settled futures contract,

$$F_{t,T} = E_t\left[\frac{1}{T}\sum_{i=1}^{T}S_i\right] \qquad for\ t \leq T.$$

Equation 2a

Expanding Equation 2, we get the following relationships:

$$F_{t,T} = \frac{1}{T}E_t[S_1 + S_2 + S_3 + S_4 + ... + S_{T-1} + S_T],$$

Equation 2b

$$= \frac{1}{T}[E_t(S_1) + E_t(S_2) + E_t(S_3) + E_t(S_4) + ... + E_t(S_{T-1}) + E_t(S_T)].$$

Equation 2c

Much of the intuition in understanding the trading characteristics comes from Equation 2c. This equation clearly shows that a futures contract that expires to a T-day average is equivalent to a strip or equally weighted portfolio of T daily contracts, each expiring on their respective day during the expiration interval to the underlying spot index.

Based on simple arbitrage arguments the following relationships hold for storable commodities (Hull, p. 53):

$$E_t(S_K) = S_K \quad for\ K \leq t \quad (known\ past\ and\ present\ values),$$

Equation 3a

$$E_t(S_K) = S_t e^{c(K-t)} \ for\ K > t \quad (unknown\ future\ values).$$

Equation 3b

In Equation 3a, the expectation for past and present spot prices is simply their realized values. While in Equation 3b, the expectation for unknown future values must equal the current spot price adjusted for carrying costs; otherwise, arbitrage opportunities would exist. For the purposes of this paper, c represents the total daily carry costs (storage, insurance, interest, and convenience yield) as a proportion of the price and it is assumed to be constant. The time in storage is then K-t days.

Equations 2 and 3 can be used to analyze the trading characteristics of a futures contract that settles to a T-day average as defined in Equation 1. It is interesting to examine the futures price at a few key points in time (see the above timeline). Prior to entering the expiration interval (t<1), the futures pricing expression can first be written directly from Equations 2b and 2c:

$$F_{t,T} = \frac{1}{T} E_t \left[S_1 + S_2 + S_3 + S_4 + \ldots + S_{T-1} + S_T \right],$$

$$= \frac{1}{T} \left[E_t(S_1) + E_t(S_2) + E_t(S_3) + E_t(S_4) + \ldots + E_t(S_{T-1}) + E_t(S_T) \right].$$

Then, substituting Equation 3b for the expected spot prices, we get Equation 4,

$$= \frac{1}{T} \left[S_t e^{c(1-t)} + S_t e^{c(2-t)} + S_t e^{c(3-t)} + S_t e^{c(4-t)} + \ldots + S_t e^{c(T-1-t)} + S_t e^{c(T-t)} \right],$$

$$= S_t \frac{1}{T} \sum_{i=1}^{T} e^{c(i-t)}. \qquad\qquad \text{Equation 4}$$

So, prior to entering the expiration interval (t<1), the futures price, $F_{t,T}$, will equal the current spot price, S_t, times the average proportional cost of carry to and through the expiration interval.

For example, assume it is 90 days prior to entering a 20 day expiration interval (t=-90 and T=20). Using Equation 4, we get the following relationship,

$$F_{-90,20} = \frac{1}{20} \left[S_{-90} e^{c(91)} + S_{-90} e^{c(92)} + S_{-90} e^{c(93)} + S_{-90} e^{c(94)} + \ldots + S_{-90} e^{c(109)} + S_{-90} e^{c(110)} \right],$$

$$= S_{-90} \frac{1}{20} \left[e^{c(91)} + e^{c(92)} + e^{c(93)} + e^{c(94)} + \ldots + e^{c(109)} + e^{c(110)} \right].$$

If the current spot price is 200.0 ($S_{-90} = 200.0$) and c=0.05%, then the futures price is equal to 210.3—the current spot price times the average proportional cost to carry to and through the expiration interval.

The second time period of interest is the futures price behavior during the expiration interval ($1 \leq t \leq T$). Again, utilizing Equations 2 and 3, the futures price can be presented as follows:

$$F_{t,T} = \frac{1}{T} E_t \left[S_1 + S_2 + \ldots + S_t + \ldots + S_{T-1} + S_T \right],$$

$$= \frac{1}{T} \left[E_t(S_1) + E_t(S_2) + \ldots + E_t(S_t) + \ldots + E_t(S_{T-1}) + E_t(S_T) \right],$$

$$= \frac{1}{T} \left[S_1 + S_2 + \ldots + S_{t-1} + S_t + S_t e^{c(t+1-t)} + \ldots + S_t e^{c(T-1-t)} + S_t e^{c(T-t)} \right],$$

$$= \frac{1}{T} \left[\sum_{i=1}^{t} S_i + \sum_{j=t+1}^{T} S_t e^{c(j-t)} \right]. \qquad\qquad \text{Equation 5}$$

Intuitively, as we move through the expiration interval (t=1,2,3,…,T-1,T) the actual prices to be used in calculating the final time T settlement price are becoming known. So, at any arbitrary time t, the futures price will equal a weighted average of the known spot prices (first term in

Equation 5) plus the expected spot prices for the remainder of the expiration interval (second term in Equation 5). As time passes, an increasing number of spot prices used in calculating the cash settlement average become known. Thus, the futures price converges toward the T-day average settlement price, and volatility, relative to the underlying spot price, declines. Importantly, the correlation between the futures and the underlying spot cash price will also decline during the expiration interval.

To illustrate this process, it is useful to work through a couple of key days and examples. Consider the first day of the expiration interval, t=1 in Equation 5,

$$
\begin{aligned}
F_{1,T} &= \frac{1}{T}\left[\sum_{i=1}^{1} S_i + \sum_{j=2}^{T} S_1 e^{c(j-1)}\right], \\
&= \frac{1}{T}\left[S_1 + S_1 \sum_{j=2}^{T} e^{c(j-1)}\right], \\
&= \frac{1}{T}\left[S_1(1 + \sum_{j=2}^{T} e^{c(j-1)})\right], \\
&= S_1\left[\frac{(1 + \sum_{j=2}^{T} e^{c(j-1)})}{T}\right].
\end{aligned}
$$

So, on the first day of the settlement period (t=1), the futures price will equal the spot price on that day, S_1, times the average proportional cost of carry through the remaining (T-1) days in the expiration interval. All else equal, this is the day when futures price will have the narrowest basis to the underlying cash or spot price. Prior to this day the futures price will reflect additional carrying costs. After this date, the futures price incorporates known spot prices used in calculating the ultimate settlement price; thus, it will not fully respond to subsequent changes in spot market prices.

For example, consider day 5 in a 20-day expiration interval,

$$
\begin{aligned}
F_{5,20} &= \frac{1}{20}\left[\sum_{i=1}^{5} S_i + \sum_{j=6}^{20} S_5 e^{c(j-1)}\right], \\
&= \frac{1}{20}\left[S_1 + S_2 + S_3 + S_4 + S_5 + \sum_{j=6}^{20} S_5 e^{c(j-1)}\right].
\end{aligned}
$$

At the end of day 5, the futures price will equal the average of the first five days of the expiration interval plus the expected spot price for the remaining 15 days. The values for the first five days are known and fixed. Therefore, the futures price is less responsive to changes in the spot price than it was on (say) the first day of the expiration interval.

Now, consider day 15 in a 20 day expiration interval,

$$F_{15,20} = \frac{1}{20}\left[\sum_{i=1}^{15} S_i + \sum_{j=16}^{20} S_{15} e^{c(j-1)}\right],$$

$$= \frac{1}{20}\left[S_1 + S_2 + S_3 + \ldots + S_{14} + S_{15} + \sum_{j=16}^{20} S_{15} e^{c(j-1)}\right].$$

The first 15 days used in calculating the cash settlement price are known and fixed. Therefore, the change in the spot price over the remaining five days has a relatively small impact on the settlement price.

Finally, on the actual expiration day, t=20. The futures price equals the cash settlement index as defined in Equation 1,

$$F_{20,20} = \frac{1}{20}\left[\sum_{i=1}^{20} S_i\right],$$

$$= \frac{1}{20}\left[S_1 + S_2 + S_3 + \ldots + S_{19} + S_{20}\right]$$

The futures ultimately cash settle to the average underlying cash or spot price reported over the T-day expiration interval.

The Case of Non-Storable Commodities
In this section, we examine futures contracts on non-storable commodities that cash settle to the average of a spot index, S, over a T-day expiration interval. The same notation and naming conventions are utilized as in the prior section.

Clearly, the biggest difference between storable and non-storable commodities is that the economic arbitrage conditions in Equation 3b do not apply to non-storable commodities. So, the futures price will purely reflect the spot price expected to prevail over the expiration interval—this may or may not be closely aligned with the current spot price.

Again, define a futures contract that settles to the average spot price, S, over the proceeding T-day expiration interval (Equation 1):

$$F_{t,T} = F_{T,T} = \frac{1}{T}\sum_{i=1}^{T} S_i \quad \text{for } t = T. \qquad\qquad \text{Equation 1}$$

Then the futures price is again defined by Equation 2,

$$F_{t,T} = E_t\left[\frac{1}{T}\sum_{i=1}^{T} S_i\right] \qquad \text{for } t \leq T, \qquad\qquad \text{Equation 2a}$$

$$= \frac{1}{T} E_t\left[S_1 + S_2 + S_3 + S_4 + \ldots + S_{T-1} + S_T\right], \qquad\qquad \text{Equation 2b}$$

$$= \frac{1}{T}\left[E_t(S_1) + E_t(S_2) + E_t(S_3) + E_t(S_4) + \ldots + E_t(S_{T-1}) + E_t(S_T)\right]. \qquad\qquad \text{Equation 2c}$$

Note, however, that we cannot impose the arbitrage condition in Equation 3b. This can lead to very different conclusions concerning the length of the expiration interval, T, and the behavior of the futures price (e.g., variability of F)

Unlike for storable commodities, Equation 2c cannot be simplified to a function of the current spot price as it is in Equation 4 for storable commodities. Therefore, the futures price will, in fact, reflect the average expected price during the expiration interval.

As previously stated, a futures contract that expires over a T-day interval is equivalent to an equally weighted portfolio of T daily-contracts. In the case of storable commodities, arbitrage opportunities guarantee that there is really only one variable asset price in the portfolio—the spot cash market—and it is carried through time. In contrast, because storage arbitrage is not possible, the futures contract for non-storable commodities represents T distinctly priced assets. As you increase T, the portfolio diversifies, and volatility of the portfolio—the futures contract— declines. This is an important distinction in the design of cash-settled contracts.

Finally, the behavior of the futures price during the expiration interval will be similar to that expressed in Equation 5, without the arbitrage restrictions. So, for $1 \leq t \leq T$, the following equation applies:

$$F_{t,T} = \frac{1}{T} E_t\left[S_1 + S_2 + \ldots + S_t + \ldots + S_{T-1} + S_T\right],$$

$$= \frac{1}{T}\left[E_t(S_1) + E_t(S_2) + \ldots + E_t(S_t) + \ldots + E_t(S_{T-1}) + E_t(S_T)\right],$$

$$\frac{1}{T}\left[S_1 + S_2 + \ldots + S_t + E_t(S_{t+1}) + \ldots + E_t(S_{T-1}) + E_t(S_T)\right],$$

$$= \frac{1}{T}\left[\sum_{i=1}^{t} S_i + \sum_{j=t+1}^{T} E_t(S_j)\right]. \qquad\qquad \text{Equation 6}$$

As with storable commodities, during the expiration interval ($1 \leq t \leq T$) the futures contract will essentially be a weighted average of known prices (S_i for $i \leq t$) and those prices expected to prevail during the remainder of the expiration interval (S_j for $t+1 \leq j \leq T$). Again, as time elapses through the expiration interval, the contract will converge to the T-day average and volatility will decline.

Summary and Conclusions

This paper examines the trading characteristics of a futures contract that cash settles to an underlying spot or cash index. In particular, we define a *settlement time* on which the futures contract settles to a simple average of the underlying spot index over the proceeding T-day *expiration interval*. In this framework, a simple pricing equation is defined for both storable and non-storable commodities. The pricing equations are intuitive in the sense that the defined futures contract is simply a strip or equally weighted portfolio of T daily contracts, each expiring on their respective day during the expiration interval to the underlying spot price.

The pricing equations suggest the following results for storable commodities (assuming a constant cost of carry). First, on any day prior to entering the expiration month, the futures price will equal the spot price on that day plus the cost-of-carry to and through the expiration month. That is, for storable commodities, arbitrage opportunities guarantee that there is really only one variable price in the portfolio—the spot cash market—and it is carried through time. Therefore, the assets held in the strip are highly correlated with little diversification. It stands to reason, that futures price will exhibit day-to-day volatility roughly equivalent to the daily volatility of the underlying spot index.

Second, the narrowest basis between the futures and the underlying spot index will occur on the first day of the expiration interval. Prior to that date, the futures will reflect more carrying costs. After that day, the futures will respond sluggishly to changes in the underlying spot index as known prices are being incorporated into the expected average settlement price.

Third, during the expiration interval, the futures price will increasingly behave like an average of the underlying spot index; where, the number of days used in calculating the average increases as we move through the expiration month. Therefore, the daily sensitivity of the futures price to daily changes in the spot index will decline through the expiration month. Thus, price volatility will generally decline through the expiration interval.

In contrast to storable commodities, non-storable assets cannot be carried or arbitraged through time. Therefore, a futures contract that cash settles over a T-day expiration interval is really a strip of daily contracts, where each daily contract could be a rather uniquely priced asset depending on the perishability of the underlying commodity. For instance, the price of live cattle today is not necessarily tied to the expected price thirty days hence. Therefore, their prices may behave quite differently. This is especially true in markets that display strong seasonality (e.g., hogs). So, a thirty day expiration interval (T=30) may include non-correlated prices; whereas, a two day expiration interval would probably reflect much more highly correlated prices. Thus, the shorter expiration interval for non-storable markets would result in a more volatile futures price.

Cash settled futures contracts appear to be gaining in popularity (see recent New York Mercantile Exchange and Minneapolis Grain Exchange announcements). So, it is important that researchers devote additional effort to this area. This will help to assure that proper settlement choices are made to meet regulatory requirements, and it can also increase the probability of introducing successful futures contracts which can ultimately increase social welfare (Silber).

The presented research contributes to this area by providing pricing equations for cash settled futures contracts. These equations, while relatively simple and intuitive, provide a starting point for further research. This research can include additional analytical work to solve explicitly for relationships among such things as the length of the expiration interval and the volatility of cash settled futures. Or, the pricing equations can be used in simulations for proposed futures contracts. In either case, these issues are important to contract developers, and the ideas presented here may assist in the introduction and understanding the price behavior of new cash settled futures contracts.

References

Black, D.G. "Success and Failure of Futures Contracts: Theory and Empirical Evidence." *Monograph Series in Finance and Economics*, 1986-1, Salomon Brothers Center for the Study of Financial Institutions, Graduate School of Business Administration, New York University, 1986.

Bollman, K., S. Thompson, and P. Garcia. "An Analysis of the Performance of the Diammonium Phosphate Futures Contract." *NCR-134 Conference, Applied Commodity Price Analysis, Forecasting, and Market Risk Management*, Proceedings, 1996, pp.388-401.

Carlton, D.W. "Futures Markets: Their Purpose, Their History, Their Growth, Their Successes and Failures." *Journal of Futures Markets*. 4(1984):237-271.

Cita, J. and D. Lien. "Estimating Cash Settlement Price: The Bootstrap and Other Estimators." *Journal of Futures Markets*. 17(1997):617-632.

Commodity Futures Trading Commission. "Application for Cash Settled Futures Contracts." http://www.cftc.gov/files/dea/analysis/deab2.doc, September 14, 2001.

Garbade, K.D. and W.L. Silber. "Cash Settlement of Futures Contracts: An Economic Analysis." *Journal of Futures Markets*. 4(1983):451-472.

Gray, R.W. "Why Does Futures Trading Succeed or Fail: An Analysis of Selected Commodities." *Readings in Futures Markets: Views from the Trade*. Edited by A.E. Peck, Chicago Board of Trade, 1978.

Hull, J.C. *Introduction to Futures and Options Markets*. Third Edition. Saddle River, New Jersey. Prentice-Hall, 1998.

Jones, F.J. "The Economics of Futures and Options Contracts Based on Cash Settlement." *Journal of Futures Markets*. 2(1982):63-82.

Leuthold, R.M., J.C. Junkus, and J.E. Cordier. *The Theory and Practice of Futures Markets*. Lexington, Massachusetts. Lexington Books, 1989.

Martinez-Garmendia, J. and J.L. Anderson. "Hedging Performance of Shrimp Futures Contracts with Multiple Deliverable Grades." *Journal of Futures Markets*. 19(1999):957-990.

Minneapolis Grain Exchange. "Minneapolis Grain Exchange Owners Approve Cash-Settled Corn and Soybean Futures; Spring Wheat Serial Options and Five-Cent Strike Prices Begin Trading Today." http://www.mgex.com/news/news_releases/release16.htm. June 1, 2001.

New York Mercantile Exchange. "New York Mercantile Exchange Board Announces Plans to Introduce Brent Crude Oil Futures Contract." http://www.nymex.com/, April 11, 2001.

Peterson, P. "Observations on Cash Settlement." *NCR-134 Conference Applied Commodity Price Analysis, Forecasting, and Market Risk Management*, Proceedings, 1995, pp.1-2.

Sandor, R.L. "Innovation by an Exchange: A Case Study of the Development of the Plywood Futures Contract." *Readings in Futures Markets: Views from the Trade*. Edited by A.E. Peck, Chicago Board of Trade, 1978.

Silber, W.L. "Innovation, Competition, and New Contract Design in Futures Markets*." Journal of Futures Markets*. 1(1981):123-155.

Tashjian, E. "Optimal Futures Contract Design*." Quarterly Review of Economics and Finance*. 35(1995):153-162.

Thompson, S., P. Garcia, and L. Wildman. "The Demise of the High Fructose Corn Syrup Futures Contract: A Case Study." *Journal of Futures Markets*, 16(1996): 697-724.