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**Portfolio Diversification with Commodity Futures:
Properties of Levered Futures**

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Portfolio Diversification with Commodity Futures:
Properties of Levered Futures

This study extends previous work on the impact of commodity futures on portfolio performance by explicitly incorporating levered futures into the portfolio optimization problem. Using data on nine individual commodity futures and one aggregate index from 1994-2003, we find that collateralized and levered futures strategies perform similarly in an ex-post context. Significant differences between the approaches emerge however when constraints on investment behavior exist. Further, levered futures do not result in a prohibitive number of margin calls. The investment performances of the collateralized and the levered strategies vary little across different rebalancing intervals, and frequent portfolio rebalancing does not necessarily result in superior performance.

Keywords: Levered futures, optimal portfolio performance, constraint investments

Introduction

Investors and fund managers have long viewed commodities as prospective investment alternatives. The dramatic growth of the hedge fund industry in recent years has further fueled this interest. The low correlation of commodities with other asset classes and the fact that futures contracts are traded at organized exchanges with very low transactions costs make them potentially attractive portfolio components. In the short-run, the particular benefits of including commodity futures in an otherwise well-diversified portfolio vary (Fortenberry and Hauser, 1990; Becker and Finnerty, 1997), but in the long-run they tend to improve its financial performance (Bodie and Rosansky, 1980; Jensen et al., 2000).

The results of this research are based on the use of fully collateralized (i.e. unlevered) commodity futures contracts. Though this procedure avoids potential problems associated with daily marking positions to market, it ignores an essential characteristic of futures contracts – their potential to leverage investments - and is inconsistent with the practice of many fund managers that use them as diversification tools. Holding fully collateralized commodity futures may constitute an opportunity cost, because posting T-Bills in excess of the minimum performance bond might not be an efficient allocation of capital. For example, if the margin requirement on a futures contract is 10% of the underlying value, a more efficient allocation of the remaining 90% might be an investment into stocks and bonds rather than T-Bills only. Moreover, the futures' leverage allows investors to take positions that garner market exposure in excess of total portfolio value.

Past researchers have paid little attention to including levered commodity futures in the portfolio optimization problem. Only Becker and Finnerty (1997) attempt to incorporate futures' leverage into the analysis by constructing levered indices, which scale futures returns by a multiplier. They find that commodity futures serve as an inflation hedge, with the degree of protection increasing as the commodity futures are levered. Hence, Becker and Finnerty's (1997) results indicate that the use of levered futures investments may improve portfolio performance. However, their approach of incorporating leverage into the analysis does have two shortcomings. First, the index of levered futures returns disregards margin calls, and second, they do not optimize the portfolio shares but rather assume that a fixed proportion (0.10) of the portfolio is allocated to commodity futures.

This paper evaluates the effects of including commodity futures into a portfolio of stocks and bonds in a more comprehensive and structured framework. We thereby extend Becker and Finnerty's (1997) study in several important dimensions. First, we incorporate fully levered futures into the analysis, specifically accounting for the possibility of margin calls, and contrast our findings to the fully collateralized approach to futures investments. Second, rather than assuming a fixed allocation to commodities, we determine the optimal portfolio weights in a Markowitz mean-variance framework. Third, we assess how the levered and collateralized approaches differ when portfolio weights are constrained. We also investigate the effect of alternate intervals for portfolio rebalancing, and explore the sensitivity of our findings to differences in economic conditions. Finally, our analysis relies on a greater subset of nine different individual commodity futures contracts and one aggregate index from 1994-2003.

Literature

Numerous researchers have evaluated the effects of including commodity futures in a well-diversified portfolio of stocks, bonds, and other assets. Bodie and Rosansky (1980), for example, conduct a comprehensive analysis of the performance of 23 commodity futures during the 1950-1976 period and find that investors who switched from a stock only portfolio to one that contained 60% stocks and 40% commodities reduced their risk by 30% without giving up returns. These results, however, are based on an aggregate measure of commodity futures performance, and no examination of the optimal portfolio allocations of individual commodities is conducted. Comparing portfolios of stocks and agricultural futures over a nine year time horizon, Fortenbery and Hauser (1990) report a reduction in the non-systematic risk of portfolios for two of the four commodity futures analyzed.

In a more recent study, Jensen et al. (2000) use an aggregate measure, the Goldman Sachs Commodity Index (GSCI), to examine the value of commodity futures in a portfolio consisting of stocks, bonds, T-Bills, and real estate. Over the period 1973-1997, portfolios containing commodity futures displayed greater returns than those that did not, but the effect varied depending on monetary policy. During periods of restrictive monetary policy, commodity futures comprised a significant portion of the portfolios, while during periods of expansive monetary policy their proportion in portfolios was small. These and other studies confirm that commodity futures tend to improve the financial performance of investment portfolios. Yet, the effect of individual commodities on portfolio performance has received considerably less attention and warrants further investigation.

The above analyses are based on fully collateralized futures positions, approximated by adding the T-Bill rate to the return on the futures contract. This approach eliminates the need to mark-to-market futures on a daily basis, but disregards their leverage. Investors that enter the futures market are only required to post a small performance margin, not the entire contract value. The difference between this performance margin and the actual contract value can then be invested in alternative assets, permitting market exposure in excess of the total portfolio value. Becker and Finnerty (1997) account for the leverage dimension of futures to some degree. However, their largest multiplier of two corresponds to a margin of approximately 50% of the contract value, far greater than the typical performance margins of about 5-10%. Moreover, the multiplier approach cannot incorporate margin calls and hence prevents analysis of fully levered futures investments.

Investors and funds managers frequently impose upper bounds on the proportion of the portfolio that can be allocated to particular asset classes. Whether these bounds are set voluntarily or are mandated by the institution's policy, they may improve portfolio performance by lowering investment risk or increasing returns (Cohen and Pogue, 1967; Frost and Savarino, 1986; Jorion, 1986). Using data from seven different stock markets, Frost and Savarino (1986) and Jorion (1986) employ empirical Bayes approaches, which hamper extreme portfolio weights, and enhance portfolio performance. Frost and Savarino (1988) find that portfolios perform better when the weights of single stocks are restricted. This study extends previous research by Becker and Finnerty (1997) by investigating the value of commodity futures in portfolios of stocks and bonds, under particular consideration of futures unique leverage and margin properties as well as possible constraints placed on portfolio weights.

Methods

The benefits of adding commodity futures to an efficient portfolio of stocks and bonds are measured by modeling an investor's actual trading environment. Consequently, futures positions are marked-to-market daily, and margin calls are accounted for. Moreover, two alternative strategies are considered for the futures investment – a fully collateralized and a levered strategy. In the fully collateralized approach, the percentage of the portfolio allocated to futures simply earns the three-month T-Bill rate on the full contract value. Under the levered approach, the portion of the portfolio assigned to commodity futures contract represents only the margin, not the full contract value. For the long-only strategy examined in this study, this means that the investor fully levers his/her commodity exposure. The levered approach hence differs from the fully collateralized approach because it allows investors to take positions that garner market exposure in excess of the total portfolio value and incorporates collateral management into the analysis.

Through time the values of the stocks, bonds, and futures in the portfolio change because the returns to the individual assets differ. As a result, their allocations in the portfolio deviate from the original weights. The portfolio is therefore rebalanced monthly to adjust the proportions back to their initial shares. This involves selling a fraction of those assets that outperformed other assets in the portfolio during the previous month, and reallocating the proceeds to the assets that underperformed. Consistent with previous research by Fortenbery and Hauser (1990) and Jensen et al. (2000), we rebalance the portfolio on the first trading day of each month. In the intermittent time periods, daily margin calls from the levered futures positions are met by borrowing at the three-month T-Bill rate using the total portfolio as collateral, and excess funds beyond the minimum margin requirement earn interest at the same rate.

The optimal allocations of stocks, bonds, and commodity futures in the portfolio are obtained by modeling how an investor following these investment strategies would have performed over the data period. A grid search is conducted in one percent increments across all possible allocations of stocks, bonds, and futures, to identify the optimal portfolios for both fully collateralized and levered strategies. The maximum Sharpe ratio served as the optimization criterion in identifying the optimal weights of stocks, bonds, and commodities.

Data

The analysis is conducted using daily data over the period from January 2, 1994, to December 31, 2003. Following previous research by Fortenbery and Hauser (1990) and Jensen et al. (2000), the stock and bond components of the portfolio are modeled using the S&P 500 index and the Lehman U.S. Aggregate Bond Index. These indices represent a broad range of stocks as well as government and non-government bonds. The S&P 500 index data was obtained from DataStream, and the Lehman U.S. Aggregate Bond Index was provided by Lehman Brothers, Inc. To capture the returns of commodity investments, we used daily futures closing prices for the Goldman Sachs Commodity Index (GSCI) as well as individual commodities (crude oil, copper, gold, silver, corn, soybeans, wheat, lean hogs, and live cattle). The individual contracts were selected because they represent the five commodity subclasses that comprise the GSCI (energy, precious metals, industrial metals, agriculture, and livestock). The commodity futures data were supplied by the New York Mercantile Exchange (NYMEX), the Chicago Mercantile Exchange (CME), and the Chicago Board of Trade (CBOT). All futures are rolled over to the next contract one month prior to the expiration of the current futures. This means that the current futures position is closed out, and reinitiated in the subsequent contract. Further, the analysis assumes a futures margin of 10% of the contract value (Lee and Leuthold, 1983).[1]

[1] This approach was adopted because data on the actual margins was not available for all commodities. However, analyzing several commodities including corn, soybeans, and wheat, we find that results are essentially the same when using the actual margin and an assumed margin of 10% of the contract value.

Results and Discussion

Fully Collateralized versus Levered Portfolios

During 1994-2003, the portfolios containing fully collateralized commodity futures display the largest Sharpe ratios when holding between 76.5% and 90% in bonds (Table 1). Stocks are always part of the optimal portfolio, but constitute at most 10%, whereas individual commodities are not always included. GSCI and the crude oil futures occupy the largest portfolio shares with 15.0% and 12.0%, causing the Sharpe ratio to increase from 0.68 to 0.86 and to 1.00. This increase, driven by a proportionally greater change in returns relative to risk, reflects the heavy weight of the energy futures in the GSCI and the strong performance of the energy markets during the data period. Copper, soybeans, and cattle comprise only marginal shares of the portfolio (5.0%, 7.5%, and 3.5%) increasing the Sharpe Ratio by 0.03, 0.05, and less than 0.01. Silver makes up a share of 1.00%, but does not significantly impact portfolio performance, and gold, corn, wheat, and hogs never enter the portfolio.

The positive contributions of the GSCI and crude oil futures to portfolio performance are in part related to returns arising from rolling the long futures positions forward over time. These returns capture a liquidity premium through an increased convenience yield during periods of high volatility of the underlying commodity (i.e. rolling long contracts forward may capture a risk premium over time if futures prices are a downwardly biased forecast of future spot prices). This relationship between spot volatility and roll return is not observed for all commodity groups, but it is quite pronounced in the cases of energy and industrial metals. Earlier studies have found that the effect of spot price volatility on the mean roll return for agricultural, non-energy, and precious metals is insignificant (Erb and Harvey, 2005). For crude oil futures however, Litzenberger and Rabinowitz (1995) argue that backwardation of crude oil prices is a necessary condition for crude oil production, and that greater uncertainty regarding future crude oil prices will lead to stronger backwardation. This backwardation allows for positive roll returns as investments in expiring contracts are rolled over to cheaper outstanding contracts.

Contrasting levered with fully collateralized futures, the amount of capital that is invested into commodities decreases for all contracts evaluated (Table 2). Total commodity exposure, as measured by the contract value of the futures position relative to the total portfolio value, however, increases modestly for the GSCI, crude oil, copper, and soybeans. This increase results from the greater degree of diversification provided by these commodities in an optimal portfolio with larger shares of stocks and bonds. Portfolios containing levered futures exhibit both higher returns and risk than portfolios containing collateralized futures, but only marginally different Sharpe ratios (difference less than 0.005) indicating that investors seeking greater returns must also assume greater risk. This effect is illustrated in Figure 1, which displays the efficient frontiers of the stock and bond only portfolio and of the portfolios containing collateralized and levered GSCI futures. When the GSCI futures are levered, the efficient frontier rotates counter-clockwise and becomes steeper.

The small differences in the results between collateralized and levered approach of futures investments are attributable to the fact that the levered approach is conceptually similar to traditional applications of the popular CAPM framework. Yet, the frequency with which the portfolio is rebalanced periodically and the direct incorporation of margin calls do have a modest impact on the performance measures, resulting in similar but not identical Sharpe ratios. The analysis of individual commodity futures shows that not all components of the index enter into the optimal portfolio. Hogs and live cattle futures, for example, are either not included or constitute only a small proportion, which is in contrast to Fortenbery and Hauser (1990) who report optimal portfolio weights of 0.18 and 0.26 for these commodities. Hence, the greatest benefits of adding commodities to an optimal portfolio of stocks and bonds are offered by GSCI and crude oil futures whereas the other commodities do not significantly contribute to enhancing portfolio returns or reducing risk.

Unconstrained versus Constrained Portfolios

Commodity futures' leverage may, however, be important to fund managers who seek to enhance portfolio returns by limiting their investment in traditional low risk and low return assets (i.e. bonds). As collateralized futures require the full contract value to be placed in T-Bills as performance bonds, the share of interest-bearing instruments (bonds + performance T-Bills) in the portfolio may increase substantially, leading to risk-and return profiles that are inconsistent with fund managers' individual preferences. To investigate the performance of levered and collateralized futures investments with portfolio constraints, we limit the weight of interest-bearing instruments (U.S. Aggregate Lehman Bond Index + excess T-Bills over initial margin) to 0.80, 0.60, and 0.40.

If these constraints are imposed, the restrictions on the interest-bearing instruments become binding for all three weight levels (Tables 3 and 4). As a result, the optimal portfolio proportions of commodities increase for both, levered and collateralized, approaches to futures investment. Though the actual share of commodities is smaller for the levered approach, the portfolio's commodity exposure (futures contract value relative to the total portfolio value) is greater than in the collateralized case. Portfolios containing levered futures therefore display greater risk and returns than those comprised of fully collateralized contracts. Despite this increase in risk, portfolios with levered commodity futures exhibit superior risk and return characteristics, as reflected by the increase in the Sharpe ratio. For example, a portfolio with a bond constraint of 0.60 that includes crude oil futures, displays an optimal Sharpe ratio of 0.85 and a commodity exposure of 0.28 in the levered case, but only 0.80 and 0.20 in the collateralized case. Furthermore, the difference in performance between levered and collateralized futures widens with stricter constraints on the interest-bearing instruments, and asset allocations in the collateralized approach become increasingly suboptimal (Tables 3 and 4). These findings show that under binding bond weight constraints the levered approach to investing in commodity futures outperforms the fully collateralized method.

Frequency of Portfolio Rebalancing

The results presented to this point are based on the assumption that fund managers rebalance their portfolios once a month. Yet, leverage increases the relative magnitude of the returns from the futures position. Hence, the investment performance of the funds that can be withdrawn from the futures' margin account (in the levered case) during the time period between portfolio rebalancing can have a significant impact on overall portfolio performance. Using levered GSCI futures—the contract that is most representative of aggregate commodity performance, we find, as expected, that altering the frequency of portfolio rebalancing changes the optimal Sharpe ratio from 0.86 (monthly) to 0.87 (bi-monthly), 0.88 (semiannually),0.94 (annually), and 0.85 (bi-annually). The differences in the optimal Sharpe ratios between levered and collateralized approach vary, but tend to be small. Hence, rebalancing affects portfolio performance over time to a much greater extent than it influences the difference between levered and collateralized approaches.

Sensitivity Analysis

During the initial part of the data period, both bonds and stocks displayed strong returns, while bonds dominated stocks in performance during the later years. Examining the optimal portfolio allocations for collateralized and levered strategies in two subperiods, 1994-1998 and 1999-2003, does not change the nature of our results (not displayed).[2] Moreover, our findings vary little when transactions costs are included into the analysis, or when the strategy for incorporating margin calls during the intermittent time periods between portfolio rebalancing is altered from earning or borrowing at the T-Bill rate to meeting daily margin calls by selling stocks and bonds and reinvesting excess funds beyond the minimum margin into stocks and bonds. This latter result may in part be due to the fact that the number of margin calls associated with the levered futures investment is limited. There are for example, only 17 margin calls for the GSCI futures during the sample period. Finally, partitioning the data period according to different criteria (economic expansion/economic recession) had little impact on the overall character of our findings.

Conclusion

This study revisits the issue of commodity investment performance using daily data from 1994-2003. We assess the impact of incorporating levered futures into the portfolio optimization problem with binding weight constraints in the EV framework. In addition, we investigate the investment performance of individual commodity futures, and conduct sensitivity analyses.   The levered and fully collateralized approaches differ minimally at the ex-post optimal portfolio, as it is somewhat similar to a levered CAPM. However, in the presence of binding constraints that force risk into the portfolio the use of levered futures result in greater risk adjusted returns. Interestingly, Becker and Finnerty also find that levered futures contracts are of value in their analysis that fixes the proportion (0.10) of the portfolio allocated to commodity futures which imposes greater risk. Furthermore, our results indicate that most of the individual commodity futures contracts considered were not elements of the optimal portfolio. This result was most pronounced for

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[2] While little differences emerge between the collateralized and levered strategies the compositions of the optimal portfolios change. In the first period, only GSCI and crude oil enter, but in the second period most commodities are part of the portfolios.

agricultural and precious metal futures. Crude oil was the only individual commodity that contributed significantly to portfolio performance. We also find that the use of levered futures does not result in a prohibitive number of margin calls. Portfolio performance for both approaches does not vary predictably across different rebalancing intervals, and more frequent portfolio rebalancing does not necessarily result in superior performance. Finally, partitioning the data period according to different criteria (economic expansion/economic recession) had little impact on the overall character of our findings.

Overall, our findings suggest that including levered futures into the portfolio optimization problem under the EV framework is of greatest importance in the presence of constraints on behavior which may exist for a variety of reasons, for example differences in investors' risk preferences. Future research should focus on investigating the effect of levered futures on investment performance when portfolios are optimized ex ante in a dynamic framework, where decision rules or constraints may be particularly important. Finally, the evidence that at times different futures contracts enter into well-diversified portfolios, as manifested here and in previous research, emphasizes the importance understanding the interactive dynamics of the various portfolio components.

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Table 1. Optimal asset allocations, returns, and standard deviations of the portfolio with stocks, bonds, and fully collateralized commodity futures, 1994-2003 (portfolio shares, returns, and standard deviations in percent)

	Stocks[a]	Bonds[a]	Commodities[a]	Annual Return	Annual Stdev.	Sharpe Ratio	FCV/ TPV[b]
Stock, Bond	10.0	90.0	0.0	7.15	4.19	0.68	
Stock, Bond, GSCI	8.5	76.5	15.0	8.05	4.31	0.86	0.150
Stock, Bond, Crude	9.7	78.3	12.0	9.51	5.19	1.00	0.120
Stock, Bond, Copper	8.6	86.5	5.0	7.19	4.03	0.71	0.050
Stock, Bond, Silver	9.9	89.1	1.0	7.12	4.13	0.68	0.010
Stock, Bond, Gold	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Corn	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Soybeans	9.3	83.3	7.5	7.33	4.14	0.73	0.075
Stock, Bond, Wheat	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Cattle	9.7	86.9	3.5	7.08	4.05	0.68	0.035
Stock, Bond, Hogs	10.0	90.0	0.0	7.15	4.19	0.68	0.000

[a]Values may not add up to 100% due to rounding.
[b]Futures contract value / total portfolio value.

Table 2. Optimal asset allocations, returns, and standard deviations of the portfolio with stocks, bonds, and levered commodity futures, 1994-2003 (portfolio shares, returns, and standard deviations in percent)

	Stocks[a]	Bonds[a]	Commodities[a]	Annual Return	Annual Stdev.	Sharpe Ratio	FCV/ TPV[b]
Stock, Bond	10.0	90.0	0.0	7.15	4.19	0.68	
Stock, Bond, GSCI	9.8	88.5	1.7	8.58	4.95	0.86	0.170
Stock, Bond, Crude	10.9	87.8	1.3	10.02	5.72	1.00	0.130
Stock, Bond, Copper	8.9	90.5	0.6	7.35	4.26	0.71	0.060
Stock, Bond, Silver	10.0	89.9	0.1	7.15	4.17	0.68	0.010
Stock, Bond, Gold	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Corn	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Soybeans	9.9	89.3	0.8	7.55	4.44	0.73	0.080
Stock, Bond, Wheat	10.0	90.0	0.0	7.15	4.19	0.68	0.000
Stock, Bond, Cattle	10.0	89.7	0.4	7.17	4.18	0.68	0.035
Stock, Bond, Hogs	10.0	90.0	0.0	7.15	4.19	0.68	0.000

[a]Values may not add up to 100% due to rounding.
[b]Futures contract value / total portfolio value.

Table 3. Optimal asset allocations, returns, and standard deviations of the portfolio with stocks, bonds, and fully collateralized commodity futures and constrained bond weights, 1994-2003 (portfolio shares, returns, and standard deviations in percent)

	Stocks[a]	Bonds[a]	Commodities[a]	Annual Return	Annual Stdev.	Sharpe Ratio	FCV/TPV[b]
				Bonds = 0.80			
Stock, Bond	20.0	80.0		7.43	4.93	0.63	
Stock, Bond, GSCI	18.2	63.8	18.0	8.49	5.15	0.81	0.180
Stock, Bond, Crude	18.6	67.4	14.0	10.13	6.08	0.96	0.140
Stock, Bond, Copper	19.5	75.5	5.0	7.48	4.87	0.65	0.050
Stock, Bond, Silver	19.9	79.1	1.0	7.40	4.87	0.63	0.010
Stock, Bond, Gold	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Corn	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Soybeans	19.2	72.8	8.0	7.63	4.89	0.67	0.080
Stock, Bond, Wheat	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Cattle	19.8	78.2	2.0	7.38	4.85	0.63	0.020
Stock, Bond, Hogs	20.0	80.0	0.0	7.43	4.93	0.63	0.000
				Bonds = 0.60			
Stock, Bond	40.0	60.0		7.89	7.63	0.47	
Stock, Bond, GSCI	37.0	33.0	30.0	9.61	8.46	0.62	0.300
Stock, Bond, Crude	37.5	37.5	25.0	12.60	10.31	0.80	0.250
Stock, Bond, Copper	39.6	56.4	4.0	7.93	7.63	0.47	0.040
Stock, Bond, Silver	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Gold	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Corn	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Soybeans	38.7	48.3	13.0	8.20	7.76	0.50	0.130
Stock, Bond, Wheat	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Cattle	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Hogs	40.0	60.0	0.0	7.89	7.63	0.47	0.000
				Bonds = 0.40			
Stock, Bond	60.0	40.0		8.25	10.96	0.36	
Stock, Bond, GSCI	55.5	0.0	44.5	10.70	12.53	0.51	0.445
Stock, Bond, Crude	56.1	4.9	39.0	15.29	15.65	0.70	0.390
Stock, Bond, Copper	59.6	36.4	4.0	8.28	10.99	0.36	0.040
Stock, Bond, Silver	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Gold	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Corn	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Soybeans	58.2	23.8	18.0	8.66	11.23	0.39	0.180
Stock, Bond, Wheat	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Cattle	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Hogs	60.0	40.0	0.0	8.25	10.96	0.36	0.000

[a]Values may not add up to 100% due to rounding.
[b]Futures contract value / total portfolio value.

Table 4. Optimal asset allocations, returns, and standard deviations of the portfolio with stocks, bonds, and levered commodity futures and constrained bond weights, 1994-2003 (portfolio shares, returns, and standard deviations in percent)

	Stocks[a]	Bonds[a]	Commodities[a]	Annual Return	Annual Stdev.	Sharpe Ratio	FCV/ TPV[b]
				Bonds = 0.80			
Stock, Bond	20.0	80.0		7.43	4.93	0.63	
Stock, Bond, GSCI	17.9	80.0	2.1	9.13	5.78	0.83	0.210
Stock, Bond, Crude	18.5	80.0	1.5	10.66	6.53	0.97	0.150
Stock, Bond, Copper	19.4	80.0	0.6	7.63	5.03	0.66	0.060
Stock, Bond, Silver	19.8	80.0	0.2	7.41	4.88	0.63	0.020
Stock, Bond, Gold	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Corn	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Soybeans	19.0	80.0	1.0	7.90	5.21	0.69	0.100
Stock, Bond, Wheat	20.0	80.0	0.0	7.43	4.93	0.63	0.000
Stock, Bond, Cattle	19.6	80.0	0.4	7.43	4.89	0.63	0.040
Stock, Bond, Hogs	20.0	80.0	0.0	7.43	4.93	0.63	0.000
				Bonds = 0.60			
Stock, Bond	40.0	60.0		7.89	7.63	0.47	
Stock, Bond, GSCI	36.2	60.0	3.8	10.88	9.45	0.69	0.380
Stock, Bond, Crude	37.2	60.0	2.8	13.75	11.14	0.85	0.280
Stock, Bond, Copper	39.1	60.0	0.9	8.16	7.88	0.49	0.090
Stock, Bond, Silver	39.8	60.0	0.2	7.87	7.58	0.47	0.020
Stock, Bond, Gold	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Corn	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Soybeans	38.2	60.0	1.8	8.70	8.23	0.53	0.180
Stock, Bond, Wheat	40.0	60.0	0.0	7.89	7.63	0.47	0.000
Stock, Bond, Cattle	39.4	60.0	0.6	7.90	7.59	0.47	0.060
Stock, Bond, Hogs	40.0	60.0	0.0	7.89	7.63	0.47	0.000
				Bonds = 0.40			
Stock, Bond	60.0	40.0		8.25	10.96	0.36	
Stock, Bond, GSCI	54.1	40.0	5.9	12.72	14.05	0.60	0.590
Stock, Bond, Crude	55.7	40.0	4.3	16.96	16.66	0.76	0.430
Stock, Bond, Copper	58.6	40.0	1.4	8.63	11.44	0.38	0.140
Stock, Bond, Silver	59.9	40.0	0.1	8.24	10.93	0.36	0.010
Stock, Bond, Gold	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Corn	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Soybeans	57.0	40.0	3.0	9.52	12.15	0.43	0.300
Stock, Bond, Wheat	60.0	40.0	0.0	8.25	10.96	0.36	0.000
Stock, Bond, Cattle	59.1	40.0	0.9	8.26	10.90	0.36	0.090
Stock, Bond, Hogs	60.0	40.0	0.0	8.25	10.96	0.36	0.000

[a]Values may not add up to 100% due to rounding.
[b]Futures contract value / total portfolio value.

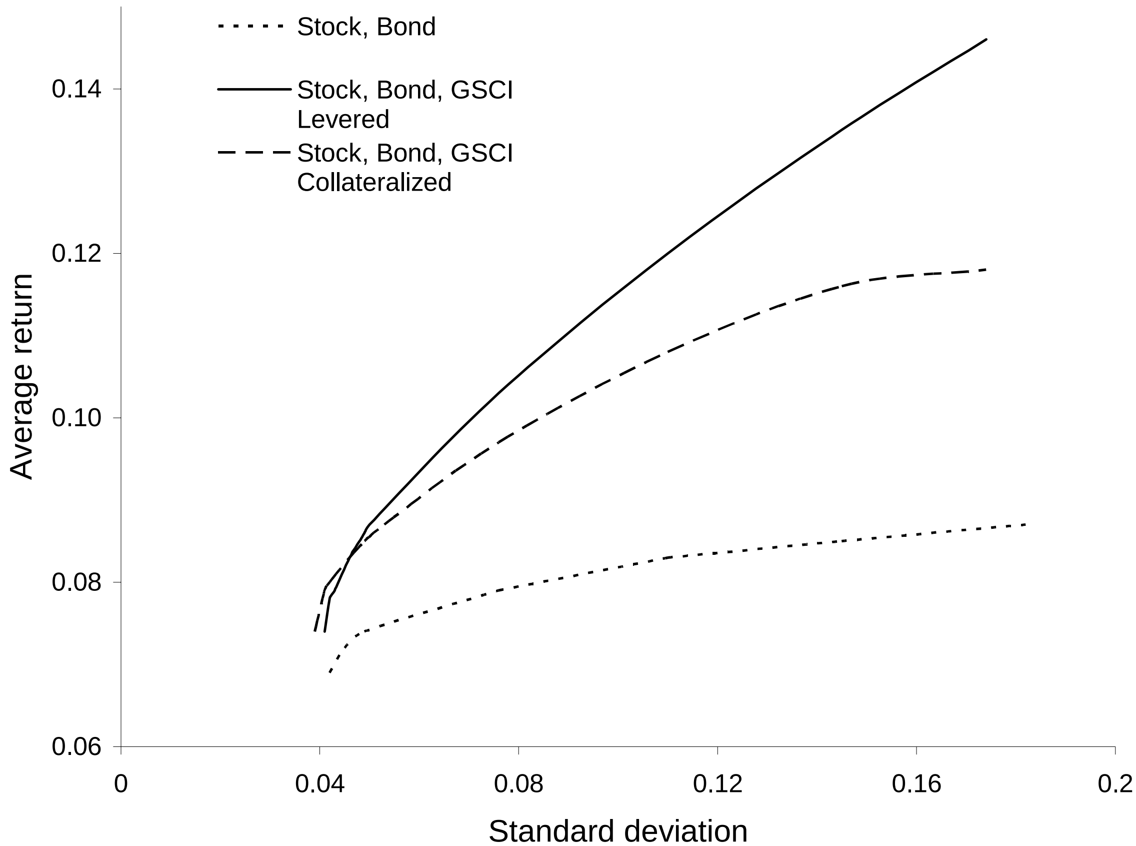

Figure 1. Efficient frontiers of portfolios of stocks and bonds as well as stocks, bonds, and GSCI futures (collateralized and levered), 1994-2003